WYSE | ENGINEERING AT ILLINOIS

2018 Academic Challenge

MATHEMATICS TEST – STATE

– This Test Consists of 30 Questions –

Mathematics Test Production Team
Kevin Boyer, Illinois State University – Author/Team Leader
Matthew Childers, Illinois State University – Author
Linda Wiggins, Illinois State University - Author
Ryan Bunge, Illinois State University – Reviewer
Sahid L. Rosado Lausell, WYSE – Coordinator of Test Production

GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as ●, not ⊙, ◍, ◑, etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

*** **Time: 40 Minutes** ***

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

1.   Which of the following is an antiderivative of $x^2 \ln x$ with respect to x?

   a) $\dfrac{x^2}{2}$

   b) $\dfrac{x^3}{3}(\ln x)^2$

   c) $2x\ln x + x$

   d) $\dfrac{x^3}{3}\ln x + \dfrac{x^3}{9}$

   e) $\dfrac{x^3}{3}\ln x - \dfrac{x^3}{9}$

2.   A lodge has a circular base with a diameter of 6 feet and a saddle-shaped roof. The roof
      and sides at the ends of the north/south diameter create a six-foot square cross section.
      Each cross-sectional plane parallel to the central cross section is also square. Determine
      the amount of space contained within the lodge. Round to the nearest cubic foot.

   a) 84 ft^3 b) 144 ft^3 c) 222 ft^3 d) 228 ft^3 e) 240 ft^3

3.   An angle x has the property that 8 tan 2x = 5 tan x. Which of the following is a possible
      value for x? Round to the nearest hundredth of a radian.

   a) –1.14   b) 0.98   c) 1.01   d) 1.14   e) 3.14

4.   Martha flies a Minion kite in the air that is keeping an altitude of 300 ft. The kite is being
      blown horizontally at a rate of 20 ft. per second away from Martha who is holding the kite
      string at ground level. At what rate is the string being extended when 500 ft. of string
      have already been extended?

   a) 6 ft/sec   b) 8 ft/sec   c) 10 ft/sec   d) 12 ft/sec   e) 16 ft/sec

5.   How many vertices does a regular icosahedron have?

   a) 12   b) 15   c) 20   d) 30   e) 60

6.   What is the sum of the coordinates of the cross product of $\langle 6, 9, 4 \rangle$ and $\langle 2, 1, 8 \rangle$?

   a) 16   b) 35   c) 53   d) 96   e) 120

7.   Eliminate the parameter and write a Cartesian equation for the curve defined by
      $x = \sec(t)$ and $y = \tan(t)$.

   a) $y = x^2 + 1$

   b) $y^2 = x^2 - 4$

   c) $x^2 + y^2 = 4$

   d) $x^2 - y^2 = 1$

   e) $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$

8. A bin contains 3 blue balls, 2 red balls, 4 yellow balls, 6 black balls, and 2 green balls. A set of three balls is randomly selected. How many ways are there to pull three balls and **not** get a set consisting of solely primary colors (red, yellow, or blue)?

   a) 56   b) 84   c) 273   d) 596   e) 680

9. Let matrix $A = \begin{bmatrix} s & t \\ u & v \end{bmatrix}$ where A is not singular. Find A^{-1}.

   a) $A^{-1} = \begin{bmatrix} v & -t \\ -u & s \end{bmatrix}$

   b) $A^{-1} = \begin{bmatrix} \dfrac{v}{sv-tu} & \dfrac{t}{sv-tu} \\ \dfrac{u}{sv-tu} & \dfrac{s}{sv-tu} \end{bmatrix}$

   c) $A^{-1} = \begin{bmatrix} \dfrac{v}{sv-tu} & \dfrac{t}{tu-sv} \\ \dfrac{u}{tu-sv} & \dfrac{s}{sv-tu} \end{bmatrix}$

   d) $A^{-1} = \begin{bmatrix} \dfrac{-s}{sv-tu} & \dfrac{u}{sv-tu} \\ \dfrac{t}{sv-tu} & \dfrac{-v}{sv-tu} \end{bmatrix}$

   e) $A^{-1} = \begin{bmatrix} -s & u \\ t & -v \end{bmatrix}$

10. A train travelling at a velocity of 60 feet per second needs to stop to avoid a collision at an upcoming crosswalk. If the train's brakes can provide a constant deceleration of 0.5 feet per second per second, how far does the train travel until it comes to a complete stop? Round to the nearest foot.

    a) 600 feet   b) 900 feet   c) 1,200 feet d) 1,800 feet e) 3,600 feet

11. Given the hyperbola described by $\dfrac{(x-2)^2}{9} - \dfrac{(y+5)^2}{16} = 1$, what is the difference in lengths (longer minus shorter) of the transverse and conjugate axes?

    a) 1   b) 2   c) 7
    d) 14   e) Hyperbolas do not have these axes

12. Rewrite $3\log(x^2-1) - 2\log(x+1)$ as a single logarithm.

    a) $\dfrac{3}{2}\log(x-1)$ b) $\log\left((x^2-1)(x-1)^2\right)$ c) $\log\sqrt{(x-1)^3}$

    d) $\log(x+1)^2$ e) $\log\left((x+1)^3(x-1)^2\right)$

13. Given the function $y = -\dfrac{1}{2}\cos\left(\dfrac{3x}{4} + \dfrac{\pi}{4}\right) + 2$, find the phase shift and then the period.

    a) $\dfrac{\pi}{4}; 2\pi$ b) $\dfrac{-\pi}{3}; \dfrac{7\pi}{3}$ c) $\dfrac{\pi}{3}; \dfrac{8\pi}{3}$ d) $\dfrac{-\pi}{4}; 2\pi$ e) $\dfrac{-\pi}{3}; \dfrac{8\pi}{3}$

14. A stationary camera is set up to record a balloon's launch 100 feet away from the balloon's launching pad and at the same height. Ten seconds after the balloon's launch, the camera has rotated upward 72 degrees to maintain focus on the balloon. Assuming the balloon has been ascending at a constant velocity, what has been its rate of ascent? Round to the nearest foot per second.

   a) 3 feet per second   b) 10 feet per second   c) 15 feet per second
   d) 31 feet per second   e) 32 feet per second

15. Which of the following sets of transformations, taken in the order in which they are written, will transform $y = e^t$ into the function $y = 7e^{-2t+4} - 6$?

   I.   Vertical stretch by a factor of 7, horizontal compression by a factor of 2, reflection over the y-axis, shift left 4 and down 6.
   II. Shift down 6, vertical stretch by a factor of 7, horizontal compression by a factor of 2, reflect over the y-axis, shift right 2 and up 36.
   III. Vertical stretch by a factor of 7, horizontal compression by a factor of 2, reflect over the y-axis, shift right 2 and down 6.

   a) I only   b) II only   c) III only
   d) I and II only   e) II and III only

16. A fair, six-sided die is rolled 20 times. What is the probability that a 6 is rolled three or fewer times? Round to the nearest thousandth.

   a) 0.238   b) 0.300   c) 0.329   d) 0.567   e) 0.671

17. A contestant going through a mirror maze wins the grand prize if he/she can finish within 3 minutes. There are 3 routes available to successfully get from the beginning to the final destination. There is an 80% probability of getting trapped on Route 1, a 60% probability on Route 2, and a 30% probability on Route 3. Because of other factors, such as distance and turns, contestants choose Route 1 approximately 50% of the time, and Routes 2 and 3 each 25% of the time. When a contestant gets trapped, the host is called to rescue the contestant. If the contestant calls the host to inform the host that he/she is trapped, find the probability that Route 1 was selected. Round to the nearest thousandth.

   a) 0.640   b) 0.625   c) 0.425   d) 0.240   e) 0.120

18. The length of the edges of a cube are increasing at a constant rate of 4 cm per second. What is the rate of increase in the surface area at the time when the cube has a volume of 1000 cm³? (All answers are given in cm² per second.)

   a) 0.013   b) 0.033   c) 480   d) 600   e) 1200

19. There are two adjacent parking lots, A and B, for downtown parking as shown below. Parking Lot A, shaped like a right triangle, is for compact cars and Lot B, shaped like a right trapezoid, is for oversized vehicles. Lot A has frontage on Main Street of 140 ft. Lot B has frontage on Main Street of 84 ft. The boundary between the lots is 120 ft. Find the perimeter of Lot B. Round to the nearest foot.

First Street

B

A

Main Street
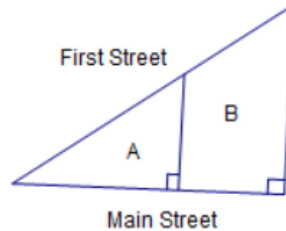
a) 691 ft.   b) 620 ft.   c) 559 ft.   d) 528 ft.   e) 507 ft.

20. Find the surface area of a ball whose circumference is 25 centimeters. Round to the nearest square centimeter.

a) 1250 cm^2 b) 625 cm^2 c) 309 cm^2 d) 199 cm^2 e) 105 cm^2

21. At noon, Bill starts at the starting line of a 400-meter track and begins walking clockwise around the track at a speed of 2 meters per second. Fifteen minutes later (at 12:15 PM), Andy arrives at the starting line of the same 400-meter track and begins jogging clockwise around the track at a speed of 4 meters per second. What is the first time that the two runners are at the same point on the track? Round to the nearest whole minute.

a) 12:15 PM b) 12:16 PM c) 12:17 PM d) 12:18 PM e) 12:20 PM

22. A chemistry lab has 43 L of 25% saline solution and virtually unlimited distilled/pure water. If a chemist needs 2,000 L of 2% saline, is there enough of the 25% solution to create it? And if so, how much water would be necessary to create the 2% solution?

a) Yes, 80 L   b) Yes, 1,920 L   c) Yes 37 L
d) Yes, 1,963 L   e) No, insufficient quantity of saline solution

23. How many solutions to $\tan^2 x - 1 = 0$ lie between x = 0 and $x = 2\pi$?

a) 0   b) 1   c) 2   d) 3   e) 4+

24. In the diagram below, H is the midpoint of BD, CH = 2, CA = 14.5, ED = 6, and EF = 8. Find FG. The diagram below is not drawn to scale.
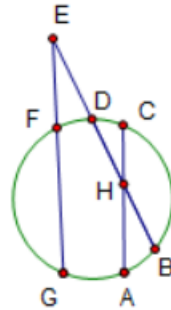
    a) 12   b) 8   c) 4   d) 3.5   e) 2.5

25. Anne and Beth needed to distribute flyers for an upcoming charity sale. From the initial stack of flyers, Anne took some of them, and Beth took the rest. After one day of distributing flyers, Anne had distributed 70% of her flyers, and Beth had distributed 40% of hers, but they each had the same <u>number</u> of flyers left. What combined percentage of the original number of flyers have the two girls distributed so far? Round to the nearest whole percent.

    a) 55%   b) 57%   c) 60%   d) 67%   e) Insufficient information

26. For the equation $4^x = 32^y$, which of the following must be true about the relationship between x and y?

    a) x is 2 less than y   b) x is 2 more than y   c) x is 150% of y
    d) x is 150% more than y   e) x is 40% of y

27. For the equation $\sqrt{2x-3} - \sqrt{x+7} = 2$, find the extraneous root.

    a) 1   b) 2   c) 7   d) 42   e) 114

28. Every minute, twice as many rabbits drink one ounce of water as in the minute before. During the third minute, there are 800 rabbits that each drink 1 ounce. How many minutes will it take for the rabbits to drink a **total** of 10,000 gallons? Note that there are 128 fluid ounces per gallon. Round **up** to the next whole minute.

    a) 6   b) 8   c) 11   d) 13   e) 14

29. Melanie is standing at point A on the bank of a 2.5 km wide river and wants to reach point B, which is 15 km downstream on the opposite bank. She plans to row directly to a point C on the opposite shore and then jog to point B, as shown in the figure below. She rows at a rate of 4 km/hr and jogs at a rate of 8 km/hr. If the entire trip takes her 3 hours, how many minutes will she spend jogging? Round to the nearest minute. The figure is not drawn to scale.

A

River

D   C   B
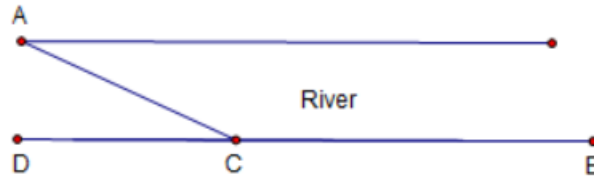
a) 51 min   b) 60 min   c) 89 min   d) 108 min   e) 120 min

30. The eight contestants at a local ping pong tournament, Amy (f), Bob (m), Cheri (f), Dave (m), Erin (f), Fred (m), Gina (f), and Hal (m), each played three rounds of ping pong. No one faced the same opponent twice, and every game ended with one winner and one loser. Only one person won all three games (and therefore won the tournament), and only one person lost all three. The following are also true:

I.   Hal won his first game against Bob, but lost to Dave and then to Cheri.
II.   Fred lost to Gina and then to Amy, but won his last game against Bob.
III. Erin lost to women in rounds 1 and 2, but beat a woman in round 3.
IV. The women had two more victories overall than the men.
V.   Two games each round had same-gender pairings.
VI. Gina won her second match.

Who won the tournament?

a) Amy   b) Cheri   c) Dave   d) Gina   e) Insufficient information