2018 Academic Challenge

MATHEMATICS TEST – SECTIONAL

– This Test Consists of 30 Questions –

Mathematics Test Production Team

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as ● , not ⊙ , ⦶ , ◑ , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

*** **Time: 40 Minutes** ***

**DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!**

1. A square baseball diamond has sides that are 90 feet long. When a runner going from second to third base is 106 feet from home plate (shortest distance) the runners distance from home plate decreases at a rate of 16 ft/sec. At what speed is the runner running in ft/sec? Round to two decimal places.

   a) 8 ft/sec
   d) 36.03 ft/sec

   b) 20.24 ft/sec
   e) 38 ft/sec

   c) 30.29 ft/sec

2. A regular octagon has an apothem of 6 units. Which of the following is its area? Round to the nearest tenth of a square unit.

   a) 39.8
   d) 238.6

   b) 119.3
   e) None of the above/Impossible to find.

   c) 124.7

3. Paul, Peter, and Rachel roll a 6-sided die in turn. Paul rolls first, Peter rolls second, and Rachel rolls third, repeating as necessary until one of them wins the game by being the first one to roll a six. Find the probability that either Peter or Rachel will win.

   a) $\dfrac{2}{3}$
   b) $\dfrac{36}{91}$
   c) $\dfrac{244}{273}$
   d) $\dfrac{5}{6}$
   e) $\dfrac{55}{91}$

4. If $x(t) = 3t^2 + 4t - 1$ and $y(t) = 8t - 7$, what is the shortest distance from the point where $t = 1$ to the point where $t = 3$? If necessary, round to the nearest full unit.

   a) 16
   b) 32
   c) 36
   d) 48
   e) 74

5. An equilateral triangle is inscribed inside a square as shown below. The triangle shares one vertex with the square, and the corresponding median of the triangle overlays the corresponding diagonal of the square. If the area of the square is one square inch, determine the area of the equilateral triangle. Round to the nearest hundredth of a square inch.
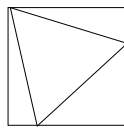
   a) 0.40 in^2
   b) 0.42 in^2
   c) 0.46 in^2
   d) 0.48 in^2
   e) 0.50 in^2

6. Express $x^2 + 5x + y^2 = 0$ in polar form.

   a) $r = -5\cos\theta$
   b) $r = 5\sin\theta$
   c) $r = -\dfrac{1}{\cos\theta}$

   d) $r = \dfrac{1}{5\cos\theta}$
   e) $r = \dfrac{1}{5\sin\theta}$

7.   What is the magnitude of 5 + 5i raised to the 8th power?

    a) 390,625   b) 781,250   c) 6,250,000
    d) 100,000,000   e) None of these

8.   Given the system of equations $\begin{cases} 2x & + & ky & + & z & = & 4 \\ 3x & - & & & z & = & -3 \\ x & - & 2y & + & 2z & = & -5 \end{cases}$ where $\begin{vmatrix} 2 & k & 1 \\ 3 & 0 & -1 \\ 1 & -2 & 2 \end{vmatrix} = -31$,

    find the value for x using determinants and Cramer's Rule.

    a) 0   b) –1   c) 2   d) –3   e) 3

9.   What is $\dfrac{d}{dz} 3e^{z^2+z-1}$?

    a) $(6z+3)e^{z^2+z-1}$ b) $(3z^2+3z-3)e^{z^2+z-1}$ c) $3e^{2z+1}$

    d) $(6z+3)e^{2z+1}$ e) None of these

10.   At the time of preparation, a petri dish contains a colony of 5 bacteria that will double every six hours for the next 120 hours. In other words B, the number of bacteria T hours after preparation, is modeled by the function $B = 5e^{0.1155T}$ for the domain [0,120]. Is there a time that the bacteria grow at a rate of 20 bacteria per hour? If so, round the correct time to the nearest hour.

    a) Yes, at T = 6 hours   b) Yes, at T = 12 hours   c) Yes, at T = 31 hours
    d) Yes, at T = 50 hours   e) No such time exists.

11.   Given $3(2^{x+1}) = 1 + \dfrac{2}{2^x}$, find x.

    a) $\dfrac{\log 3 - \log 2}{\log 2 - \log 3} + \dfrac{3}{2}\log 2$ b) $\dfrac{2 - \log 3}{-3\log 2}$ c) $\dfrac{\log 3 + \log 2}{\log(3 \cdot 2^{\log 3} - 1)}$

    d) $\dfrac{\log 2 - \log 3}{\log 2}$ e) $\log 3 + \log 2 - \log(3 \cdot 2^{\log 3} - 1)$

12.   Which of the following is true about the graph of $y = \dfrac{x^3 - 2}{x^2 + 1}$?

    a) It has only an oblique asymptote.
    b) It has only a horizontal asymptote.
    c) It has both a vertical and oblique asymptote.
    d) It has both a vertical and horizontal asymptote.
    e) It has no vertical, horizontal, or oblique asymptotes.

13. Two straight roads cross at an angle of 135.65°. A car on one road is currently 5 miles from the intersection and is moving away from it at 60 mph. There is also currently a truck on the other road that is 10 miles from the intersection and is moving away from it at 50 mph. If the car and truck maintain their constant speeds, what will be the distance between them 45 minutes from now? Round to the nearest mile.

    a) 69 miles   b) 71 miles   c) 85 miles   d) 90 miles   e) 98 miles

14. A culture of bacteria starts with 535 bacteria, and grows at an instantaneous exponential growth rate of 7.1% per hour. How many hours, rounded **up** to the next whole hour, will it take to get to 1,000,000 bacteria?

    a) 29   b) 71   c) 95   d) 107   e) 110

15. How many edges are there on a regular pentagonal prism?

    a) 5   b) 7   c) 10   d) 15   e) 30

16. A shipment of two boxes, each containing a variety of 6 pairs of skates, is received by a store. Box 1 contains exactly 1 pair of freestyle skates, and Box 2 contains exactly 2 pairs of freestyle skates. After the boxes are unpacked, a pair of skates is selected and found to be a freestyle pair of skates. Find the probability that the freestyle pair of skates chosen came from Box 2.

    a) $\dfrac{2}{3}$ b) $\dfrac{5}{9}$ c) $\dfrac{1}{12}$ d) $\dfrac{4}{9}$ e) $\dfrac{1}{3}$

17. If $f'(x) = x^3 - 4x$ and $f(2) = 4$, what is $f(4)$?

    a) –4   b) 8   c) 36   d) 40   e) 52

18. In the diagram below, find the length of the chord that consists of c and d, where $a = 2x + 3$, $b = x + 4$, $c = 2x - 1$, and $d = 3x + 2$. The diagram is not to scale.
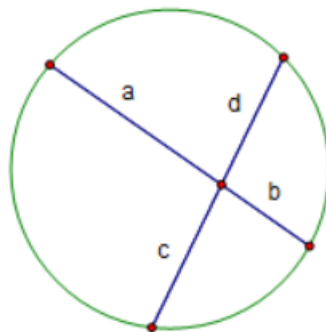
    a) 22 units   b) 18.5 units   c) 17.5 units   d) 9.8 units   e) 7 units

19. How many solutions to the system $\begin{cases} x^2 - 5xy + 2y^2 = 1 \\ 3xy = 1 \end{cases}$ exist on the unit circle?

    a) 0   b) 1   c) 2   d) 3   e) 4 or more

20. A farmer uses 1/4 of the feed in her truck to feed her cows, 2/5 of the remaining feed to feed her pigs, and then 1/3 of the remaining feed to feed her chickens. If she has 60 pounds of feed left in the end, how much did she start with? Round your answer to the nearest whole pound.

    a) 121 lbs   b) 200 lbs   c) 1800 lbs   d) 2000 lbs   e) 3600 lbs

21. A plot of land has been donated to Home Sweet Home Ministries (HSHM) to develop a vegetable garden to raise money for the ministry. The plot of land has the following characteristics: The north side of the garden runs from point A to point B, where point B is directly 4 meters east of point A. Point D is directly south of point A. Point C is 7 meters directly east of D. The boundary line from point B to point C has a bearing of $30°$ east of true south. HSHM expects to make \$7 per square meter for this plot of land. How much money is HSHM expecting to make from the donated plot of land?

    a) \$115.50   b) \$155.31   c) \$200.05   d) \$231.00   e) \$400.10

22. If $\sin x = \dfrac{7}{9}$, what is $1 - \cos^2 x$?

    a) $\dfrac{49}{81}$ b) $\dfrac{32}{81}$ c) $\dfrac{81 - 4\sqrt{2}}{81}$ d) $\dfrac{65}{81}$ e) Insufficient information

23. Simplify the expression: $\dfrac{1}{x^3 + 2x^2 + x} + \dfrac{1}{x^2 + 2x + 1} - \dfrac{x}{x^3 + x^2 + x + 1}$

    a) $-\dfrac{1}{2x\left(x^2 + 1\right)\left(x + 1\right)}$ b) $\dfrac{1}{x\left(x^2 + 1\right)\left(x + 1\right)^2}$ c) $\dfrac{x}{\left(x^2 + 1\right)\left(x + 1\right)}$

    d) $\dfrac{1}{x\left(x^2 + 1\right)\left(x + 1\right)}$ e) $\dfrac{2 - x}{x\left(x^2 + 1\right)\left(x + 1\right)}$

24. I am studying for WYSE and have 6 biology books, 3 math books and 5 English books. In how many ways can I arrange them on my bookshelf so that they're **not** all together by subject? (Some can be arranged in entire subject groups, but there must be **at least one** book that is not with the rest of its subject group.)

    a) 168,168   b) 518,400   c) 3,110,400
    d) 87,175,180,800   e) 87,178,291,200

25. It takes Jane 4 minutes to sort one pound of recycled material, and it takes Ken 6 minutes per pound. The two of them need to sort a total of 100 pounds. If Jane starts at 9:00 AM and then Ken joins in an hour later at 10:00, when will the two of them be done with the entire 100 pounds? Round to the nearest minute.

   a) 12:24 PM b) 12:59 PM c) 1:24 PM   d) 1:59 PM   e) 7:00 PM

26. Joe sells game consoles on the weekends to earn extra cash. He can sell x consoles per weekend at a price of p dollars per console where $x = 65 - \frac{2}{5}p$. Joe purchases the consoles from a supplier who charges C dollars for x consoles according to the equation $C = 400 + 15x + \frac{1}{5}x^2$. Determine the profit made when the price for the console is $125.

   a) $1,875   b) $1,718   c) $1,404   d) $1,247   e) $1,205

27. What is the sum of all real k-values that make the vectors $\langle k, 6, 4 \rangle$ and $\langle 2k - 5, 6, -17 \rangle$ orthogonal? If necessary, round your answer to the nearest hundredth.

   a) –2.94   b) 2.50   c) 5.00   d) 10.88   e) There are none.

28. Two stock investments cost $20,000. One had a 35% gain while the other a 15% loss. If the net profit is $3362.50, how much was invested in the stock with the 35% gain?

   a) $15,000   b) $12,725   c) $11,500   d) $8,500   e) $7,275

29. Find the length of the semiminor axis in an ellipse with a center at (7,6), a focus at (11,6) and a vertex at (–13,6). Round your answer to the nearest tenth of a unit.

   a) 4.0   b) 19.6   c) 20.0   d) 20.4   e) Impossible to tell.

30. Five students from the same home town, Amy (F), Ben (M), Carl (M), Dan (M), and Emily (F), each go to five different colleges during the school year, and each studies a different major. Over the summer, two were lifeguards at the pool, two others painted, and the fifth one was a cashier at a grocery store.

   I.   Emily worked at the pool with the brother of the art major from NIU.
   II. Carl, the math major, worked with the brother of the girl going to ISU.
   III. The girl majoring in economics worked with the biology major from EIU.
   IV. The boy going to the U of I worked with Ben, the chemistry major from SIU.

   Does the person working as a cashier at the grocery store have a sibling, and if so, what is that sibling's major?

   a) No, the grocery store cashier doesn't have a sibling.
   b) Yes, the cashier's sibling is majoring in art.
   c) Yes, the cashier's sibling is majoring in biology.
   d) Yes, the cashier's sibling is majoring in chemistry.
   e) Not enough information is given.