WYSE – Academic Challenge Physics Solutions (Regional) – 2018

1. Correct Response: B

$$\frac{\text{Pressure}}{(\text{velocity})^2} = \frac{\text{Force/area}}{(\text{length/time})^2} = \frac{\frac{\text{mass} \times \text{length}}{\text{time}^2} \times (\text{time})^2}{(\text{length})^2 (\text{length})^2} = \frac{\text{mass}}{(\text{length})^3}$$

Micrograms per cubic meter has dimensions mass/(length)³.

2. Correct Response: E

$$d = v_1 t_1 + v_2 t_2 + v_3 t_3 =$$

$$\left(11.4 \ \frac{m}{s}\right)(1.48 \ s) + \left(0.54 \ \frac{m}{s}\right)(25.3 \ s) + \left(16.1 \ \frac{m}{s}\right)(0.30 \ s) = 35.4 \ m$$

3. Correct Response: D

The downward force of gravity along with the normal force are the only forces acting. The sum of those forces must equal the mass times the acceleration.

$$F_N - mg = ma \implies$$

$$F_N = mg + ma = (43.0 \text{ kg})(9.80 \text{ m/s}^2) + (43.0 \text{ kg})(0.800 \text{ m/s}^2) = 456 \text{ N}$$

4. Correct Response: D

At this point, the velocity is constant and the acceleration is zero.

$$F_N - mg = ma \implies F_N = mg + ma = (43.0 \text{ kg}) \left(9.80 \frac{\text{m}}{\text{s}^2}\right) + 0 = 421 \text{ N}$$

5. Correct Response: B

The center of mass of this object would be in the center if the piece weren't cut out. Cutting the piece out reduces the mass on that side and therefore moves the center of mass slightly away from the center and the cut out.

6. Correct Response: C

$$v_f = v_i + a_1 \Delta t_1 + a_2 \Delta t_2 + a_3 \Delta t_3 =$$

$$v_f = 0 + \left(2 \frac{m}{s^2}\right)(3 s) + \left(-1 \frac{m}{s^2}\right)(2 s) + \left(3 \frac{m}{s^2}\right)(1 s) = 7 \frac{m}{s}$$

7. Correct Response: A

$V_{1x} = 3.00 \times \cos 140^\circ = -2.30$	$V_{1y} = 3.00 \times \sin 140^\circ = 1.93$
$V_{2x} = 2.05 \times \cos 250^\circ = -0.70$	$V_{2y} = 2.05 \times \sin 250^\circ = -1.93$
$V_{totx} = V_{1x} + V_{2x} = -3.00$	$V_{tot y} = V_{1y} + V_{2y} = 0.00$

This vector has a magnitude of 3.00 and the negative sign means that it is along the negative x axis or 180°.

8. Correct Response: E

The total travel time is the time it takes to drop from a height of 1.10 m.

$$t = \sqrt{\frac{2H}{g}} = 0.474s$$

The distance it travels is 142 m in that time. So the x velocity at the muzzle is

$$v_{\chi} = \frac{142 \text{ m}}{0.474 \text{ s}} = 300 \frac{\text{m}}{\text{s}}$$

The momentum given to the cannon ball is also the impulse given to the cannon by conservation of momentum. So

$$\Delta p = I = 3.20 \ kg \times 300 \ \frac{\mathrm{m}}{\mathrm{s}} = 959 \frac{\mathrm{kg \, m}}{\mathrm{s}}$$

9. Correct Response: A

It is the difference in gravitational pull from the Moon between the near and far sides of the Earth that causes water near the Moon to be pulled closer and water farther from the Moon to not be pulled as much.

10. Correct Response: C

$$v_f = \sqrt{2gh + v_o^2} = \sqrt{2\left(9.80 \ \frac{m}{s^2}\right)(38.0 \ m) + \left(14.5 \ \frac{m}{s}\right)^2} = 30.9 \ \frac{m}{s}$$

11. Correct Response: A

If we use the top of the ladder as the axis of rotation, measure the distance along the ladder to the point where the string is attached as *x*, call the tension *T*, and use the torque equation in static equilibrium we get:

$$-xT\sin 70 - mg\frac{l}{2}\sin(90 - 70) + mgl\sin(90 - 70) = 0$$

This can become:

$$xT = mg\frac{l}{2}\cot 70 = constant$$

If xT is constant then larger x means smaller T so the largest x (at the bottom) will be the smallest T.

12. Correct Response: A

The mass of the block is *M* and the mass of the board is *m*. The distance from the fulcrum to the mass is *x* and the distance from the fulcrum to the center of mass of the board is $\left(\frac{4.2}{2} - (0.5 + x)\right)$. With these definitions and the axis of rotation being the fulcrum the torque equation in static equilibrium is:

$$Mgx - mg\left(\frac{4.2}{2} - (0.5 + x)\right) = 0$$

Solving for x we get 0.43 m.

13. Correct Response: B

The force of gravity creates an acceleration down the incline of $9.8x\sin 22^{\circ} = 3.67 \text{ m/s}^2$. This will be the acceleration throughout the trajectory (including when the object stops at the top of its motion).

14. Correct Response: C

The radius, r_{40} , from the axis of rotation at 40° latitude is $R_{earth} \cos 40^\circ$. The speed at the equator is $v_o = R_{earth} \omega$. The speed at 40° is

$$v_{40} = r_{40}\omega = R_{earth}\omega\cos 40^\circ = v_o\cos 40^\circ = (464 \text{ m/s})\cos 40^\circ = 355 \text{ m/s}$$

15. Correct Response: C

Nano is the prefix for 10⁻⁹.

16. Correct Response: E

As the initial and final heights of the projectile are the same, the initial and final vertical velocity components are of the same magnitude but opposite sign.

$$v_{yf} - v_{yi} = a_y \Delta t \implies \Delta t = \frac{v_{yf} - v_{yi}}{a} = \frac{-v_0 \sin \theta - v_0 \sin \theta}{-g} = 2 \frac{v_0 \sin \theta}{g}$$

Applying this to the batted ball,

$$\Delta t_{batted \ ball} = 2 \frac{(30.0 \text{ m/s}) \sin 45^{\circ}}{9.8 \text{ m/s}^2} = 4.33 \text{ s}$$

and to the thrown ball,

$$\Delta t_{thrown \ ball} = 2 \frac{(22.81 \text{ m/s}) \sin 45^{\circ}}{9.8 \text{ m/s}^2} = 3.29 \text{ s}$$

The total time is the sum of these time times plus the 0.50 seconds to catch and throw:

$$\Delta t = \Delta t_{batted \ ball} + \Delta t_{thrown \ ball} + 0.50 \ s = 4.33 \ s + 3.29 \ s + 0.50 \ s = 8.12 \ s$$

17. Correct Response: A

The average velocity is

$$\frac{\Delta x}{\Delta t} = \frac{x_f - x_o}{t_f - t_o} = \frac{2 \text{ m}}{7 \text{ s}} = 0.286 \text{ m}$$

18. Correct Response: D

$$W = \Delta KE$$
 and $F = \mu_k F_N$

where the normal force, F_N , equals mg in this situation. The work done is $F \cdot d$ where d is the distance travelled during breaking. The negative sign is because the force is in the opposite direction to the displacement.

$$W = -Fd = -\mu_k F_N = -\mu_k mgd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_o^2$$

Solving for μ_k we get

$$\mu_k = \frac{v_o^2}{2gd} = 0.469$$

19. Correct Response: C

The work is the area under the curve:

$$W = \frac{1}{2}bh = \frac{1}{2}$$
 3.00 m × 5.00 N = 7.50 J

20. Correct Response: D

Conservation of momentum gives us

$$m_1 v_{1o} + m_2 v_{2o} = m_1 v_{1f} + m_2 v_{2f} \implies v_{2f} = \frac{m_1 v_{1o} + m_2 v_{2o} - m_1 v_{1f}}{m_2}$$
$$v_{2f} = \frac{(7.25 \text{ kg})(4.12 \text{ m/s}) + (1.50 \text{ kg})(0.00 \text{ m/s}) - (7.25 \text{ kg})(4.12 \text{ m/s})}{(1.50 \text{ kg})} = 6.82 \text{ m/s}$$

21. Correct Response: C

The tension in the string is what the scale measures and this tension is the same all along the string because of the pulley. A force diagram would show two tension arrows pointed upwards along both sections of string leaving the pulley. The force downward is mg and this force is balanced by the two tensions that are evenly distributed. Thus the tension must be $\frac{mg}{2} = 98.0 N$.

22. Correct Response: B

Conservation of energy in this system that includes rotational motion will result in the equation:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Substituting $\omega = v/r$ and solving for *v*:

$$v = \sqrt{\frac{2mgh}{m + I/r^2}} = \sqrt{\frac{2(0.200 \text{ kg})(9.80 \text{ m/s}^2)(0.76 \text{ m})}{(0.200 \text{ kg}) + (4.20 \times 10^{-4} \text{ kg} \cdot \text{m}^2)/(0.0065 \text{ m})^2}} = 0.542 \text{ m/s}$$

23. Correct Response: C

Using Newton's 2nd Law in the horizontal direction:

$$16.7 \text{ N} - F_{friction} = Ma \quad \Longrightarrow \quad$$

$$F_{friction} = 16.7 \text{ N} - Ma = 16.7 \text{ N} - (21.3 \text{ kg}) \left(0.411 \frac{\text{m}}{\text{s}^2}\right) = 7.95 \text{ N}$$

24. Correct Response: E

$$\alpha = \frac{a}{R} = \frac{0.411 \text{ m/s}^2}{(0.825 \text{ m})/2} = .996 \frac{\text{rad}}{\text{s}^2}$$

25. Correct Response: C

$$V = IR \implies I = V/R = (12 \text{ V})/(6 \Omega) = 2.0 \text{ A}$$

26. Correct Response: B

The potential is related to the force by:

$$\mathbf{F} = -\nabla U = -\mathbf{i}\frac{\partial U}{\partial x} - \mathbf{j}\frac{\partial U}{\partial y} - \mathbf{k}\frac{\partial U}{\partial z} = -\mathbf{i}ay^2z^3 - \mathbf{j}ax^2yz^3 - \mathbf{k}axy^23z^2$$

27. Correct Response: B

$$P = Fv = (2090 \text{ N})(51.4 \frac{\text{m}}{\text{s}}) = 1.07 \times 10^5 \text{ W}$$

28. Correct Response: B

$$\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2} \implies I_2 = I_1 \frac{r_1^2}{r_2^2} = \left(2.50 \times 10^{-6} \frac{\text{W}}{\text{m}^2}\right) \left(\frac{12.0 \text{ m}}{185. \text{ m}}\right)^2 = 1.05 \times 10^{-8} \frac{\text{W}}{\text{m}^2}$$
$$\beta = 10 \log \frac{I}{I_o} = 10 \log \frac{1.05 \times 10^{-8} \frac{\text{W}}{\text{m}^2}}{1.00 \times 10^{-12} \frac{\text{W}}{\text{m}^2}} = 40.2 \text{ dB}$$

29. Correct Response: E

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} \implies R = \sqrt[3]{\frac{3M}{4\pi\rho}} = \sqrt[3]{\frac{3(60.\,\text{kg})}{4\pi\left(19.5 \times 10^3 \frac{\text{kg}}{\text{m}^3}\right)}} = 0.0902\,\text{m}$$

30. Correct Response: A

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \text{ and } M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \implies$$

$$f = \left(-\frac{M}{d_i} + \frac{1}{d_i}\right)^{-1} = \left(-\frac{h_i}{d_i h_o} + \frac{1}{d_i}\right)^{-1} = \left(-\frac{1.5 \text{ cm}}{(-12.0 \text{ cm})(7.2 \text{ cm})} + \frac{1}{-12.0 \text{ cm}}\right)^{-1} = -15.2 \text{ cm}$$
2018 Regional

31. Correct Response: D

Michael Faraday developed Faraday's Law as stated.

32. Correct Response: E

Electrons and muons are leptons and not hadrons.

33. Correct Response: D

$$Q = \Delta E + W = \Delta E + P\Delta V = 1.73 \times 10^{6} \text{ J} + (2.30 \times 10^{5} \text{ Pa})(7.50 \text{ m}^{3} - 2.50 \text{ m}^{3})$$
$$= 2.88 \times 10^{6} \text{ J}$$

 $\Delta E = Q - W$

34. Correct Response: B

Considering the equilibrium of the upper horizontal rod:

$$m_{left}gr_{left} - m_{right}gr_{right} = 0.00$$

$$(200 \text{ g})g(20 \text{ cm}) - Mg(50 \text{ cm}) = 0.00 \implies$$

$$(200 \text{ g})g(20 \text{ cm}) - Mg(50 \text{ cm}) = 0.00 \implies$$

$$M = \frac{(200 \text{ g})(20 \text{ cm})}{(50 \text{ cm})} = 80 \text{ g}$$

35. Correct Response: D

The sum of the torques about any point is zero. Summing the torques about the elbow joint applied to the forearm:



 $T_{biceps}(\cos 10^{\circ})(x) - W_{forearm}d - W_{ball}L = 0.00 \implies$

$$T_{biceps} = \frac{W_{forearm}d + W_{ball}L}{(\cos 10^{\circ})(x)} = \frac{(32.0 \text{ N})(0.150 \text{ m}) + (112.0 \text{ N})(0.320 \text{ m})}{(\cos 10^{\circ})(0.0470 \text{ m})} = 878 \text{ N}$$