WYSE – Academic Challenge Physics Test Solutions (State) – 2017

1. Correct Response: E

Young's Modulus has dimensions of force per area, which has SI units N/m²:

$$\frac{N}{m^2} = \frac{kg \cdot m/s^2}{m^2} = \frac{kg}{s^2 \cdot m}$$

2. Correct Response: B

$$\left\lceil \frac{P}{A} \right\rceil = \left\lceil \frac{E}{A \cdot t} \right\rceil = \frac{M \cdot L^2 / T^2}{L^2 \cdot T} = \frac{M}{T^3}$$

3. Correct Response: B

Using a coordinate system with the +x-direction toward the east and the +y direction toward the north, the total of the first two displacements is

$$|d_1 + d_2| = |(4.00 \text{ mi})\hat{i} + (3.00 \text{ mi})\hat{j}| = \sqrt{(4.00 \text{ mi})^2 + (3.00 \text{ mi})^2} = 5.00 \text{ mi}$$

The average speed is the total path length divided by the total duration,

$$\overline{v} = \frac{|d_1| + |d_2| + |d_3|}{t_1 + t_2 + t_3} = \frac{4.00 \text{ mi} + 3.00 \text{ mi} + 5.00 \text{ mi}}{1.00 \text{ hr} + 0.750 \text{ hr} + 1.667 \text{ hr}} = 3.51 \text{ mi/hr}$$

4. Correct Response: C

$$v_f^2 = v_i^2 + 2a(x - x_0)$$
 \Rightarrow $x - x_0 = \frac{v_f^2 - v_i^2}{2a} = \frac{(1.00 \text{ m/s})^2 - (3.00 \text{ m/s})^2}{2(-0.200 \text{ m/s}^2)} = 20.0 \text{ m}$

5. Correct Response: E

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

In this case,

$$x_f = x_i \implies 0 = v_i t + \frac{1}{2} a t^2 \implies t = 0 \text{ or } t = -\frac{2v_i}{a} = -\frac{2(3.00 \text{ m/s})}{-0.200 \text{ m/s}^2} = 30.0 \text{ s}$$

6. Correct Response: D

Using a coordinate system with the +x axis to the East and the +y axis to the North,

$$\vec{v}_{boat to \ bank} = \vec{v}_{boat to \ river} + \vec{v}_{river \ to \ bank} \implies \vec{v}_{boat to \ river} = \vec{v}_{boat to \ bank} - \vec{v}_{river \ to \ bank}$$

$$\implies \vec{v}_{boat to \ river} = 12.0 \text{ m/s } \hat{\mathbf{i}} - 5.00 \text{ m/s } \hat{\mathbf{j}} \implies v_{boat to \ river} = \sqrt{(12.0 \text{ m/s})^2 + (5.00 \text{ m/s})^2} = 13.0 \text{ m/s}$$

7. Correct Response: E

Using a coordinate system with horizontal as the *x* direction and vertical upward as the *y* direction,

$$v_{y}^{2} = v_{0y}^{2} + 2a_{y}(y - y_{0})$$

$$\Rightarrow v_{0y} = \sqrt{v_{y}^{2} - 2a_{y}(y - y_{0})} = \sqrt{0 - 2(-9.80 \, \text{m/s}^{2})(30.0 \, \text{m})} = 24.25 \, \text{m/s}$$

$$0 = y - y_{0} = v_{0y}t + \frac{1}{2}a_{y}t^{2} \quad \Rightarrow \quad t = 0 \quad \text{or} \quad t = -\frac{2v_{0y}}{a_{y}} = -\frac{2(24.25 \, \text{m/s})}{-9.80 \, \text{m/s}^{2}} = 4.949 \, \text{s}$$

$$\tan \theta_{0} = \frac{v_{0y}}{v_{0x}} \quad \Rightarrow \quad v_{0x} = \frac{v_{0y}}{\tan \theta_{0}} = \frac{24.25 \, \text{m/s}}{\tan 50.0^{\circ}} = 20.35 \, \text{m/s}$$

$$x - x_{0} = v_{0x}t = (20.35 \, \text{m/s})(4.949 \, \text{s}) = 101 \, \text{m}$$

8. Correct Response: D

Using the same coordinate system as in the previous situation,

$$v_y^2 - v_{0y}^2 = 2a(y - y_0)$$
 and $v_{0y} = v_0 \sin \theta_0 \implies$

$$\Rightarrow v_0 = \frac{\sqrt{v_y^2 - 2a(y - y_0)}}{\sin \theta_0} = \frac{\sqrt{0^2 - 2(-9.8 \text{ m/s}^2)(30.0 \text{ m})}}{\sin 50.0^\circ} = 31.7 \text{ m/s}$$

9. Correct Response: A

$$F = \frac{\Delta p}{\Delta t} = m_{cart} a_{cart} + m_{person} a_{person} + m_{block} a_{block} \implies a_{cart} = \frac{F - m_{person} a_{person} - m_{block} a_{block}}{m_{cart}}$$

$$\Rightarrow a_{cart} = \frac{1200 \text{ N} - (100 \text{ kg})(2.00 \text{ m/s}^2) - (200 \text{ kg})(2.00 \text{ m/s}^2)}{800 \text{ kg}} = 0.750 \text{ m/s}^2$$

10. Correct Response: B

The only horizontal component of the force on the 6.00 kg block is the 30.0 N tension force. The horizontal acceleration of the 6.00 kg block is

$$a = \frac{F}{m} = \frac{30.0 \text{ N}}{6.00 \text{ kg}} = 5.00 \text{ m/s}^2$$

The block of mass m has the same magnitude acceleration, but in the downward direction. The total vertical force on that block is the upward tension force and the downward gravitational force. By Newton's 2^{nd} Law,

$$30.0 \text{ N} - mg = ma \implies m = \frac{30.0 \text{ N}}{a+g} = \frac{30.0 \text{ N}}{-5.00 \text{ m/s}^2 + 9.80 \text{ m/s}^2} = 6.25 \text{ kg}$$

11. Correct Response: A

Using a coordinate system with the +x-axis downhill parallel to the incline,

$$\sum F_x = ma_x \implies F \sin 25^\circ + mg \sin 25^\circ = ma_x \implies F = \frac{ma_x}{\sin 25^\circ} - mg$$

$$\implies F = \frac{\left(8.00 \text{ N}/9.80 \text{ m/s}^2\right) \left(6.00 \text{ m/s}^2\right)}{\sin 25^\circ} - \left(8.00 \text{ N}\right) = 3.59 \text{ N}$$

12. Correct Response: C

$$\sum F_x = ma_x = 0 \implies -F_{applied} + F \sin 25^\circ + mg \sin 25^\circ = ma_x = 0$$

But from problem 11,

$$F \sin 25^{\circ} + mg \sin 25^{\circ} = m(6.00 \text{ m/s}^2) \implies -F_{applied} + m(6.00 \text{ m/s}^2) = 0$$

$$\implies F_{applied} = m(6.00 \text{ m/s}^2) = \frac{8.00 \text{ N}}{9.80 \text{ m/s}^2} (6.00 \text{ m/s}^2) = 4.90 \text{ N}$$

13. Correct Response: B

There are no external forces acting on the system, so the center of mass moves with constant velocity. From the initial conditions, the center of mass velocity is

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(3.00 \text{ kg})(4.00 \text{ m/s}) + (2.00 \text{ kg})(-3.00 \text{ m/s})}{(3.00 \text{ kg}) + (2.00 \text{ kg})} = 1.20 \text{ m/s}$$
$$x_{cm} - x_{0cm} = v_{cm}t = (1.20 \text{ m/s})(5.00 \text{ s}) = 6.00 \text{ m}$$

14. Correct Response: A

In a perfectly elastic collision, the total kinetic energy is conserved.

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \implies v_{2f} = \sqrt{\frac{m_1}{m_2}}\left(v_{1i}^2 - v_{1f}^2\right) + v_{2i}^2$$

$$\Rightarrow v_{2f} = \sqrt{\frac{5.00 \text{ kg}}{4.00 \text{ kg}}\left[30.0 \text{ m/s}^2 - [10.0 \text{ m/s}]^2\right) + 0} = 31.6 \text{ m/s}$$

15. Correct Response: C

The metric prefix pico is equivalent to 10^{-12} .

16. Correct Response: B

$$P = \frac{W}{t} = \frac{\Delta K}{t} = \frac{600 \,\text{J}}{12.0 \,\text{s}} = 50.0 \,\text{W}$$

17. Correct Response: B

$$T - m_1 g = m_1 a$$
 and $T - m_2 g = -m_2 a$

$$\Rightarrow T = \frac{2m_1 m_2}{m_1 + m_2} g = \frac{2(4.00 \text{ kg})(6.00 \text{ kg})}{4.00 \text{ kg} + 6.00 \text{ kg}} (9.80 \text{ m/s}^2) = 47.0 \text{ N}$$

18. Correct Response: E

The entire original piece can be considered two masses, the mass m_1 that ends up with the hole in it and the mass m_2 circular piece that is moved.

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} = \frac{(10.0 \text{ kg})(\vec{0}) + (2.00 \text{ kg})(3.00 \text{ m} \,\hat{\mathbf{i}} + 2.00 \text{ m} \,\hat{\mathbf{j}})}{10.0 \text{ kg} + 2.00 \text{ kg}} = 0.500 \text{ m} \,\hat{\mathbf{i}} + 0.333 \text{ m} \,\hat{\mathbf{j}}$$

19. Correct Response: D

Assuming a Cartesian coordinate system with positive x horizontally to the right and positive y vertically downward. On the 45.0° line, the x coordinate is equal to the y coordinate.

$$x = v_{0x}t = v_{0y}t + \frac{1}{2}at^2 \implies t = 2\frac{v_{0x} - v_{0y}}{a} = 2\frac{30.0 \text{ m/s} - 0.00 \text{ m/s}}{9.80 \text{ m/s}^2} = 6.122\text{s}$$

$$\implies x = v_{0x}t = (30.0 \text{ m/s})(6.122\text{s}) = 184 \text{ m}$$

20. Correct Response: C

Calculate the moment of inertia of a solid circular plate of the same density and subtract the moment of inertia of four circles that are cut from the plate.

$$I = I_{solid} - 4I_{cutout} = \frac{1}{2} m_{solid} R^2 - 4 \left(\frac{1}{2} m_{cutout} R_{cut-out}^2 + m_{cutout} \left[\frac{2}{3} R \right]^2 \right)$$

The mass of a solid circular plate of the same density and radius R as the object shown is

$$m_{solid} = \pi R^2 \sigma = \pi R^2 \frac{M}{\pi R^2 - 4\pi \left(\frac{1}{3}R\right)^2} = \frac{9}{5}M \quad \text{and} \quad m_{cut-out} = \pi \left(\frac{1}{3}R\right)^2 \sigma = \frac{1}{9}m_{solid} = \frac{1}{5}M$$

$$\Rightarrow I = \frac{1}{2} \left(\frac{9}{5}M\right) R^2 - 4 \left(\frac{1}{2} \left[\frac{1}{5}M\right] \left[\frac{1}{3}R\right]^2 + \left[\frac{1}{5}M\right] \left[\frac{2}{3}R\right]^2\right) = \left[\frac{9}{10} - 4 \left(\frac{1}{90} + \frac{4}{45}\right)\right] MR^2 = \frac{1}{2}MR^2$$

21. Correct Response: B

$$\frac{Gm_{star}m_{planet}}{R^2} = m_{planet}a = m_{planet}\omega^2 R = m_{planet}\frac{4\pi^2}{T^2}R \implies$$

$$m_{star} = \frac{4\pi^2}{GT^2}R^3 = \frac{4\pi^2 (6.00 \times 10^{11} \text{ m})^3}{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(20.0 \text{ yr})^2 (3.156 \times 10^7 \text{ s/yr})^2} = 3.21 \times 10^{29} \text{ kg}$$

22. Correct Response: C

$$PV = NkT$$
 $\Rightarrow \frac{P_2V_2}{P_1V_1} = \frac{NT_2}{NT_1}$ $\Rightarrow T_2 = \frac{P_2V_2}{P_1V_1}T_1 = \frac{(200 \text{ kPa})(5.00 \text{ m}^3)}{(300 \text{ kPa})(1.00 \text{ m}^3)}T_1 = 3.33T_1$

23. Correct Response: E

Work done by a gas is the area below the PV curve. That area can be partitioned into two trapezoids, one with a base from 1.0 m^3 to 2.0 m^3 and the second with a base from 2.0 m^3 to 5.0 m^3

$$W = \frac{1}{2} \left(300 \times 10^3 \text{ Pa} + 600 \times 10^3 \text{ Pa} + \cancel{1.00} \text{ m}^3 \right) + \frac{1}{2} \left(600 \times 10^3 \text{ Pa} + 200 \times 10^3 \text{ Pa} + \cancel{1.00} \text{ m}^3 \right)$$
$$= 1.65 \times 10^6 \text{ J}$$

24. Correct Response: C

$$R_{eq} = 4.00 \,\Omega = 2.00 \,\Omega + \frac{R(6.00 \,\Omega)}{R + 6.00 \,\Omega} \quad \Rightarrow \quad R = \frac{(4.00 \,\Omega)(6.00 \,\Omega) - (2.00 \,\Omega)(6.00 \,\Omega)}{6.00 \,\Omega + 2.00 \,\Omega - 4.00 \,\Omega} = 3.00 \,\Omega$$

25. Correct Response: B

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

$$= \sqrt{(9.00 \,\Omega)^2 + \left[(1.20 \times 10^3 \,\mathrm{s}^{-1}) (30.0 \times 10^{-3} \,\mathrm{H}) - 1/((1.20 \times 10^3 \,\mathrm{s}^{-1}) (20.0 \times 10^{-6} \,\mathrm{F})) \right]^2} = 10.6 \,\Omega$$

26. Correct Response: D

$$\sum \vec{F} = 0 \implies qE - mg = 0 \implies g = \frac{qE}{m} = \frac{(3.00 \times 10^{-3} \text{ C})(4000 \text{ V/m})}{2.00 \text{ kg}} = 6.00 \text{ m/s}^2$$

27. Correct Response: B

$$m\lambda = \frac{dy}{L}$$
 \Rightarrow $y = \frac{m\lambda L}{d}$ \Rightarrow $y_2 - y_{-3} = [(2) - (-3)] \frac{\lambda L}{d}$
 \Rightarrow $d = \frac{5\lambda L}{y_2 - y_{-3}} = \frac{5(500 \times 10^{-9} \text{ m})(3.00 \text{ m})}{(7.00 \times 10^{-3} \text{ m})} = 1.07 \times 10^{-3} \text{ m} = 1.07 \text{ mm}$

28. Correct Response: A

$$\frac{1}{f_2} = \frac{1}{s_2} + \frac{1}{s_2'} \implies s_2 = \frac{f s_2'}{s_2' - f} = \frac{(-80.0 \text{ cm})(2.00 \text{ m} - 60.0 \text{ cm})}{(2.00 \text{ m} - 60.0 \text{ cm}) - (-80.0 \text{ cm})} = -50.91 \text{ cm}$$

Therefore, the image from the first lens is at x = 60.0 cm + 50.91 cm = 110.91 cm.

$$\frac{1}{f_1} = \frac{1}{s_1} + \frac{1}{s_1'} \implies s_1 = \frac{f s_1'}{s_1' - f} = \frac{(40.0 \text{ cm})(110.91 \text{ cm} - 10.0 \text{ cm})}{(110.91 \text{ cm} - 10.0 \text{ cm}) - (40.0 \text{ cm})} = 66.27 \text{ cm}$$

The source is 66.27 cm from the lens located at 10.0 cm, so the x position of the source is

$$x_{\text{source}} = 10.0 \text{ cm} - 66.27 \text{ cm} = -56.3 \text{ cm}$$

29. Correct Response: C

$$\begin{split} \sum F &= 0 \quad \Rightarrow \quad F_{buoyant} - mg = 0 \quad \Rightarrow \quad \rho_{fluid} g V_{submerged} - \rho_{cylinder} g V_{cylinder} = 0 \\ &\Rightarrow \quad \rho_{cylinder} = \frac{V_{submerged}}{V_{cylinder}} \rho_{fluid} = \frac{\frac{2}{3} \pi R^2 L + 2 \left(\frac{1}{2} R \cos 60^\circ R \sin 60^\circ\right) L}{\pi R^2 L} \rho_{fluid} = 0.804 \rho_{fluid} \end{split}$$

30. Correct Response: B

$$P_{1} + \frac{1}{2}\rho v_{1}^{2} = P_{2} + \frac{1}{2}\rho v_{2}^{2} \quad \text{and} \quad A_{1}v_{1} = A_{2}v_{2} \quad \Rightarrow \quad v_{2} = \frac{A_{1}}{A_{2}}v_{1} = \frac{\pi R_{1}^{2}}{\pi R_{2}^{2}}v_{1}$$

$$\Rightarrow \quad \rho = \frac{2(P_{1} - P_{2})}{v_{2}^{2} - v_{1}^{2}} = \frac{2(P_{1} - P_{2})}{v_{1}^{2}(R_{1}^{4} / R_{2}^{4} - 1)} = \frac{2(2.00 \times 10^{5} \text{ N/m}^{2})}{(40.0 \text{ m/s})^{2}((20.0 \text{ cm})^{4} / (10.0 \text{ cm})^{4} - 1)}$$

$$= 16.7 \text{ kg/m}^{3}$$

31. Correct Response: E

By definition of change in entropy,

$$\Delta S = \frac{Q}{T}$$

32. Correct Response: A

$$\Delta U + \Delta K = 0 \implies$$

$$-mg\Delta h = \Delta K = mC\left(T_f - T_i\right) \implies T_f = T_i - \frac{g\Delta h}{C} = 40.0 \text{ °C} - \frac{\left(9.80 \text{ m/s}^2\right)\left(-20.0 \text{ m}\right)}{80.0 \text{ J/(kg} \cdot \text{C}^\circ)} = 42.5 \text{ °C}$$

33. Correct Response: E

$$\Delta E = E_1 + E_2 = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} = hc \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right)$$

$$= \left(6.63 \times 10^{-34} \text{J} \cdot \text{s} \right) \left(3.00 \times 10^8 \text{ m/s} \right) \left(\frac{1}{400 \times 10^{-9} \text{ m}} + \frac{1}{600 \times 10^{-9} \text{ m}} \right) = 8.29 \times 10^{-19} \text{ J}$$

$$\Delta E = 8.29 \times 10^{-19} \text{ J} \times \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} = 5.17 \text{ eV}$$

34. Correct Response: B

$$K = E - mc^2 = Amc^2 - mc^2 = (A - 1)mc^2$$

35. Correct Response: C

The neutron is a subatomic particle with a half-life 611 s.