WYSE – Academic Challenge
Physics Test Solutions (Sectional) – 2017

1. **Correct Response: E**

Parallel axis theorem:

$$I_T = I_{cm} + MD^2 \quad \text{and} \quad D = R$$
$$\Rightarrow \quad I_T = \tfrac{2}{5}MR^2 + MR^2 = \tfrac{7}{5}MR^2$$

2. **Correct Response: B**

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} \quad \text{and} \quad y_{cm} = \frac{\sum m_i y_i}{\sum m_i}$$

Divide the sheet up into four pieces, one in each quadrant. The center of mass of each square is at the center of that square. Thus, the locations of the centers of mass are at (0.6 m, 0.6 m), (–0.6 m, 0.6 m), (–0.6 m, –0.6 m), and (0.6 m, –0.6 m). Each of these pieces is the same and has mass *M*/4. It is only the latter three that are remaining piece of wood.

$$x_{cm} = \frac{\tfrac{1}{4}M(-0.6\,\text{m}) + \tfrac{1}{4}M(-0.6\,\text{m}) + \tfrac{1}{4}M(0.6\,\text{m})}{\tfrac{1}{4}M + \tfrac{1}{4}M + \tfrac{1}{4}M} = -0.2\,\text{m}$$
$$y_{cm} = \frac{\tfrac{1}{4}M(0.6\,\text{m}) + \tfrac{1}{4}M(-0.6\,\text{m}) + \tfrac{1}{4}M(-0.6\,\text{m})}{\tfrac{1}{4}M + \tfrac{1}{4}M + \tfrac{1}{4}M} = -0.2\,\text{m}$$

3. **Correct Response: C**

Electrons and muons are two types of leptons.

4. **Correct Response: D**

Conservation of energy:

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2 = \tfrac{7}{10}mv^2$$

Constant acceleration implies the final velocity is twice the average velocity and the average velocity is total distance divided by the total time.

$$mgd\sin\theta = \tfrac{7}{10}m\left(\frac{2d}{t}\right)^2 \quad \Rightarrow \quad g = \frac{28d}{10\sin\theta\, t^2} = \frac{28(0.50\,\text{m})}{10\sin 10^\circ (0.900\,\text{s})^2} = 9.95\,\text{m/s}^2$$

5. **Correct Response: A**

The work-energy theorem:

$$W = \Delta K \quad \Rightarrow \quad \mathbf{F} \cdot \mathbf{s} - W_{friction} = \tfrac{1}{2} m v^2$$
$$\Rightarrow \quad v = \sqrt{\frac{2\left(\mathbf{F} \cdot \mathbf{s} - W_{friction}\right)}{m}} = \sqrt{\frac{2\left[(400\,\text{N})(50.0\,\text{m}) - 18000\,\text{J}\right]}{1500\,\text{kg}}} = 1.63\,\text{m/s}$$

6. **Correct Response: B**

$$M_C v_C + M_d v_d = \left(M_C + M_d\right) v_F$$
$$\Rightarrow \quad v_F = \frac{M_C v_C + M_d v_d}{M_C + M_d} = \frac{(1500\,\text{kg})(8.00\,\text{m/s}) + (100\,\text{kg})(12.0\,\text{m/s})}{(1500\,\text{kg}) + (100\,\text{kg})} = 8.25\,\text{m/s}$$

7. **Correct Response: D**

Use a coordinate system with the positive horizontal direction to the right and the positive vertical direction upward. Newton's Second Law for those directions:

Horizontal: $F_{push} - N = 0$ and Vertical: $F_{friction} - mg = \mu N - mg = 0$

$$\Rightarrow \quad F_{push} = \frac{mg}{\mu} = \frac{(1.17\,\text{kg})(9.80\,\text{m/s})}{(0.521)} = 22.0\,\text{N}$$

8. **Correct Response: B**

$$F_P - mg \sin\theta = 0 \quad \Rightarrow \quad \theta = \sin^{-1}\frac{F_P}{mg} = \sin^{-1}\frac{40.0\,\text{N}}{(8.00\,\text{kg})(9.80\,\text{kg})} = 30.7^\circ$$

9. **Correct Response: E**

The definition of a totally inelastic collision is that there is no bounce.

10. **Correct Response: C**

The flag points in the direction of the air flow relative to the car.

$$v_{air\,to\,car} = v_{air\,to\,ground} + v_{ground\,to\,car} = v_{air\,to\,ground} - v_{car\,to\,ground}$$

Using a coordinate system with the +x direction toward the east and +y direction toward the north,

$$v_{air\,to\,car} = -(17.4\,\text{m/s})\cos 45^\circ\,\hat{\mathbf{i}} + (17.4\,\text{m/s})\sin 45^\circ\,\hat{\mathbf{j}} - (25.0\,\text{m/s})\hat{\mathbf{j}}$$
$$= (-12.3\,\text{m/s})\hat{\mathbf{i}} + (-12.7\,\text{m/s})\hat{\mathbf{j}}$$

The direction of this velocity is given by

$$\theta = \tan^{-1}\frac{-12.7\,\text{m/s}}{-12.3\,\text{m/s}} = 45.9^\circ\ (\text{South of West})$$

11. **Correct Response: D**

$$v_y^2 - v_{0y}^2 = 2a_y\left(y - y_0\right)$$

$$\Rightarrow \quad y = y_0 + \frac{v_y^2 - v_{0y}^2}{2a_y} = 30.0\,\text{m} + \frac{(0.00)^2 - ([57.6\,\text{m/s}]\sin 70.0°)^2}{2(-9.80\,\text{m/s}^2)} = 179\,\text{m}$$

12. **Correct Response: D**

$$y = y_0 + v_{0y}t + \tfrac{1}{2}at^2 \quad \Rightarrow \quad t = \frac{-v_{0y} \pm \sqrt{v_{0y}^2 - 4\left(\tfrac{1}{2}a\right)(y_0 - y)}}{a}$$

$$\Rightarrow \quad t = \frac{([57.6\,\text{m/s}]\sin 70.0°) \pm \sqrt{([57.6\,\text{m/s}]\sin 70.0°)^2 - 2(9.80\,\text{m/s}^2)(0.0 - 30.0\,\text{m})}}{9.80\,\text{m/s}^2}$$

$$\Rightarrow \quad t = 11.58\,\text{s}$$

$$x - x_0 = v_x t = ([57.6\,\text{m/s}]\cos 70.0°)(11.58\,\text{s}) = 228\,\text{m}$$

13. **Correct Response: A**

$$\tfrac{1}{2}kx_i^2 + K_i = \tfrac{1}{2}kx_f^2 + K_f \quad \Rightarrow \quad K_f = \tfrac{1}{2}kx_i^2 + K_i - \tfrac{1}{2}kx_f^2$$

$$\Rightarrow \quad K_f = \tfrac{1}{2}(4.35\,\text{N/m})(0.10\,\text{m})^2 + 0 - \tfrac{1}{2}(4.35\,\text{N/m})(0.05\,\text{m})^2 = 0.0163\,\text{J}$$

14. **Correct Response: B**

$$\Delta V = Blv\sin\theta = \left(30\times10^{-6}\,\text{T}\right)(12\,\text{m})(60\,\text{m/s})\sin 90° = 0.0216\,\text{V} = 21.6\,\text{mV}$$

15. **Correct Response: C**

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(4.99\,\Omega)^2 + (3.33\,\Omega)^2} = 6.00\,\Omega$$

16. **Correct Response: D**

$$\rho = \frac{M}{V} = \frac{1.30\,\text{kg}}{(0.080\,\text{m})^3} = 2.54\times10^3\ \text{kg/m}^3$$

17. **Correct Response: A**

$$v = \frac{\omega}{k} = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{Mg}{\mu}} \quad \Rightarrow \quad M = \frac{\mu\omega^2}{k^2 g} = \frac{(0.0150\,\text{kg/m})\left(189\ \text{s}^{-1}\right)^2}{\left(18.9\,\text{m}^{-1}\right)^2\left(9.80\,\text{m/s}^2\right)} = 0.153\,\text{kg}$$

18. **Correct Response: E**

$$[E] = \frac{M \cdot L^2}{T^2} = [f]^n [d]^m = \left(\frac{M \cdot L}{T^2 \cdot L^3}\right)^n L^m = \frac{M^n L^{-2n+m}}{T^{2n}} \quad \Rightarrow \quad n = 1,\ m = 4$$

19. **Correct Response: A**

$$P_{x'} = \mathbf{P} \cdot \hat{\mathbf{x}}' = P\cos\theta = 5.00\cos 75° = 1.29$$

20. **Correct Response: C**

$$x - x_0 = \frac{v^2 - v_0^2}{2a} \quad \Rightarrow \quad a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(80.0\text{ m/s})^2 - 0^2}{2(1.20\times 10^3\text{ m})} = 2.67\text{ m/s}^2$$

21. **Correct Response: A**

$$F = ma = m\frac{\Delta v}{\Delta t} = (3.00\times 10^6\text{ kg})\frac{(80.0\text{ m/s} - 0.0\text{ m/s})}{(12.0\text{ s})} = 2.00\times 10^7\text{ N}$$

22. **Correct Response: D**

$$a = \frac{v^2}{r} \quad \Rightarrow \quad r = \frac{v^2}{a} = \frac{(3.00\text{ m/s})^2}{4.00\text{ m/s}^2} = 2.25\text{ m}$$

23. **Correct Response: D**

$$\Delta K + \Delta U = 0 \quad \Rightarrow \quad \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 + mgh_f - mgh_i = 0 \quad \Rightarrow \quad h_f - h_i = -\frac{\tfrac{1}{2}v_f^2 - \tfrac{1}{2}v_i^2}{g}$$

$$\Rightarrow \quad h_f - h_i = -\frac{\tfrac{1}{2}(0)^2 - \tfrac{1}{2}(3.00\text{ m/s})^2}{9.80\text{ m/s}^2} = 0.459\text{ m}$$

24. **Correct Response: D**

$$\Delta K + \Delta U = 0 \quad \Rightarrow \quad \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 + mgh_f - mgh_i + \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2 = 0$$

$$\Rightarrow \quad k = -\frac{mv_f^2 - mv_i^2 + 2mgh_f - 2mgh_i}{x_f^2 - x_i^2} = -\frac{0 - 0 + 2(2.00\text{ kg})(9.80\text{ m/s}^2)(8.00\text{ cm})}{(5.00\text{ cm})^2 - (3.00\text{ cm})^2}$$

$$= 19.6\text{ N/cm}$$

25. **Correct Response: C**

$$\omega = \omega_0 + \alpha t = \omega_0 + \frac{\tau}{I}t = \omega_0 + \frac{rF\sin\theta}{I}t$$

$$= \left(10.0\ \frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\text{ radians}}{\text{rev}}\right) + \left(\frac{(0.100\text{ m})(30.0\text{ m})\sin(-110)}{2.00\text{ kg}\cdot\text{m}^2}\right)(40.0\text{ s}) = 6.45\ \frac{\text{radians}}{\text{s}} = 1.03\frac{\text{rev}}{\text{s}}$$

26. **Correct Response: C**

$$\bar{v} = \frac{d_{total}}{t_{total}} = \frac{d_{0\,s\,to\,2\,s} + d_{2\,s\,to\,3\,s} + d_{3\,s\,to\,6\,s}}{6.0\text{ s}} = \frac{6.0\text{ m} + 0.0\text{ m} + 3.0\text{ m}}{6.0\text{ s}} = 1.5\text{ m/s}$$

27. **Correct Response: D**

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{(-1.0\text{ m/s}) - (3.0\text{ m/s})}{5.0\text{ s} - 1.0\text{ s}} = -1.0\text{ m/s}^2 \quad \Rightarrow \quad |a| = 1.0\text{ m/s}^2$$

28. **Correct Response: E**

$$F = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = m_b \frac{N_b}{t}\left(v_{bf} - v_{bi}\right) = (0.0600\,\text{kg})\left(400\,\text{min}^{-1}\right)\frac{1\text{ min}}{60\text{ s}}(1500\,\text{m/s} - 0.0\text{ m/s})$$
$$\Rightarrow \quad F = 600.\,N$$

29. **Correct Response: A**

Hooke's Law: $F = -kx$

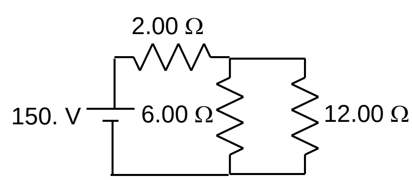
30. **Correct Response: E**

$$\Delta E = Q - W \quad \Rightarrow \quad Q = \Delta E + W = \Delta E + P\Delta V$$
$$\Rightarrow \quad Q = 9.00\times10^5\text{ J} + \left(2.00\times10^5\text{ Pa}\right)\left(5.00\text{ m}^3 - 2.00\text{ m}^3\right) = 15.0\times10^5\text{ J}$$

31. **Correct Response: B**

Recall that cyclotron period (and frequency) are a constant for a given particle and magnetic field strength independent of the radius of the trajectory.

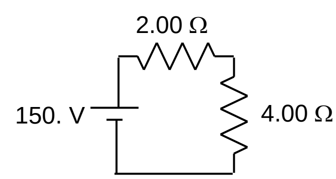
32. **Correct Response: B**



The 6.00 Ω and the 12.00 Ω resistors are in parallel. The equivalent resistance is

$$R_{eq} = \frac{(6.00\,\Omega)(12.00\,\Omega)}{6.00\,\Omega + 12.00\,\Omega} = 4.00\,\Omega$$

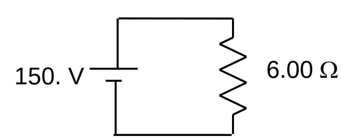
The equivalent circuit becomes:



The 2.00 Ω resistor and the 4.00 Ω equivalent resistor are in series. Their equivalent resistance is

$$R'_{eq} = 2.00\,\Omega + 4.00\,\Omega = 6.00\,\Omega$$

The equivalent circuit becomes:



By Ohm's Law, the current *I* is

$$I = \frac{V}{R} = \frac{150\,\text{V}}{6.00\,\Omega} = 25.0\text{ A}$$

This is the current through the 2.00 Ω resistor. Again by Ohm's Law,

$$V_{2.00\,\Omega} = I(2.00\,\Omega) = (25.0\text{ A})(2.00\,\Omega) = 50.0\text{ V}$$

33. **Correct Response: D**

$$\theta = \frac{y}{L} = \frac{1.22\lambda}{D} \quad \Rightarrow \quad L = \frac{yD}{1.22\lambda} = \frac{(1.40\text{ m})(8.00\times10^{-3}\text{ m})}{1.22(500\times10^{-9}\text{ m})} = 1.84\times10^{4}\text{ m}$$

34. **Correct Response: D**

$$p' = \gamma' m v' \quad and \quad p = \gamma m v \quad \Rightarrow \quad p' = \frac{\gamma' m v'}{\gamma m v} p = \frac{\sqrt{1-(0.500c/c)^2}}{\sqrt{1-(0.900c/c)^2}} \frac{0.900c}{0.500c} = 3.58p$$

35. **Correct Response: B**

$$N_1(t) = N_1(0)e^{-t/\tau_1} = N_2(t) = N_2(0)e^{-t/\tau_2} \quad \Rightarrow \quad t = \ln\frac{N_2(0)}{N_1(0)}\left(\frac{1}{\tau_2} - \frac{1}{\tau_1}\right)^{-1}$$

$$\Rightarrow \quad t = \ln\frac{4.00\times10^{16}}{8.00\times10^{16}}\left(\frac{1}{40.0\text{ s}} - \frac{1}{20.0\text{ s}}\right)^{-1} = 27.7\text{ s}$$