2017 Academic Challenge

MATHEMATICS TEST – STATE

– This Test Consists of 30 Questions –

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as ●, not ⊙, ◐, ◑, etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

*** **Time: 40 Minutes** ***

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

1. A ball is dropped to the ground from 20 feet, and it rebounds to 1/3 of its height on each bounce. If left to its own device, what is the total vertical distance that it travels?

    a) 40/3 ft.   b) 200/9 ft.   c) 40 ft.   d) 60 ft.   e) 80 ft.

2. If a rocket car is x = 3 + 4t miles east of its starting point after t minutes and y = 4 + 5t miles north of its starting point after t minutes, how fast is the car travelling in miles per hour? Round to the nearest mph, if necessary.

    a) 384 mph   b) 487 mph   c) 540 mph   d) 684 mph   e) 960 mph

3. A six-foot-long spar is connected by a hinge to the base of a six-foot-tall mast, with a rope and pulley system connecting the top of the mast to the free moving end of the spar (effectively creating an isosceles triangle whose three sides are the rigid mast, the rigid spar, and the adjustable connecting rope). Suppose the rope is pulled at a constant speed of 0.5 feet per second, raising the spar toward the mast. How quickly is the angle between the mast and spar decreasing at the moment the angle between the mast and spar is 45 degrees? Answers have been rounded to tenths of a radian per second.

    a) ≈ 0.0 radians per second
    b) 0.1 radians per second
    c) 0.2 radians per second
    d) 0.4 radians per second
    e) Not enough information given to determine angular velocity

4. Which of the following describes the graph of $x^2 - 2x + y^2 + 6y + 9 = 0$?

    a) parabola   b) ellipse   c) hyperbola d) circle   e) blank (no solution)

5. Find the area of a parallelogram determined by the vectors < 3, 4, 5 > and < 2, 1, 6 >. Round to the nearest square unit, if necessary.

    a) 15   b) 21   c) 32   d) 42   e) 45

6. The total number of people in a city who have been exposed to a new type of virus can be modeled by the function $P(t) = \dfrac{10,000}{1 + 100e^{-0.5t}}$, where t is the number of days since the virus was introduced to the city. After how many days will the number of people exposed to the virus be growing the fastest? Round your answer to the nearest whole day.

    a) 9 days   b) 10 days   c) 12 days   d) 18 days   e) No such time exists

7.   Which of the following is not possible?

   a) A cubic with no real zeros
   b) A cubic with no relative extrema
   c) A quartic with no real zeros
   d) An unshifted exponential with no real zeros
   e) All of these are possible

8.   Which of the following is the domain of $f(x) = \dfrac{5x - 4}{\sqrt{-x^2 + 6x - 8}}$?

   a) $(2, 4)$ b) $[2, 4]$ c) $(-\infty, 2) \cup (4, \infty)$

   d) $(-\infty, 2] \cup [4, \infty)$ e) Undefined

9.   In the following picture, an empty circle is inscribed inside a shaded equilateral triangle that is inscribed in a larger, empty circle. Determine what percentage of the area enclosed by the large circle is shaded. Round your answer to the nearest percent.
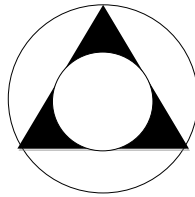
   a) 10 %   b) 13 %   c) 16 %   d) 18 %   e) 20 %

10. Which of the following is an approximation, to three significant figures, of the Golden Ratio?

   a) 1.62   b) 2.72   c) 3.14   d) 6.28   e) 6.02 x 10^{23}

11. Use a left Riemann sum with three rectangles to approximate $\int_{-1}^{2} e^{x^2+1}dx$. Round the answer to the nearest square unit, if necessary.

   a) 3   b) 17   c) 18   d) 49   e) 146

12. A soccer ball can be approximated by a truncated icosahedron with twenty regular hexagons made of white material and twelve regular pentagons made of black material. What percentage of the surface area of a soccer ball is black? Round to the nearest whole percent.

   a) 28 %   b) 30 %   c) 33 %   d) 38 %   e) 40 %

13. Use differentials to **estimate** the change in f(x) = 3x² + 5 from x = 4 to x = 4.1.

   a)   0.06   b) 2.4   c) 2.43   d) 5.3   e) 5.543

14. What is $\lim\limits_{h \to 0} \dfrac{|h|}{h}$?

    a) 0   b) 1   c) –1   d) ∞   e) Doesn't exist

15. How many total faces does a skew hexagonal pyramid have?

    a) 6   b) 7   c) 8   d) 12   e) Not enough information

16. What is the annual percentage yield (the annual simple interest rate which one would need to equate to the one-year compounded value of the investment) for 12.35% compounded continuously? Round to the nearest 10th of a percent.

    a) 12.4%   b) 12.6%   c) 12.8%   d) 13.0%   e) 13.1%

17. For a given substance, its pH can be found by taking –log [H⁺], where [H⁺] is its hydronium ion concentration (in moles per liter). What is the ratio of hydronium ion concentrations of one substance to another if the first has a pH that is 2.3 more than the other? Round to three significant digits.

    a) 2.30   b) 0.435   c) 200   d) 0.00501   e) Not enough information

18. Determine which of the following has the greatest volume:

    I. A cube with 10-inch edges
    II. A sphere with a 12-inch diameter
    III. A cone with a 15-inch diameter and 15-inch height

    a) I   b) II   c) III
    d) Two or more are tied   e) Not enough information

19. If the mean time between lightning strikes in a thunderstorm is 2.6 seconds, what is the probability of 10 strikes in 13 seconds? Assume that strikes occur at random and the time between strikes is independent of one another. Round to four decimal places.

    a) 0.0013   b) 0.0040   c) 0.0181   d) 0.0378   e) Impossible to determine

20. Given a standard deck of 52 cards with 4 suits of 13 denominations each, in how many ways can you get three of a kind (three of one denomination, one each of two others)?

    a) 3,432   b) 3,744   c) 54,912   d) 109,824   e) 113,568

21. A pool has two faucets and a drain. The first faucet can fill the pool in ten hours, the second can fill it in five hours, and the drain can empty the pool in four hours. The pool is full at noon when the drain is first opened. Two hours later (2 PM), the first faucet is turned on, and two hours after that (4 PM), the second faucet has been turned on. By the time eight more hours have passed (midnight), what is the state of the pool?

   a) Empty　　　　　　　　b) 2/3 of the way full　　　　c) 3/4 of the way full
   d) 4/5 of the way full　　　e) Full and overflowing

22. The vercosine of z, vercos z, is equal to $2\cos^2\left(\frac{1}{2}z\right)$. What is the period of vercos z?

   a) π　　　　b) 2π　　　　c) 4π　　　　d) 6π　　　　e) 8π

23. Which of the following is the period of $\sin^2(x)$?

   a) π　　　　b) 2π　　　　c) 4π　　　　d) 6π　　　　e) 8π

24. Given the equation AD + C = A + CB and B > C > D > 1, solve for A.

   a) $A = \dfrac{BC}{D}$　　　　b) $A = \dfrac{CB - C}{D}$　　　　c) $A = \dfrac{CB - C}{D - 1}$
   d) All real numbers　　　　e) No solution

25. Given the functions a = tan 2x and b = sin 2x, which of the following is an even function?

   a) a　　　　b) b　　　　c) a + b　　　　d) ab　　　　e) None of these

26. Which of the following is guaranteed to be (3 + bi)(3 – di)?

   a) 9 + bd　　　　　　　　b) 9 – bd　　　　　　　　c) 9 + bd + 3bi – 3di
   d) 9 – bd + 3bi – 3di　　　e) 9bd

27. Kaley can jog at 5 mph and Jamel can bike at 15 mph. At noon, Kaley starts at the north end of a two-mile trail and starts heading south. She plans on heading to the south end, turning around, and then returning to the north end. Fifteen minutes later, Jamel also starts at the north end and heads to the south end, turns around, and goes back north. Do Kaley and Jamel meet twice? If they do, how far apart are their meeting places? Round to the nearest tenth of a mile.

   a) No, they only meet up once
   b) Yes, the two meeting places are 0.1 miles apart
   c) Yes, the two meeting places are 0.2 miles apart
   d) Yes, the two meeting places are 0.3 miles apart
   e) Yes, the two meeting places are 0.4 miles apart

28. What is the determinant of $\begin{bmatrix} k & 5 \\ 7 & 35 \end{bmatrix} \cdot \begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix}$?

    a) 35(k – 1)   b) 0   c) 1260(k + 1)
    d) 630(k – 1)   e) None of these

29. Suppose a multiple-choice test must have each answer option equally or close to equally represented on the answer key. What is the probability that a randomly constructed multiple-choice test with five answer options (only one of which is correct) to each of its 30 questions has exactly 5, 6, or 7 questions with option A as the correct answer? Round to three decimal places.

    a) 0.154   b) 0.172   c) 0.179   d) 0.2   e) 0.506

30. Five relatives, Andrew (m), Beth (f), Carl (m), David (m), and Emily (f), recently did some volunteer work on their own to help finish building a house for charity. They planned on working from 8AM until 4PM in two hour shifts to complete five different tasks: roofing, siding, flooring, plumbing, and electrical. They created a schedule where no person performed the same task on more than one shift. Although Carl missed his first shift, Beth missed her last shift, and Emily had to leave from noon until 2 to do errands, every task was covered by at least 2 people. The following were also true:

    I. Flooring is the only task that was performed during all four shifts.
    II. David worked with his cousin from 8 until 10, with his sister from 10 until 2, and with his father from noon until 4.
    III. Andrew always worked alone. He did siding prep from 8 until 10 for the group of people doing siding from 10 until noon, and he did electrical prep from 10 until noon for the group of people who did electrical from 2 until 4.
    IV. Emily did roofing prep on her own from 10 until noon for the group doing roofing from noon until 2, and her father did touch-up work on the roofing from 2 until 4.

    Which task required the most amount of work-hours?

    a) Flooring, at a total of 10 work hours
    b) Roofing, at a total of 10 work hours
    c) Both flooring and roofing, at a total of 8 work-hours each
    d) Both flooring and roofing, at a total of 10 work-hours each
    e) Not enough information given