Academic Challenge

EASTERN ILLINOIS UNIVERSITY

2019 Academic Challenge

MATHEMATICS TEST – REGIONAL

Physics Test Production Team

Kevin Boyer, Illinois State University – Author/Team Leader
Matthew Childers, Illinois State University – Author
Linda Wiggins, Illinois State University – Author
Ryan Bunge, Illinois State University – Reviewer

GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as ● , not ⊙ , ⊕ , ◑ , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

Time: 40 Minutes Number of Questions: 30

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

Academic Challenge
Mathematics Test (Regional) – 2019

1. Let $x = \sin(\theta) + 2$, $y = \cos(\theta) - 3$ for parameter θ. Determine which of the following Cartesian equations model the same curve.

   a) $x^2 + y^2 - 4x + 6y + 12 = 0$
   b) $x^2 - y^2 - 4x - 6y - 6 = 0$
   c) $-x^2 + y^2 + 4x - 6y + 4 = 0$
   d) $2x^2y^2 - 48xy + 35 = 0$
   e) $-x^2y^2 + y^2 + 24xy - 18 = 0$

2. Which of the following is true about the graph of $r(x) = \dfrac{12x^2 - 44x + 2}{x^2 - 11x + 9}$?

   a) It has exactly one vertical asymptote.
   b) It has exactly two vertical asymptotes.
   c) It has an oblique asymptote.
   d) It has a horizontal asymptote.
   e) More than one of these are true.

3. For the arithmetic sequence $-13, -5, 3, 11, \ldots$, find the sum of the first twenty-one terms.

   a) 139 b) 147 c) 777 d) 1407 e) 1491

4. Determine the distinguishable ways to rearrange the letters of "ACRIMONIOUS".

   a) 5,040 b) 362,880 c) 3,628,800
   d) 9,979,200 e) 39,916,800

5. Determine the number of vertices of a dodecahedron.

   a) 12 b) 20 c) 30 d) 40 e) 60

6. Suppose $g(x) = \frac{1}{2} f(x - 1)$. State the graph transformations that would convert the graph of f(x) into the graph of g(x).

   a) The graph shifts right by $\frac{1}{2}$ unit and stretches horizontally.
   b) The graph shifts left by $\frac{1}{2}$ unit and stretches vertically.
   c) The graph stretches vertically and shifts right 1 unit.
   d) The graph compresses horizontally and shifts left 1 unit.
   e) The graph compresses vertically and shifts right 1 unit.

7. What is the sum of the four solutions of $x^4 - 64 = 0$?

   a) –8 b) $1 + 2\sqrt{2}i$ c) $1 - 2\sqrt{2}i$ d) 0 e) 2

8. Given $A = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & 0 \\ 0 & -1 & k \end{bmatrix}$, find det(A).

a) $-k-10$ b) $k+10$ c) k d) $-k$ e) $-3k+10$

9. What is the name of the shape described by the polar equation $r = 3 - 3\cos\theta$?

a) lemniscate b) rose c) cardioid d) ellipse e) None of these

10. A 15 foot-tall ladder is leaning up against a wall so that it makes a 9-degree angle with the wall. How far away from the wall is the base of the ladder? Round your answer to the nearest tenth of a foot.

a) 2.3 feet b) 6.0 feet c) 6.2 feet d) 12.0 feet e) 14.8 feet

11. You place $16,000 in a bank account that yields 6.4% interest per year compounded quarterly in order to save enough money to buy a new $18,000 car. How long will you have to leave the money in the account to reach your goal? Round to the nearest tenth of a year.

a) 1.3 yrs b) 1.7 yrs c) 1.9 yrs d) 1 yr e) 2.3 yrs

12. If log (x + 9) + log x = 1, which of these is a solution? Round to two decimal places.

a) –10 b) –8.89 c) –0.11 d) 1 e) More than one of these

13. A prince wants to climb up the tower to meet Rapunzel. The prince notes that when he is 50 feet from the tower, the angle of inclination from the ground to Rapunzel's window is $40°$. How long must Rapunzel's hair be if it must reach down the tower from the window to the ground? Round to the nearest foot.

a) 28 ft b) 32 ft c) 38 ft d) 42 ft e) 48 ft

14. What is the length of the latus rectum of $y = 4x^2 + 3x - 1$? Round your answer to two decimal places.

a) 0.06 b) 0.25 c) 0.50 d) 1.00 e) 4.00

15. The value of a parcel of land was $100,000 in 1990, $120,000 in 2000, and $144,000 in 2010. If this rate of exponential growth continues, in what year should the value of the parcel be $200,000? Round to the closest year.

a) 2028 b) 2033 c) 2035 d) 2038 e) 2040

16. The probability that a Netflix viewer watches “Anne with an E” is 2/3. The probability that a viewer watches “Blue Bloods” is 4/9. If the probability that a viewer watches both shows is 14/45, what is the probability that a viewer watches at least one of these two shows?

a) $\frac{4}{5}$ b) $\frac{8}{9}$ c) $\frac{26}{45}$ d) $\frac{2}{5}$ e) $\frac{38}{45}$

17. An executive committee is made up of 5 people selected randomly from a group of 10 men and 6 women. How many possible committees have more women than men?

a) 41 b) 150 c) 900 d) 1,050 e) 1,056

18. Find the remainder produced by performing the operation $\left(8x^3 - 12x + 3\right) \div \left(2x - 3\right)$.

a) 0 b) 3 c) 6x + 3 d) –6 e) 12

19. In a series of games, the 3 players in group A scored an average of 43 points per game. The 5 players in group B scored an average of 15 points per game. The players in group C scored 10 points per game. The players of the three groups scored a combined average of 14 points per game. How many players are there total in the three groups?

a) 15 b) 19 c) 21 d) 23 e) 31

20. Two friends, Sam and Pat, each pay the same dollar amount of rent for their apartment. Sam’s rent is 40% of her monthly paycheck. Pat makes more money and her rent is 25% of her monthly paycheck. The combined rent is what percent of the combined amount of Sam and Pat’s paychecks? Round your answer to the nearest tenth of a percent.

a) 30.0% b) 30.8% c) 32.5% d) 33.3% e) 34.2%

21. Determine the type of conic section produced by the equation $y^2 = 9x^2 - 9$.

a) Ellipse b) Hyperbola c) Circle d) Parabola e) Cone

22. Given 30 gallons of 62% acid and as many gallons of 10% acid as you need, what is the greatest whole number of gallons of 42% acid that you can create?

a) 22 b) 30 c) 48 d) 72 e) 80

23. Six friends stand out in an open field so that they form a trapezoid. Amy and Don form the lower base of the trapezoid and stand twice the distance as Bob and Cat who form the upper base of the trapezoid. Ron and Sue stand 24 yards apart and represent the median of the trapezoid. How far apart do Amy and Don stand?

a) 26 yards b) 28 yards c) 30 yards d) 32 yards e) 36 yards

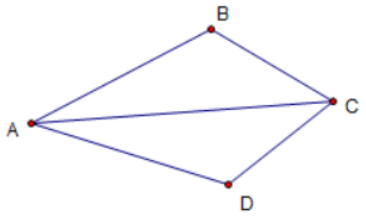
24. What is the phase shift of $y = 5 + 3\cos(2t - 4)$? If necessary, round to the nearest integer.

a) 4 b) 2 c) π d) 2π e) 4π

25. City A and City B are 100 miles apart. At 9:00, a truck leaves City A and heads toward City B at 40 mph. Thirty minutes later (at 9:30), a car leaves City A and heads toward City B at 60 mph. Does the car meet the truck before the truck reaches City B? If it does, determine what time the car and truck meet. Round to the nearest whole minute.

a) No, the car does not meet the truck in time.
b) Yes, the car meets the truck at 9:48.
c) Yes, the car meets the truck at 10:00.
d) Yes, the car meets the truck at 10:18.
e) Yes, the car meets the truck at 10:30.

26. The four vertices of a kite are A, B, C, and D. $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$. $m\angle BCD = 2m\angle BAD$. If $m\angle ABC = 120°$, find $m\angle BAC$. The diagram is not to scale.



a) 20° b) 40° c) 60° d) 80° e) 100°

27. When the solution for t of the equation $\frac{2t - 4x}{4y + z} = \frac{t + 3x}{2y + 3z}$ is written as a simplified single fraction, how many terms are in the denominator?

a) 0 b) 1 c) 2 d) 3 e) 4

28. Find the surface area of a ball whose equatorial circumference is 25.15 cm. Round to the nearest square centimeter.

a) $280\,cm^2$ b) $269\,cm^2$ c) $244\,cm^2$ d) $228\,cm^2$ e) $201cm^2$

29. What is $\lim_{x\to\infty} \frac{x^2 - 2x + 5}{-x - 1}$?

a) 0 b) 1 c) -5 d) ∞ e) $-\infty$

30. At a recent charity Olympics, eight girls from three families were paired up into four teams to compete in four events – a tug of war, a pie eating contest, thumb wrestling, and a three-legged race. Points were awarded for each of the various events, and a prize was awarded to the team with the highest total.

I. The eight girls were Amy, Beth, Claire, Denise, Emily, Faith, Grace, and Hannah; the three families were Jones, Smith, and Miller; and no sisters were paired together.

II. The tug of war was 2 vs 2. Faith Jones's team got 2 points for beating her oldest sister's team, and Amy Smith's team got 2 points for beating her twin sister's team.

III. One girl from each team competed in the pie eating contest. Beth got 2 points for 2^{nd}, behind her teammate's middle sister who got 3 points for 1^{st}, but ahead of her middle sister's teammate who got 1 point for 3^{rd} and her oldest sister's teammate who got 0 for 4^{th}.

IV. The thumb wrestling contest paired the other four girls. Hannah Miller got 2 points for beating her sister Claire, and Amy got 2 points for beating Grace Jones.

V. In the three-legged race, the oldest Miller girl and her teammate got 3 points for 1^{st}, Grace and her teammate got 2 points for 2^{nd}, and Emily and her teammate got 1 point for coming in 3^{rd}, just ahead of Claire and Denise who got 0 for 4^{th}.

Determine who was on the team that won the prize for having the highest point total.

a) Amy and Emily
b) Beth and Grace
c) Faith and Hannah
d) There was a tie for first between two teams.
e) Not enough information is given.