2017 Academic Challenge

MATHEMATICS TEST – SECTIONAL

– This Test Consists of 30 Questions –

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as ● , not ⊙ , ◍ , ◗ , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

*** **Time: 40 Minutes** ***

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

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1.   When the complex number z = 3 – 5i is written in polar form, which of the following is the
      least positive angular argument (rounded to the nearest tenth of a degree)?

      a) 31.0°   b) 59.0°   c) 121.0°   d) 159.0°   e) None of these

2.   Which of the following has the greatest z-score in a distribution taken from a population
      with mean 130 and standard deviation 50?

      a) A single data point of 30
      b) A single data point of 180
      c) A sample of size 100 whose mean is 135
      d) A sample of size 1,000 whose mean is 132
   e) Two of these are tied for the greatest z-score

3.   A pair of squares are inscribed inside a circle as shown below. If the combined area of
      the two squares is 2 m^2, determine the area of the circle. Round to the nearest tenth of
      a square meter.
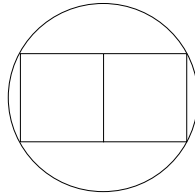
      a) 3.1 m^2 b) 3.5 m^2 c) 3.9 m^2 d) 4.1 m^2 e) 4.9 m^2

4.   If $f(x) = 5\sqrt{x}$, which of the following is the value for $\lim\limits_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$ when x = 1?

      a) 2.5   b) 5   c) 0   d) 10   e) Undefined

5.   What is the area completely enclosed between $f(x) = \dfrac{x}{3}$ and $g(x) = \sqrt{x}$?

      a) 0   b) 0.5   c) 2   d) 4.5   e) Undefined

6.   A town with a population of 100,000 people is growing exponentially at a rate of
      2,000 people per year at this instant. How fast will the population be growing in ten
      years? Round to the nearest whole person per year.

      a) 2,000 people per year   b) 2,214 people per year   c) 2,400 people per year
      d) 2,424 people per year   e) 2,443 people per year

7.   What is an angle of rotation for the axes that will eliminate the xy-term in the conic
     section whose general form equation is $3x^2 - 5xy + 4y^2 + 31 = 0$? (Please give your
     answer in degree form, rounded to the nearest tenth.)

     a) 5.7°   b) 39.3°   c) 50.7°   d) 78.7°   e) None of these

8.   Which of the following is not a property of matrices?

     a) Associative property of multiplication
     b) Distributive property
     c) Commutative property of addition
     d) Associative property of addition
     e) Commutative property of multiplication

9.   We start with 20 gallons of a drink mix that is 10% syrup. How much 75% syrup
     concentrate must be added in order to end up with a final result that is 20% syrup?
     Round to the nearest tenth of a gallon.

     a) 2.7 gal   b) 3.1 gal   c) 3.3 gal   d) 3.6 gal   e) 4.0 gal

10. What is $(1 - 2i)^6$?

     a) 117 – 44i   b) –35 + 120i   c) –117 + 44i
     d) 75 – 150i   e) None of these

11. What is the sum of all of the real solutions to the equation $\log(x + 33) + \log(x - 30) = 3$?

     a) –30   b) –3   c) 1.5
     d) $\dfrac{-3 + \sqrt{7969}}{2}$ e) Undefined

12. A regular square pyramid encloses a volume of 1000 square meters. What is the
     combined surface area of its four triangular faces? Round to the nearest whole square
     meter.

     a) 173 m²   b) 333 m²   c) 360 m²   d) 397 m²   e) 454 m²

13. Which of the following is a non-vertical asymptote of the graph of the rational function
     $r(x) = \dfrac{x^2 - 3x - 10}{x + 2}$?

     a) x = –2   b) y = x – 5   c) y = 0   d) y = 1   e) None of these

14. Jack takes 5 hours longer to paint a fence than Justin. The two of them can work
     together and paint that fence in 4 hours. How long does it take for Jack to paint it alone?
     Round to the nearest tenth of an hour.

     a) 1.8   b) 6.2   c) 11.2   d) 22.4   e) None of these

15. Lines A and B are perpendicular, lines B and C are parallel, and lines C and D are perpendicular. Which of the following must be true about A and D?

    a) A and D must be parallel
    b) A and D must be perpendicular
    c) A and D must be coincident (the same line)
    d) A and D must intersect at a forty-five degree angle
    e) A and D could be either parallel or coincident.

16. Parametric equations of the form $x = A\sin(\theta t)$, $y = B\cos(\varphi t)$ generate a graph known as a Lissajou figure. Which of the following is **guaranteed** to be true about the area of a bounding rectangle, which is a rectangle that completely contains this Lissajou figure?

    a) AB   b) $\pi^2 A^2$ c) 4A   d) $4A^2$ e) 4AB

17. What is the solution (in interval notation) to the inequality $\log(x - 2) - \log(9 - x) < 1$?

    a) (2, 9)   b) (2.5, 9)   c) (2, 2.5)   d) (92/11, 9)   e) (2, 92/11)

18. In the following diagram, the radius of circle P is 40 inches, and the radius of circle Q is 20 inches. The line AC is tangent to circle P at point A, and tangent to circle Q at point C. The line DE is tangent to circle P at point E, and tangent to circle Q at point D. The point B is the point of intersection for lines AC and DE. If the length of the segment AB is 80 inches, determine the length of the segment BC. Round your answer to the nearest whole inch.
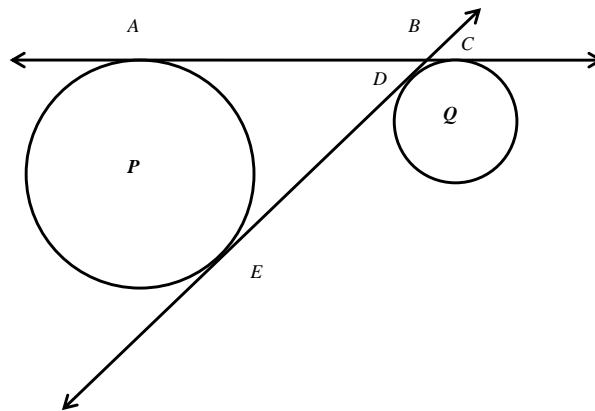
    a) 8 in   b) 10 in   c) 12 in   d) 13 in   e) 15 in

19. Which of the following is the sum of the solutions to $10^{2x} - 8 \cdot 10^x + 15 = 0$?

    a) 0.5   b) 0.7   c) 1.2   d) 1.8   e) No solutions

20. What is the linear speed (in miles per hour, rounded to the nearest hundredth) of a unicyclist who pedals a 13-inch diameter wheel at 40 revolutions per minute?

    a) 0.01   b) 0.05   c) 0.77   d) 1.55   e) 3.09

21. How many edges does a regular icosahedron have?

    a) 12   b) 15   c) 20   d) 30   e) 60

22. Say that we play a game in which we play until we win and then stop, and we accept a $32 payout if we win the first try, and half as much for each subsequent try. If the probability of winning on any single try is 0.25, what is the expected payout?

    a) $8   b) $12.80   c) $21.33   d) $32   e) $64

23. The 5th term of a geometric sequence is 8. The common ratio is 0.5. What's the 1st term?

    a) 0.5   b) 6   c) 10   d) 128   e) 256

24. Give all solutions to $\dfrac{x^2 - A^2}{x - B} > 0$ where $0 < B < A$

    a) (B, A) b) $(-B, -A) \cup (A, B)$ c) $(-\infty, -A) \cup (A, \infty)$

    d) $(-\infty, -A) \cup (B, A)$ e) $(-A, B) \cup (A, \infty)$

25. What is the remainder when $x^7 - 3x^5 + 6x^2 - 1$ is divided by x + 2?

    a) –9   b) –1   c) 0   d) 1   e) 55

26. The three different dimensions of a cube are given by (5x + 1), (3x + 3), and (2x + 5). What is the volume of this cube?

    a) $\dfrac{12167}{27}$ b) 15   c) 216   d) 729   e) Impossible

27. A hiker travels up a four-mile hill at 2 mph, and back down the same hill at 5 mph. Determine the average speed of this hike. Round to the nearest tenth of a mile per hour.

    a) 2.9 mph   b) 3.0 mph   c) 3.2 mph   d) 3.5 mph   e) Not enough info given

28. Which of the following has the smallest phase shift? Do not worry about the direction of the shift (left or right) when evaluating your answer.

    I. $\tan\left(2x - \dfrac{\pi}{6}\right)$ II. $\sin(50x - 4\pi)$ III. $\csc(75x + 6\pi)$

    a) I   b) II   c) III
    d) Two are tied   e) All three are tied

29. A spring moves in accordance to the damped sinusoidal function $y = 5e^{-0.011t}\sin(6\pi t)$ inches after t seconds. When is its amplitude equal to 2 inches? Round your answers to the nearest hundredth of a second.

    a) 0.02 sec   b) 0.14 sec   c) 0.92 sec   d) 1.02 sec   e) 83.30 sec

30. At a recent farmers' market, Aaron (m), Bethany (f), Craig (m), Diana (f), and Eric (m) all had vegetable stands. Each one sold a different type of vegetable, and the five types of vegetables were corn, beans, squash, lettuce, and tomatoes. They had a total of fifteen unique customers, each seller getting at least one customer and no shared customers. The following statements are also true:

    i. Bethany had more customers than the woman selling squash, but fewer customers than the man selling beans
    ii. Eric had the same number of customers as the woman selling corn
    iii. The man selling lettuce had twice as many customers as Diana
    iv. Craig had three times as many customers as Aaron

    Who was selling the tomatoes, and how many customers did this seller have?

    a) Aaron – one customer
    b) Aaron – two customers
    c) Craig – three customers
    d) Eric – two customers
    e) Eric – three customers