WYSE – Academic Challenge
Physics Solutions (State) – 2018

1. **Correct Response: E**

Both work and torque have dimensions

$$\frac{\mathrm{M}\cdot\mathrm{L}^2}{\mathrm{T}^2}$$

2. **Correct Response: C**

$$(30.0\ \mathrm{km/hr})\left(\frac{1000\ \mathrm{m}}{\mathrm{km}}\right)\left(\frac{100\ \mathrm{cm}}{\mathrm{m}}\right)\left(\frac{1.00\ \mathrm{in}}{2.54\ \mathrm{cm}}\right)\left(\frac{1\ \mathrm{hr}}{3600\ \mathrm{s}}\right) = 328\ \mathrm{in/s}$$

3. **Correct Response: A**

Using a coordinate system with the +*x* direction toward the east and the +*y* direction toward the north

$$\mathbf{d}_{BA} = \mathbf{r}_B - \mathbf{r}_B = 800\ \mathrm{m}\ \hat{\mathbf{j}} - 600\ \mathrm{m}\ \hat{\mathbf{i}}$$

$$d_{BA} = \sqrt{d_{BAx}^2 + d_{BAy}^2} = \sqrt{(-600\ \mathrm{m})^2 + (800\ \mathrm{m})^2} = 1000\ \mathrm{m}$$

$$\theta_{BA} = \tan^{-1}\frac{d_{BAy}}{d_{BAx}} + (180°\ if\ d_{BAx} < 0)$$

$$= \tan^{-1}\frac{800\ \mathrm{m}}{-600\ \mathrm{m}} + 180° = 126.9°\ (\mathrm{CCW\ from + x\ axis}) = 36.9°\ \mathrm{west\ of\ north}$$

4. **Correct Response: C**

In the reference frame of the speeding car, the position of the speeding car is always zero. The position of the patrol car in that reference frame is

$$x_{pc} = x_{pc0} + v_{pc0}t + \tfrac{1}{2}a_{pc}t^2 \quad \Longrightarrow \quad 0 = 0 - \left(40.0\frac{\mathrm{m}}{\mathrm{s}}\right)t + \tfrac{1}{2}\left(3.00\frac{\mathrm{m}}{\mathrm{s}^2}\right)t^2$$

$$\Longrightarrow \quad t = 0 \quad \text{or} \quad t = \frac{40.0\frac{\mathrm{m}}{\mathrm{s}}}{\frac{1}{2}\left(3.00\frac{\mathrm{m}}{\mathrm{s}^2}\right)} = 26.7\ \mathrm{s}$$

5. **Correct Response: A**

Using a coordinate system with the +x direction toward the east and the +y direction toward the north,

$$\mathbf{v}_{DT} = \mathbf{v}_D - \mathbf{v}_T \quad \Longrightarrow \quad v_{DTx} = v_{Dx} - v_{Tx} \quad and \quad v_{DTy} = v_{Dy} - v_{Ty} \quad \Longrightarrow$$

$$v_{DTx} = (30.0\ \mathrm{knots})\cos 225° - (20.0\ \mathrm{knots})\cos 60.0° = -31.2\ \mathrm{knots}$$

$$v_{DTy} = (30.0\ \mathrm{knots})\sin 225° - (20.0\ \mathrm{knots})\sin 60.0° = -38.5\ \mathrm{knots}$$

$$v_{DT} = \sqrt{v_{DTx}^2 + v_{DTx}^2} = \sqrt{(-31.2\ \mathrm{knots})^2 + (-38.5\ \mathrm{knots})^2} = 49.6\ \mathrm{knots}$$

$$\theta_{DT} = \tan^{-1}\left(\frac{v_{DTy}}{v_{DTx}}\right) + 180° = \tan^{-1}\left(\frac{-38.5\ \mathrm{knots}}{-31.2\ \mathrm{knots}}\right) + 180° = 231.0°\ \ (51.1°\ \mathrm{S\ of\ W})$$

6. **Correct Response: D**

$$y = y_0 + v_0 t + \tfrac{1}{2} a_y t^2 \quad \Longrightarrow$$

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 2a_y(y_0 - y)}}{a_y} \quad \Longrightarrow$$

$$t = \frac{-20.0\ \text{m/s} \pm \sqrt{(20.0\ \text{m/s})^2 - 2(-9.80\ \text{m/s}^2)(2.00\ \text{m} - 0\ \text{m})}}{(-9.80\ \text{m/s}^2)} = 4.18\ \text{s} \quad \text{or} \quad -0.099\ \text{s}$$

7. **Correct Response: C**

$$v^2 - v_0^2 = 2a(y - y_0) \quad \Longrightarrow \quad y = y_0 + \frac{v^2 - v_0^2}{2a} = 2.00\ \text{m} + \frac{0 - (20.0\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 22.4\ \text{m}$$

8. **Correct Response: B**

Recalling that an equation of constant accelerated motion is

$$x(t) = x_0 + v_0 t + \tfrac{1}{2} a t^2 \quad \Longrightarrow \quad \tfrac{1}{2} a = 2.00\ \text{m/s}^2 \quad \Longrightarrow \quad a = 4.00\ \text{m/s}^2$$

9. **Correct Response: B**

This is a two-body problem. For the sliding mass, use a coordinate system with the $+x_b$ direction parallel to and up the inclined plane. For the mass *m*, use a coordinate system with the $+y_m$ direction vertically upward. Then the constraint of the string is

$$a_{xb} = -a_{ym}$$

$$\text{6.00 kg block:} \quad \sum F_{on\ Bx} = m_b a_{xb} \quad \Longrightarrow \quad -m_b g \sin 30.0° + T = m_b a_{xb}$$

$$\text{mass m:} \quad \sum F_{on\ mx} = m_m a_{ym} \quad \Longrightarrow \quad -m_m g + T = -m_m a_{xb}$$

$$\Longrightarrow \quad m_b g \sin 30.0° - m_m g = -m_b a_{xb} - m_m a_{xb} \quad \Longrightarrow \quad m_m = \frac{m_b g \sin 30.0° + m_b a_{xb}}{g - a_{xb}}$$

$$\Longrightarrow \quad m_m = \frac{(6.00\ \text{kg})(9.80\ \text{m}/s^2)\sin 30.0° + (6.00\ \text{kg})(2.00\ \text{m}/s^2)}{(9.80\ \text{m}/s^2) - (2.00\ \text{m}/s^2)} = 5.31\ \text{kg}$$

10. **Correct Response: B**

$$W = \Delta K + \Delta U \quad \Longrightarrow \quad -\mu m g d \cos\theta = \tfrac{1}{2} m (v^2 - v_0^2) + m g \Delta h \quad \Longrightarrow$$

$$\mu = -\frac{\tfrac{1}{2} m (v^2 - v_0^2) + m g \Delta h}{m g d \cos\theta} = \frac{g d \sin\theta - \tfrac{1}{2}(v^2 - v_0^2)}{g d \cos\theta}$$

$$\mu = \frac{(9.8\ \text{m/s}^2)(1.50\ \text{m})\sin 30.0° - \tfrac{1}{2}[(3.00\ \text{m/s})^2 - (1.00\ \text{m/s})^2]}{(9.8\ \text{m/s}^2)(1.50\ \text{m})\cos 30.0°} = 0.263$$

11. **Correct Response: E**

The drag force of the parachute is in the upward direction and has a magnitude

$$F_{parachute} = \tfrac{1}{2}C_d\rho A v^2 = \tfrac{1}{2}(0.800)(1.29\ \text{kg/m}^3)(30.0\ \text{m}^2)(20.0\ \text{m/s}^2)^2 = 6.19 \times 10^3\ \text{N}$$

Using a coordinate system with upward as the positive *y* direction,

$$\sum F_y = ma_y \quad \Longrightarrow \quad F_{parachute} - mg = ma_y$$

$$\Longrightarrow \quad a_y = \frac{F_{parachute}}{m} - g = \frac{6.19 \times 10^3\ \text{N}}{80.0\ \text{kg}} - 9.80\ \text{m/s}^2 = 67.6\ \text{m/s}^2$$

12. **Correct Response: D**

At terminal velocity, the net force is zero.

$$\tfrac{1}{2}C_d\rho A v^2 - mg = 0 \quad \Longrightarrow \quad v = \sqrt{\frac{2mg}{C_d\rho A}} = \sqrt{\frac{2(80.0\ \text{kg})(9.80\ \text{m/s}^2)}{(0.800)(1.29\ \text{kg/m}^3)(30.0\ \text{m}^2)}} = 7.12\ \text{m/s}$$

13. **Correct Response: B**

There are no external forces acting on the system, so the center of mass moves with constant velocity. From the initial conditions, the center of mass velocity is

$$v_{cm} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{(4.00\ \text{kg})(2.00\ \text{m/s}) + (3.00\ \text{kg})(-4.00\ \text{m/s})}{(4.00\ \text{kg}) + (3.00\ \text{kg})} = -0.5714\ \text{m/s}$$

$$x_{cm} - x_{0cm} = v_{cm}t = (-0.5714\ \text{m/s})(3.00\ \text{s} + 6.00\ \text{s}) = -5.14\ \text{m}$$

14. **Correct Response: E**

A totally elastic collision is one in which the total kinetic energy of the system is the same after the collision as it was before the collision.

15. **Correct Response: D**

The T in the units represents the metric prefix tera = 10^{12}.

$$1\ \text{TeV} = 10^{12}\ \text{eV}$$

16. **Correct Response: D**

$$a = \sqrt{a_{centripetal}^2 + a_{tangential}^2} = \sqrt{\frac{v^2}{R} + a_{tangential}^2} = \sqrt{\left[\frac{(10.0\ \text{m/s}^2)}{25.0\ \text{m}}\right]^2 + [3.00\ \text{m/s}^2]^2}$$

$$= 5.00\ \text{m/s}^2$$

17. **Correct Response: B**

$$\Delta K + \Delta U = 0 \quad \Longrightarrow \quad \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + mgh_2 - mgh_1 + \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2 = 0$$

$$0 - 0 + (3.00\text{ kg})\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)(-d - 0.40\text{ m}) + \tfrac{1}{2}(2000\ \text{N/m})(d^2 - 0) = 0 \ \Longrightarrow$$

$$(1000\ \text{N/m})d^2 - (29.4\text{ N})d - 11.76\text{ N}\cdot\text{m} = 0 \quad \Longrightarrow$$

$$d = \frac{29.4\,N \pm \sqrt{(29.4\text{ N})^2 - 4(1000\ \text{N/m})(-11.76\text{ N}\cdot\text{m})}}{2(1000\ \text{N/m})} = \quad 0.124\text{ m} \quad \text{or} \quad -0.095\text{ m}$$

18. **Correct Response: A**

As the initial object is at rest, the total momentum before the explosion is zero. By conservation of momentum, the total momentum after the explosion must also be zero.

$$\sum \mathbf{p} = 0 \quad \Longrightarrow \quad m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + m_3\mathbf{v}_3 = 0 \quad \Longrightarrow \quad \mathbf{v}_3 = -\frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_3}$$

$$\mathbf{v}_3 = -\frac{(3.00\text{ kg})\left(200\,\hat{\mathbf{i}} + 300\,\hat{\mathbf{j}} + 200\,\hat{\mathbf{k}}\right)\text{m/s} + (2.00\text{ kg})\left(\left(100\,\hat{\mathbf{i}} - 400\,\hat{\mathbf{j}} - 200\,\hat{\mathbf{k}}\right)\text{m/s}\right)}{(4.00\text{ kg})}$$

$$\mathbf{v}_3 = \left(-200\,\hat{\mathbf{i}} - 25\,\hat{\mathbf{j}} - 50\,\hat{\mathbf{k}}\right)\text{m/s}$$

19. **Correct Response: E**

In static equilibrium, the net torque on the disk is zero. Using counterclockwise as the positive rotation direction,

$$\sum \tau = 0 \quad \Longrightarrow \quad mgR\sin 90.0° - F(R/2)\sin 60.0° = 0$$

$$\Longrightarrow \quad F = \frac{2mg\sin 90.0°}{\sin 60.0°} = \frac{2(20.0\text{ kg})(9.80\ \text{m/s}^2)\sin 90.0°}{\sin 60.0°} = 453\text{ N}$$

20. **Correct Response: B**

$$I = I_{cm} + md^2 \quad \Longrightarrow \quad d = \sqrt{\frac{I - I_{cm}}{m}} = \sqrt{\frac{1.200\text{ kg}\cdot\text{m}^2 - 0.800\text{ kg}\cdot\text{m}^2}{2.00\text{ kg}}} = 0.447\text{ m}$$

21. **Correct Response: C**

$$g = \frac{GM_e}{R_e^2} = \frac{GM_{planet}}{R_{planet}^2} = \frac{G(4.00)M_e}{R_{planet}^2} \quad \Longrightarrow \quad R_{planet} = \sqrt{(4.00)R_e^2} = 2.00R_e$$

22. **Correct Response: D**

$$P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \tfrac{1}{2}\rho v_2^2 \quad \Longrightarrow \quad P_2 = P_1 + \rho g(h_1 - h_2) + \tfrac{1}{2}\rho(v_1^2 - v_2^2)$$

As the fluid is incompressible and the cross-section of the pipe is uniform, $v_1 = v_2$.

$$P_2 = 200 \times 10^3\text{ Pa} + (5.00 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(2.00\text{ m} - [-8.00\text{ m}]) =$$

$$= 6.90 \times 10^5\text{ Pa} = 690\text{ kPa}$$

23. **Correct Response: C**

$$V_L = I_L X_L = I_L \omega L = (2.00\text{ A})(80.0\text{ s}^{-1})(50.0\ \times 10^{-3}\text{ H}) = 8.00\text{ V}$$

24. **Correct Response: D**

$$R_{eq} = 2.00\ \Omega + 6.00\ \Omega = 8.00\ \Omega \quad \Longrightarrow \quad I = \frac{V}{R_{eq}} = \frac{40.0\text{ V}}{8.00\ \Omega} = 5.00\text{ A}$$

$$V_{6.00\ \Omega} = I(6.00\ \Omega) = (5.00\text{ A})(6.00\ \Omega) = 30.0\text{ V}$$

25. **Correct Response: D**

$$\begin{aligned}\mathbf{E} &= \frac{kq_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{kq_2}{r_2^2}\hat{\mathbf{r}}_2 \\ &= \frac{\left(8.99\times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(20.0\times 10^{-6}\text{ C})}{(40.0\times 10^{-2}\text{ m})^2}(-\hat{\mathbf{i}}) + \frac{\left(8.99\times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(-50.0\times 10^{-6}\text{ C})}{(60.0\times 10^{-2}\text{ m})^2}(\hat{\mathbf{i}}) \\ &= -2.38\times 10^6\ \text{N/C}\ \hat{\mathbf{i}}\end{aligned}$$

26. **Correct Response: D**

$$\Delta U = q\Delta V = kq\left\{q_1\left(\frac{1}{r_{1f}} - \frac{1}{r_{1i}}\right) + q_2\left(\frac{1}{r_{2f}} - \frac{1}{r_{2i}}\right)\right\} \quad \Longrightarrow$$

$$\begin{aligned}\Delta U &= \\ &= \left(8.99\times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(30.0\times 10^{-6}\text{ C})\left\{(20.0\times 10^{-6}\text{ C})\left(\frac{1}{30.0\times 10^{-2}\text{ m}} - \frac{1}{30.0\times 10^{-2}\text{ m}}\right)\right. \\ &\qquad \left. + (-50.0\times 10^{-6}\text{ C})\left(\frac{1}{130.0\times 10^{-2}\text{ m}} - \frac{1}{70.0\times 10^{-2}\text{ m}}\right)\right\} = 8.90\text{ J}\end{aligned}$$

27. **Correct Response: B**

$$m\lambda = a\sin\theta = \frac{ay}{L} \quad \Longrightarrow \quad d = y_{+1} - y_{-2} = \frac{(+1)\lambda L}{a} - \frac{(-2)\lambda L}{a}$$

$$\Longrightarrow \quad a = \{(+1) - (-2)\}\frac{\lambda L}{d} = 3\,\frac{(600\times 10^{-9}\text{ m})(2.00\text{ m})}{(5.00\times 10^{-3}\text{ m})} = 7.20\times 10^{-4}\text{ m} = 720\ \mu\text{m}$$

28. **Correct Response: E**

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \quad \textit{and} \quad m = -\frac{s'}{s} \quad \Longrightarrow \quad m = \frac{f}{f-s} = \frac{30.0\text{ cm}}{30.0\text{ cm} - 20.0\text{ cm}} = 3.00$$

29. **Correct Response: D**

$$P = \frac{Q}{\Delta t} = \frac{mC\Delta T}{\Delta t} = (2.00\text{ kg})(800\ \text{J/kg/C}^\circ)(2.00\ \text{C}^\circ\text{/min})(1.00\ \text{min/60 s}) = 53.3\text{ W}$$

30. **Correct Response: E**

$$|EMF| = \left|\frac{\Delta\phi_B}{\Delta t}\right| = \left|\frac{\Delta B \cdot A \cdot \cos\theta}{\Delta t}\right| = \left|\frac{\Delta B \cdot A \cdot \cos\theta}{\Delta t}\right| = \left|\frac{(0 - 3.00\ \mathrm{T})\pi(0.200\ \mathrm{m})^2}{40.0 \times 10^{-3}\ \mathrm{s}}\right| = \ 9.42\ \mathrm{V}$$

31. **Correct Response: B**

$$\frac{P_1V_1}{P_2V_2} = \frac{n_1RT_1}{n_2RT_2} \quad \Longrightarrow \quad P_2 = \frac{P_1V_1}{V_2}\frac{n_2RT_2}{n_1RT_1} = \frac{P_1V_1}{V_2} = \frac{PV}{2.00V} = 0.500P$$

32. **Correct Response: A**

The change in internal energy of an ideal gas is proportional to the change in absolute temperature. The process described is isothermal, meaning at constant temperature. As there is no change in temperature, there is no change in internal energy.

33. **Correct Response: E**

Each of the phenomena listed in responses a. through d. cannot be explained in terms of purely classical physics.

34. **Correct Response: C**

$$L = \frac{L_0}{\gamma} = L_0\sqrt{1 - \frac{v^2}{c^2}} \quad \Longrightarrow \quad v = c\sqrt{1 - \frac{L^2}{{L_0}^2}} = c\sqrt{1 - \frac{(60.0\ \mathrm{m})^2}{(80.0\ \mathrm{m})}} = 0.661c$$

35. **Correct Response: E**

$$\lambda = \frac{1}{\tau} = \frac{1}{T_{1/2}/\ln 2} = \frac{\ln 2}{T_{1/2}}$$