

WYSE – Academic Challenge
Mathematics Solutions (Regional) – 2019

1. Ans A: First rearrange $x = \sin\theta + 2 \Rightarrow \sin\theta = x - 2$; $y = \cos\theta - 3 \Rightarrow \cos\theta = y + 3$. Then the identity $\sin^2\theta + \cos^2\theta = 1$ becomes $(x - 2)^2 + (y + 3)^2 = 1$, which simplifies to $x^2 + y^2 - 4x + 6y + 12 = 0$.
2. Ans E: Since the numerator and denominator share no factors, it is in reduced form. Since the degrees of the numerator and denominator are the same, the graph has a horizontal asymptote. Since the denominator has two zeroes, the function has two vertical asymptotes. This means b and d are both true, making the correct answer e.
3. Ans D: An arithmetic sequence of the form $a_n = a_0 + r \cdot n$ has a sum of $S_n = \frac{n+1}{2}(a_0 + a_n)$. We have been given $a_0 = -13$ and $r = 8$. Because we start at $n = 0$, the twenty-first term would be $n = 20$, giving us $a_{20} = -13 + 8(20) = 147$ and $S_{20} = \frac{21}{2}(-13 + 147) = 1407$.
4. Ans D: There are two repeated letters in acrimonious, each of which is repeated only once, so we divide $11!$ by $2!$ and then $2!$ again to get $11!/(2!2!) = 9,979,200$.
5. Ans B: A dodecahedron has 12 pentagonal faces, each with five 2D face vertices. This yields a total count of $5 \cdot 12 = 60$ "2D face vertices". Because three faces meet at each 3D vertex of the solid, the number of 3D vertices for the solid is $60/3 = 20$.
6. Ans E: In $\frac{1}{2}f(x - 1)$, the leading factor $\frac{1}{2}$ is between 0 and 1, which represents a vertical compression. The "1" in $(x - 1)$ implies that the graph shifts 1 unit to the right.
7. Ans D: Factor the left side: $(x^2 - 8)(x^2 - (-8)) = (x + \sqrt{8})(x - \sqrt{8})(x + i\sqrt{8})(x - i\sqrt{8})$. The four solutions $-\sqrt{8}$, $\sqrt{8}$, $-i\sqrt{8}$, and $i\sqrt{8}$ add up to 0.
8. Ans B: $\det A = 1 \begin{vmatrix} 1 & 0 \\ -1 & k \end{vmatrix} - 0 \begin{vmatrix} -2 & 0 \\ 0 & k \end{vmatrix} + 5 \begin{vmatrix} -2 & 1 \\ 0 & -1 \end{vmatrix} = 1(k - 0) - 0 + 5(2 - 0) = k + 10$.
9. Ans C: Because the constant term and the coefficient of the cosine term are same magnitude and opposite sign, this is the equation of a cardioid.
10. Ans A: If x is the distance from the base of the ladder to the wall, $\sin 9 = \frac{x}{15}$, $x \approx 2.3465$.
11. Ans C: Given $18,000 = 16,000 \left(1 + \frac{0.064}{4}\right)^{4t}$, $t = \frac{\ln\left(\frac{18,000}{16,000}\right)}{4 \ln\left(1 + \frac{0.064}{4}\right)} \approx 1.855 \approx 1.9$.

12. Ans D: combine the two logarithms and multiply the arguments to get $\log(x^2 + 9x) = 1$.
If we raise 10 to the power of both sides, we get $x^2 + 9x = 10$. Solving $x^2 + 9x - 10 = 0$ gives zeroes of -10 and 1 . However, -10 is not a solution of the original equation.
13. Ans D: If x is the length of Rapunzel's hair, then $\tan(40^\circ) = \frac{x}{50}$;
 $x = 50 \tan(40^\circ) = 41.95 \approx 42$.
14. Ans B: The coefficient of x^2 indicates this equation is a translated version of $y = 4x^2$, which has the same length latus rectum. Knowing that our equation has the specific form $y = 4(x - \frac{3}{8})^2 - \frac{25}{16}$ isn't necessary. Rewrite $y = 4x^2$ into $x^2 = \frac{1}{4}y$. In that form, the length of the latus rectum is the same as the coefficient of y .
15. Ans A: The periodic growth factor is $144000/120000 = 120000/100000 = 1.2$ every 10 years. If we let x be the number of years after 2000, then we can find when the parcel of land will be worth \$200,000 by solving $200000 = 120000(1.2^{0.1x})$. This gives a value of $x = \frac{\log\left(\frac{200000}{120000}\right)}{0.1 \cdot \log 1.2} \approx 28.02$, which is in the year 2028.
16. Ans A: Let A be the event that the viewer watches "Anne with an E" and B is the event that the viewer watches "Blue Bloods". Then $P(A) = \frac{2}{3}$, $P(B) = \frac{4}{9}$, and $P(A \cap B) = \frac{14}{45}$.
This means $P(A \cup B) = \frac{2}{3} + \frac{4}{9} - \frac{14}{45} = \frac{4}{5}$.
17. Ans E: The committee will have three, four, or five women with two, one, or zero men respectively completing the committee. Three women and two men can be selected in $C(6, 3) * C(10, 2) = 900$ possible ways. Four women and one man can be selected in $C(6, 4) * 10 = 150$ possible ways. Five women (and no men) can be selected $C(6, 5) = 6$ ways. Add these up to get 1,056 total possible committees.
18. Ans E: Using polynomial long division, we get:
- $$\begin{array}{r} 4x^2 + 6x + 3 \\ 2x - 3 \overline{) 8x^3 + 0x^2 - 12x + 3} \\ \underline{-(8x^3 - 12x^2)} \\ 12x^2 - 12x + 3 \\ \underline{-(12x^2 - 18x)} \\ 6x + 3 \\ \underline{-(6x - 9)} \\ 12 \end{array}$$

19. Ans E: If x is the players in group C, the total points is $43 \cdot 3 + 15 \cdot 5 + 10x = 204 + 10x$, the number of players is $3 + 5 + x = 8 + x$, and the overall average is $\frac{204 + 10x}{8 + x} = 14$, which simplifies to $204 + 10x = 112 + 14x$. Continue solving to get $10x = -92 + 14x$, which becomes $-4x = -92$, so $x = 23$. The number of players is $8 + x$, which equals 31.
20. Ans B: Let R be the combined rent, S be Sam's paycheck, and P be Pat's paycheck, giving a combined paycheck total of $S + P$. Based on $0.4S = 0.5R$ and $0.25P = 0.5R$, we get $S = 1.25R$, $P = 2R$, and the combined check total is $S + P = 3.25R$. The rent per combined paycheck percentage is thus $R / 3.25R \approx 0.30769$, or $\approx 30.8\%$.
21. Ans B: Rearrange the equation to $\frac{x^2}{1} - \frac{y^2}{9} = 1$, which is the standard form of a hyperbola.
22. Ans C: If x is the number of gallons of 42% acid, we use $x - 30$ gallons of 10% acid and all 30 gallons of the 62% acid. The gallons of acid in the 42% acid would be $0.42x$, the gallons of acid coming from the 62% acid would be $0.62(30) = 18.6$, and the gallons from the 10% acid would be $0.10(x - 30) = 0.10x - 3$. So $0.42x = 18.6 + 0.10x - 3$, and $0.32x = 15.6$. Thus we can make 48.75 gallons of 42% acid. Round down to 48.
23. Ans D: Let AD represent the distance between Amy and Don and BC represent the distance between Bob and Cat. Then $AD = 2BC$. Let RS represent the distance between Ron and Sue. Then $RS = \frac{1}{2}(2BC + BC) = \frac{3}{2}BC$. Since RS is 24, $BC = 16$ and $AD = 32$.
24. Ans B: For $y = a \cos(k(t - b)) + d$, the phase shift is b . This gives a phase shift of 2.
25. Ans E: The truck will arrive at City B at $100/40 = 2.5$ hours after 9:00, or 11:30. The car will arrive at City B at $100/60 = 1\frac{2}{3}$ hours or an hour and 40 minutes, after 9:30, which is 11:10. The car does catch up to the truck. To find the time, solve $40T = 60C$, where T is hours after 9:00 and C is hours after 9:30. Substitute $T = C + 0.5$, which will give us $40(C + 0.5) = 60C$. Solving gives us $C = 1$ hour after 9:30, which is 10:30.
26. Ans A: $2m\angle BAC = m\angle BCA$ by definition of angle bisector. The triangle sum gives us $3m\angle BAC + 120^\circ = 180^\circ$. Then $m\angle BAC = 20^\circ$.
27. Ans B: Cross multiply to get $(2t - 4x)(2y + 3z) = (t + 3x)(4y + z)$.
Expand to get $4ty + 6tz - 8xy - 12xz = 4ty + tz + 12xy + 3xz$.
Subtract $4ty$ and tz from both sides to get $5tz - 8xy - 12xz = 12xy + 3xz$.
Add $8xy$ and $12xz$ to both sides to get $5tz = 20xy + 15xz$.
Finally, divide both sides by $5z$ to get $t = \frac{4xy + 3xz}{z}$, which has one denominator term.
28. Ans E: $2\pi r = 25.15 \Rightarrow r = \frac{25.15}{2\pi} \approx 4.0027$. Surface area is $4(3.14159)(4.0027)^2 \approx 201$.

29. Ans E: For rational functions where the numerator has a degree one higher than the denominator, the graph has an oblique asymptote. The equation of the oblique asymptote is the quotient of the numerator divided by the denominator. In this case, the equation will have a negative leading coefficient, so it's a line with a negative slope. Therefore, the function goes toward negative infinity as the x-value tends toward positive infinity.
30. Ans A: First let's consider the number of siblings in the Jones, Smith, and Miller families. Statement III implies two families of 3 girls to get two youngest/middle/oldest sets, and II says the Smiths are twins. In order to have this and no teams with sisters, the only allowable distribution is 3:3:2, so there are 3 Jones girls, 2 Smith twins, and 3 Miller girls. Next let's determine pairs. We know by V that Denise and Claire are teammates. If we compare II and IV, we know Beth's two sisters are not in the pie eating contest, so they must be in the thumb wrestling competition, forcing Beth to be a Miller. Since Amy is a Smith twin and can't have a middle sister, Grace Jones must be Beth's teammate. This also brings in by II that Grace must be the oldest Jones referred to in the loss to Faith. Speaking of Faith Jones, her teammate must be Hannah Miller, since she must be partnered with a Miller (by I and II) and we know she's not partnered with Beth or Claire. The last pair must be Amy and Emily. This means Denise must be the other Smith twin and Emily must be a Jones. Hannah Miller must be the oldest Miller referred to in V (all other teams accounted for), making Claire the middle Miller sister. Emily Jones must be the "partner's middle sister" referred to in III, making Emily the middle Jones sister. By default, Faith is the youngest Jones sister. With that in mind, we can start getting point totals. Grace and Beth got 0 in the tug of war, 2 in pie eating, 0 in thumb wrestling, and 2 in the three legged race for 4 points total. Faith and Hannah got 2 in the tug of war, 0 in pie eating, 2 in thumb wrestling, and 3 in the three legged race for a total of 7 points. Claire and Denise got 0 in the tug of war, 1 in pie eating contest, 0 in thumb wrestling, and 0 in the three legged race for an awful 1 point total. Finally, Amy and Emily got 2 points in the tug of war, 3 in pie eating, 2 in thumb wrestling, and 1 in the three legged race, giving them the winning total of 8.