2017 Academic Challenge

MATHEMATICS TEST – REGIONAL

– This Test Consists of 30 Questions –

Mathematics Test Production Team

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as ● , not ◉ , ◐ , ◑ , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.
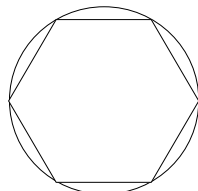
*** **Time: 40 Minutes** ***

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

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1. What is the sum of the two roots of $\sqrt{3}x^2 - \sqrt{2}x - 1 = 0$? Please put your answer in simplified form with a rational denominator.

    a) $\dfrac{\sqrt{6}}{6}$

    b) $\dfrac{\sqrt{6}}{3}$

    c) $\dfrac{\sqrt{6} + \sqrt{2\sqrt{3} + 12}}{6}$

    d) $\dfrac{\sqrt{6} + \sqrt{2\sqrt{3} + 12}}{3}$

    e) Can't be found as two of the coefficients are not rational

2. You will only defeat your archnemesis if you can answer more questions than him or her. One of these questions is "In how many distinguishable ways can you rearrange the letters in the word 'archnemesis'?"

    a) 990   b) 362,880   c) 725,760
    d) 9,979,200   e) 39,916,800

3. A regular hexagon is inscribed inside a circle as shown. If the area of the hexagon is $1\,m^2$, determine the area of the circle. Round to the nearest hundredth of a square meter.

    a) $1.07\,m^2$ b) $1.21\,m^2$ c) $1.23\,m^2$ d) $1.27\,m^2$ e) $1.31\,m^2$

4. If we had as many i's as you see in this sentence, including capital ones, what would be their product?

    a) 1   b) i   c) –i   d) –1   e) None of these

5. The nascent sport of doggie wheelchair racing includes dogs with two legs, dogs with three legs, and dogs with all four legs. In total, the 36 competitors in the Canine Wheelchair Coalition have 102 legs. If there are 10 times as many two-legged dogs as so-called "tripods", how many four-legged dogs are there?

    a) 2   b) 10   c) 14   d) 20   e) None of these

6. Angles A and B are supplementary, angles B and C are complementary, and angles C and D are supplementary. What must be true about angles A and D?

   a) A and D must be supplementary
   b) A and D must be complementary
   c) A and D must have the same angle measure
   d) A and D must have measures that add up to 270°
   e) Not enough information given to determine any of these as true

7. What is the solution set for $\log_2 x = \log_4 x^2$?

   a) All reals
   b) $(-\infty, 0) \cup (0, \infty)$
   c) $(0, \infty)$
   d) 1
   e) 4

8. A culture of bacteria starts with 10 bacteria, and grows by 4.1% per hour. How many hours, rounded **up** to the nearest hour, will it take to get to 100,000 bacteria?

   a) 17   b) 23   c) 40   d) 225   e) 281

9. A cylindrical paint can's diameter is equal to its height, giving it a square cross section. If the can has a total surface area of 1 square foot (top, bottom, and sides), calculate its volume. Round to the nearest hundredth of a cubic foot.

   a) 0.04 ft^3 b) 0.05 ft^3 c) 0.06 ft^3 d) 0.07 ft^3 e) 0.08 ft^3

10. Which of the following is **exactly** equivalent to $\ln x^8$?

    a) $8 \cdot \ln x$ b) $|8 \cdot \ln x|$ c) $8 \cdot \ln|x|$ d) $\ln 8x$ e) $\ln|8x|$

11. We're each given eight keys. What is the probability that we arrange them in the same order on our key chains or can roll them around or flip our key rings in order to match?

    a) $\dfrac{1}{40,320}$ b) $\dfrac{1}{20,160}$ c) $\dfrac{1}{5,040}$ d) $\dfrac{1}{2,520}$ e) $\dfrac{1}{720}$

12. If a quadrilateral has three angles of the same measure, which of the following three statements must be true about the quadrilateral?

    I. At least one pair of opposite sides must be parallel
    II. At least one pair of adjacent sides must have the same length
    III. At least one pair of opposite sides must have the same length

    a) I must be true   b) II must be true   c) III must be true
    d) Both I & III must be true   e) None of these must be true

13. If $\cos x = \dfrac{4}{5}$, what is $\tan^2 x + 1$?

   a) $\dfrac{25}{16}$ b) $\dfrac{5}{4}$ c) $\dfrac{9}{16}$ d) $\dfrac{3}{4}$ e) Insufficient information

14. Which of the following has the smallest period?

   I. $\tan\left(\dfrac{\pi}{4}x\right)$ II. $\csc\left(\dfrac{\pi}{2}x\right)$ III. $\sec\left(\dfrac{\pi}{16}x\right)$

   a) I only   b) II only   c) III only   d) I & II only   e) All three

15. How many faces does a regular hexagonal prism have?

   a) 4   b) 6   c) 8   d) 12   e) Not enough information

16. Point A is 45 feet from point B and 88 feet from point C. B and C are 83 feet apart. What is the measure of $\angle B$? All answers are in degrees and rounded to the nearest tenth of a degree.

   a) 30.3   b) 30.8   c) 68.7   d) 70.6   e) 81.0

17. What is the length of the latus rectum of $y = x^2 - 8x + 11$?

   a) 0.25   b) 1   c) 4   d) 16   e) 64

18. Given the equation $\sqrt{2x+1} = x - 7$, solve for x.

   a) $x = 7$ b) $x = 12$ c) $x = 1 + \frac{\sqrt{204}}{2}$
   d) $x = -0.5, 7$ e) $x = 4, 12$

19. Jake makes an air journey to his homeland 3000 miles away and then spends an hour to refuel and get cookies from his mom. On the way there, he takes 24 hours and is working against a wind of 12.5 mph. If he works with a wind of the same speed on the way back from whence he came, how many hours will the entire journey take? If need be, round to the nearest hour.

   a) 20   b) 22   c) 44   d) 45   e) 47

20. $r = 4\cos\theta + 4\sin\theta$ takes on which of the following shapes?

   a) cardioid   b) lemniscate c) limacon   d) rose   e) none of these

21. A swimming pool has two pumps. Pump A can fill the entire pool in 40 minutes, and pump B can fill the entire pool in 50 minutes. Starting with an empty pool, suppose we turn pump B on at noon and then pump A on at 12:05. When is the pool completely full? Round your answer to the nearest whole minute.

    a) 12:19   b) 12:20   c) 12:24   d) 12:25   e) 12:27

22. What is the sum of $2 + 1 + \dfrac{2}{3} + \dfrac{1}{2} + \dfrac{2}{5} + \dfrac{1}{3} + \dfrac{2}{7} + \cdots$? Round to the nearest whole number if necessary.

    a) ∞ b) 4   c) 6   d) 8   e) 12

23. Orange is the New Banana, a nutrition-based show that nobody watches, notes that if Jackson eats 15 bananas and 10 oranges, she'll reach 1,800 calories. If she wants 2,160 calories, she could eat 20 bananas and 8 oranges. What is her caloric intake if she eats just two of each?

    a) 90   b) 135   c) 180   d) 270   e) Undeterminable

24. City A and City B are 200 miles apart. At noon, a truck leaves City A and heads toward City B at 60 mph. At 12:30, a car leaves City A and heads toward City B at 75 mph. Does the car pass the truck before it reaches City B? If it does, determine what time the car passes the truck. Round to the nearest whole minute.

    a) No, the car does not pass the truck in time
    b) Yes, the car passes the truck at 1:15
    c) Yes, the car passes the truck at 1:45
    d) Yes, the car passes the truck at 2:00
    e) Yes, the car passes the truck at 2:30

25. On a certain planet, a skydiver falls at a rate of $r(t) = 117\left(4 - 4e^{-0.07t}\right)$ miles per hour after t seconds. What is his terminal velocity (i.e., long-term velocity), in miles per hour?

    a) 0   b) 117   c) 236   d) 468   e) 1872

26. Per the Rational Zeroes Theorem, which of the following is **always** a **possible** rational zero of a polynomial **with integral coefficients**?

    a) 0   b) –1   c) 1/3   d) 1/2   e) 2

27. Determine the domain of $f(x) = \dfrac{\sqrt{2-x}}{1 - \sqrt{x+3}}$

    a) $[-3, -2) \cup (-2, 2]$ b) $(-3, -2) \cup (-2, 2)$ c) $(-\infty, -2) \cup (-2, \infty)$
    d) $[-3, 2]$ e) All real numbers

28.   If x = 4 cos t and y = 9 sin t, which of the following shapes do we have?

   a) ellipse   b) parabola   c) line   d) circle   e) cardioid

29.   What is the set of all k-value(s) that make(s) the matrix $\begin{bmatrix} 3 & 0 & 4 \\ 6 & k & 2 \\ 7 & -k & 1 \end{bmatrix}$ singular?

   a) 0   b) ±1   c) ±3   d) All reals   e) None

30.   Four sets of twins and their grandfather recently went to a carnival. The children's names are Amy (f), Bill (m), Carl (m), Deb (f), Emily (f), Fred (m), George (m), and Hannah (f). Each of the three rides required that three people ride together. Each person rode one ride at noon, one at 1:00, and one at 2:00. Everyone rode each ride once, and only with people from their group. The following facts were also true.

   I.   Each set of twins rode together exactly once, except for Amy, who rode with her twin brother all three times
   II. Fred rode the Twister at 1:00 with the pair of twin girls
   III. Hannah rode the Wipeout at 2:00 with the pair of twin boys
   IV. Emily rode the Xenophobia at 2:00 with Carl and his twin sister
   V.   The one time Deb rode with George, their grandfather had to sit between them to keep the cousins from fighting

   Determine whether or not Bill was on a ride with anyone more than once.

   a) No, he was grouped with six different people
   b) Yes, he was on a ride with Amy all three times
   c) Yes, he was on a ride with Deb twice
   d) Yes, he was on a ride with Emily twice
   e) Not enough information given