2018 Academic Challenge

MATHEMATICS TEST – REGIONAL

– This Test Consists of 30 Questions –

Mathematics Test Production Team

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as ●, not ⊙, ◍, ◐, etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

*** **Time: 40 Minutes** ***

**DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!**

1.   Which of the following is true about $r(x) = \dfrac{12x^3 - 44x + 2}{x^2 - 3x + 9}$?

   a) It has two vertical asymptotes and one horizontal asymptote.
   b) It has no vertical asymptotes and one horizontal asymptote.
   c) It has two vertical asymptotes and one oblique asymptote.
   d) It has no vertical asymptotes and one oblique asymptote.
   e) None of the above.

2.   Let the ray BG lie in the interior of $\angle ABC$ such that the measure of $\angle ABG$ is $\frac{7}{12}$ the measure of $\angle ABC$. If $\angle ABC$ is twice the measure of its complement, find the measure of $\angle GBC$.

   a) 20°   b) 25°   c) 30°   d) 55°   e) 70°

3.   Love it or hate it, Desiigner's *Panda* exists. How many distinguishable ways can you rearrange all of the letters of "Desiigner"?

   a) 990   b) 5,040   c) 90,720   d) 181,440   e) 362,880

4.   Two parallel lines are cut by a transversal with the given angles 60°, 100°, and x° as shown in the diagram below. Find the value of x. The diagram is not drawn to scale.
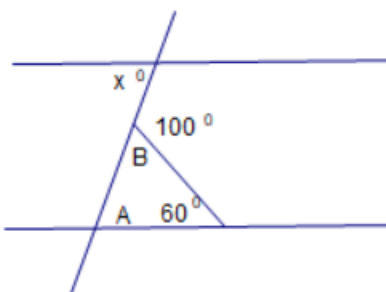
   a) 80   b) 70   c) 60   d) 50   e) 40

5.   A regular pentagon has a perimeter of 10 inches. Determine the area of the pentagon. Round to the nearest tenth of a square inch.

   a) 6.9 in²   b) 7.2 in²   c) 8.7 in²   d) 9.1 in²   e) 9.5 in²

6.   In your Precalculus course, you may not have to deal with me, but you have to deal with i. Which of the following is i^{99}?

   a) 1   b) i   c) –i   d) –1   e) None of these

7.   A triangle bounded by the lines $y = 0$, $y = 2x$, and $y = -0.5x + k$, where $k > 0$, has an area of 180 square units. Find the value of k.

a) 10   b) 15   c) 20   d) 45   e) 100

8.   What is the name of the polar shape described by $r^2 = a^2 \cos 2\theta$?

a) lemniscate   b) rose   c) cardioid
d) limaçon   e) None of these

9.   Find the quotient of the quantity $3x^3 + 19x^2 + 16x - 20$ and the quantity $x^2 + 7x + 10$.

a) $3x - 2$ b) $x + 5$ c) $x + 2$ d) $3x + 5$ e) $x - 5$

10.   Bill and Cherise are standing on a level field observing a balloon launch. Cherise stands 50 meters further away in a straight line from the launch pad than Bill. A few seconds after the balloon has risen vertically from the launch pad, Bill has to look up at an angle of 72 degrees to see the balloon, while Cherise only has to look up 65 degrees. How far above the launch pad is the balloon? Round your answer to the nearest whole meter.

a) 102 m   b) 115 m   c) 165 m   d) 354 m   e) 372 m

11.   If $\log_x 1 = 0$, what are the possible values for x?

a) All reals   b) $(0,1) \cup (1,\infty)$ c) $(0,\infty)$
d) $(-\infty,0) \cup (0,\infty)$ e) None

12.   Ken designs an electrical circuit that contains 44 resistors in series with a total resistance of 78 Ω by using two different resistances, 1.5 Ω and 2.0 Ω. How many 1.5 Ω resistors did Ken use in the design?

a) 24   b) 22   c) 20   d) 18   e) 16

13.   A radioactive substance follows an exponential decay model and has a third-life of 20 hours. (Every 20 hours, 2/3 of the substance will decay into another substance and 1/3 will be left.) How long will it take for there to be 10% of the original substance to remain? Round to the nearest hour.

a) 2   b) 42   c) 60   d) 84   e) 114

14.   The equation $2x^2 - y^2 = 4(x + y + 1)$ represents which type of curve?

a) cardioid   b) circle   c) ellipse   d) hyperbola e) parabola

15. If an exponentially growing population doubles every 10 years, how frequently does the population triple? Round your final answer to the nearest tenth of a year.

   a) Every 15.0 years   b) Every 15.5 years   c) Every 15.8 years
   d) Every 15.9 years   e) Insufficient information

16. Which of the following distributions is best suited for finding the probability of observing a total of 35 people entering a bank in a 40 minute period?

   a) Poisson   b) Normal   c) Hypergeometric
   d) Uniform   e) Binomial

17. Fifty ounces of mocha coffee is 40% chocolate. How much chocolate should be added for the resulting mixture of mocha coffee to be 60% chocolate?

   a) 10 oz   b) 15 oz   c) 20 oz   d) 25 oz   e) 30 oz

18. If the radius of a cone is increased by 60% but its volume remains the same, by what percentage must the height be decreased? Round to the nearest percent.

   a) 36%   b) 39%   c) 61%   d) 77%   e) Impossible to tell

19. Translate the English written expression to a mathematical expression:

   "Twice a number cubed less than the product of 5 and the number"

   a) $(2n)^3 - (5+n)$ b) $(2n)^3 - 5n$ c) $2n^3 < 5n$
   d) $5n - (2n)^3$ e) $5n - 2n^3$

20. A water tank has two drains. Drain A can empty the entire tank in 40 minutes, and drain B can empty the entire tank in 60 minutes. Starting with a full tank, how many minutes does it take to empty the tank if we open both drains? Round your answer to the nearest whole minute.

   a) 20 minutes   b) 24 minutes   c) 25 minutes
   d) 30 minutes   e) 50 minutes

21. If $\sin x = \dfrac{4}{5}$, what is $\cot^2 x$?

   a) $\dfrac{25}{16}$ b) $\dfrac{5}{4}$ c) $\dfrac{9}{16}$ d) $\dfrac{3}{4}$ e) Insufficient information

22. A fishing boat was purchased on March 1, 2015 for $10,000. The fishing boat depreciates in value exponentially at an instantaneous rate of 18%. How many years after the purchase will the boat's value be a fourth of the original value? Round to two decimal places.

   a) 1.60 yrs   b) 2.35 yrs   c) 3.85 yrs   d) 7.70 yrs   e) 10.00 yrs

23. What is the sum of the phase shift and period of $y = 4 + 3\cos\left(\dfrac{\pi t}{4} - \dfrac{\pi}{2}\right)$? If necessary, round to the nearest integer.

a) 1   b) 2   c) 3   d) 6   e) 10

24. Which one of the following equations would produce the shown graph below:
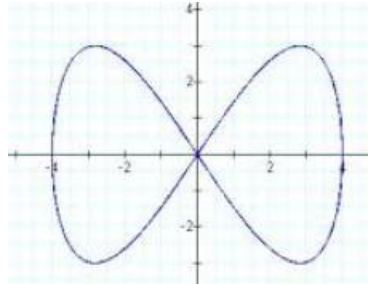
a) $x(t) = 4\cos(4t)$; $y(t) = 3\sin(3t)$
b) $x(t) = 4\cos(3t)$; $y(t) = 3\sin(3t)$
c) $x(t) = 3\cos(4t)$; $y(t) = 4\sin(3t)$
d) $x(t) = 4\cos t$; $y(t) = 3\sin t$
e) $x(t) = 4\cos t$; $y(t) = 3\sin(2t)$

25. City A and City B are 100 miles apart. At 9:00 AM, a truck leaves City A and heads toward City B at 40 mph. At 9:30 AM, a car leaves City B and heads toward City A at 60 mph. Does the car meet the truck on the road before the truck reaches City B? If it does, determine what time the car and truck meet. Round to the nearest whole minute.

a) No, the car does not meet the truck in time.
b) Yes, the car meets the truck at 9:48 AM.
c) Yes, the car meets the truck at 10:10 AM.
d) Yes, the car meets the truck at 10:18 AM.
e) Yes, the car meets the truck at 10:30 AM.

26. For which of the following values of N does 3^N and 7^N have the same value in the ones place?

a) N = 1,937   b) N = 1,995   c) N = 2,525
d) N = 65,536   e) N = 43,046,721

27. Let $A = \begin{bmatrix} 2 & 1 \\ -3 & 0 \\ 1 & k \end{bmatrix}$ $B = \begin{bmatrix} -1 & 6 & 5 & -2 \\ 3 & 0 & 1 & -4 \end{bmatrix}$. Let C = AB. Find the value of k if $c_{34} = -10$.

a) –2   b) –1   c) 0   d) 1   e) 2

28.   What is $\lim\limits_{x\to\infty} \dfrac{x^2 - 2x + 5}{x^2 - 1}$?

   a) 0   b) 1   c) –5   d) ∞   e) –∞

29.   An old well continuously collects water such that during a given hour, only 90% as much water is collected as in the previous hour. If 30 gallons of water are collected in the first hour, what must be the minimum capacity of the well?

   a) 810 gal   b) 487.5 gal   c) 300 gal   d) 232.5 gal   e) 165 gal

30.   Four sets of twins, two identical and two fraternal, recently went to a carnival. They rode four rides that each required two people ride together. Each of the twins rode one ride at 2:00, one at 3:00, one at 4:00, and one at 5:00. Everyone rode each ride once, and only with a twin from their group. No identical twin ever rode with another identical twin, and no fraternal twin ever rode with another fraternal twin.

   I.   David rode the Rocket at 2 with Brian's sister and the Spiral at 3 with Frank's sister.
   II. Erica rode the Tornado at 3 with Cheryl's sister and the Whirlwind at 4 with Henry.
   III. Amy rode the Whirlwind at 2 with Frank's sister and the Rocket at 5 with Brian.
   IV. Henry rode the Tornado at the same time his brother was riding with Frank.

   Determine who rode with Gina on the Spiral and when.

   a) Gina rode the Spiral at 2:00 with Henry.
   b) Gina rode the Spiral at 3:00 with David.
   c) Gina rode the Spiral at 4:00 with Amy.
   d) Gina rode the Spiral at 5:00 with Cheryl.
   e) Insufficient information