

Academic Challenge



EASTERN ILLINOIS UNIVERSITY

2019 Academic Challenge

MATHEMATICS TEST – SECTIONAL

Physics Test Production Team

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

Time: 40 Minutes

Number of Questions: 30

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

WYSE – Academic Challenge
Mathematics Test (Sectional) – 2019

- A pole is erected in the middle of a field. Two people stand 1,000 feet apart at opposite ends of the field with the base of the pole on the line between them. If the angle of elevation from the ground to the top of the pole is 23 degrees for the first person and 35 degrees for the second, how tall is the pole? Round to the nearest hundredth foot.

a) 22.97 ft b) 264.27 ft c) 640.61 ft d) 889.14 ft e) 9,613.33 ft
- A candy shop is making a new type of candy that is 35% chocolate by mixing two types of candies together. The first type of candy is 40% chocolate, and the other type is 20% chocolate. How much of the first type of candy should be mixed to produce 200 grams of the new type of candy? Round to the nearest gram.

a) 100 g b) 150 g c) 175 g d) 200 g e) 250 g
- Given the natural numbers k , m , and n where $k \geq m$, $k \geq n$, and $m \neq n$, what is the sum of m and n if $C(k,m)=C(k,n)$?

a) k b) $2k$ c) 0
d) $\min(2m+k, 2n+k)$ e) There is no consistent sum.
- A bag of coins contains quarters, dimes, and nickels with a total value of \$64. There are 3 times as many dimes as nickels, and the number of quarters is 3 less than twice the number of dimes. How many coins are in the collection?

a) 98 b) 107 c) 347 d) 442 e) 513
- A population of ants that currently has 10,000 individuals is growing exponentially. If its current growth rate is 200 ants per day, how fast will it be growing 20 days from now? Round to the nearest ant per day.

a) 200 ants per day b) 246 ants per day c) 298 ants per day
d) 2,000 ants per day e) 4,000 ants per day
- If $x = t + 3$ and $y = t^2 + 5t - 1$, which of the following could represent y as a function of x alone?

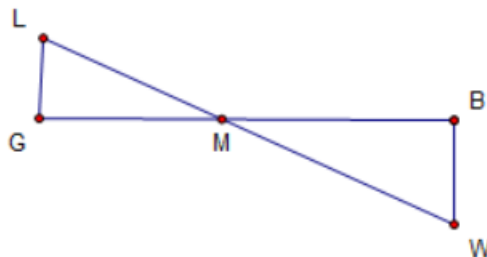
a) $y = x^2 - t - 10$ b) $y = x^2$ c) $y = x^2 + 5x - 1$
d) $y = x^2 - 5x - 1$ e) $y = x^2 - x - 7$
- Paul is traveling due east on his bike at 30 km/hr while Sarah travels due south on her bike at 45 km/hr. Both are heading for the intersection of their two roads. At what rate are they approaching each other when Paul is 0.5 km and Sarah is 1.2 km from the intersection? Round to the nearest km/hr.

a) 38 km/hr b) 53 km c) 64 km d) 75 km/hr e) 90 km/hr

8. A committee of 6 people is being convened. There are 10 men and 9 women available for the committee. In how many ways can the committee have an unequal number of women and men?

a) 7,014 b) 10,038 c) 10,080 d) 17,052 e) 27,132

9. In the figure below let $m\angle LGB = m\angle WBG = 90^\circ$, $GB = 15$, $MB = 9$, and $MW = 12$. Find the length of LM .



a) 4.5 b) 8 c) 3 d) 7.5 e) 4

10. Determine the focal length of the ellipse given by $4x^2 + 16x + 9y^2 - 18y + 16 = 0$. Round your answer to the nearest hundredth of a unit.

a) 0.75 units b) 2.14 units c) 2.24 units
d) 3.61 units e) No solution – this isn't an equation of an ellipse

11. Consider the vectors $\langle 6, -5, 0 \rangle$, $\langle 3, 2, -27 \rangle$, $\langle 4, -1, 4 \rangle$, and $\langle 0, 0, 0 \rangle$. How many of these are orthogonal to $\langle 5, 6, 1 \rangle$?

a) 0 b) 1 c) 2 d) 3 e) 4

12. An airplane flying due south makes a 90° left turn to head due east. Later, the pilot makes another left turn of 120° . In order to fly the plane due south again, what angle of a third left turn would be required?

a) 30° b) 45° c) 90° d) 108° e) 150°

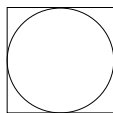
13. Given $\frac{x-4}{x+2} = \frac{x-5}{2x+5}$, solve for x .

a) 0 b) $\pm\sqrt{10}$ c) $\pm\sqrt{30}$ d) -4 e) No solution

14. First place, second place, and third place are to be awarded at random to three different people out of Amy, Barb, Carrie, Deb, Ed, Fred, Greg, and Hans. Find the probability that Fred wins first place and Carrie wins nothing, or Carrie wins second place and Fred wins nothing. Round to two decimal places.

a) 0.18 b) 0.29 c) 0.54 d) 0.62 e) 0.80

15. A circle with circumference 1 inch is inscribed inside a square as shown below. What is the area of the square? Round your answer to the nearest hundredth of a square inch.



- a) 0.10 square inches b) 0.31 square inches c) 0.79 square inches
d) 1.27 square inches e) 1.57 square inches
16. Find the sum of all solutions to the equation $3 \cdot 49^x - 7 \cdot 7^x + 2 = 0$. Round to two decimal places.
- a) -0.56 b) -0.21 c) 0.36 d) 2.33 e) No solutions
17. Oliver is floating in a hot air balloon, while Mary and Nancy each stand by a tether anchoring the balloon to the ground. Mary's tether is 60 feet long and has an angle of inclination of 27° . Nancy's tether is 50 feet long and has an angle of inclination of 33° . Assuming Mary and Nancy are standing opposite the point directly below the balloon, how many feet away is Mary from Nancy? Answers are rounded to the nearest foot.
- a) 45 ft b) 70 ft c) 95 ft d) 100 ft e) 115 ft
18. Assuming x can only take real values, which of the following statements would be true about $(x - 3 - i)$ and $(x - 3 + i)$?
- I. Their sum is a real number.
II. Their product is a real number.
III. They are complex conjugates of each other.
- a) All three b) I only c) III only d) I and II e) None
19. Randy deposits \$15,000 in an interest-bearing account. If the interest earned is 6% compounded twice each month, how much interest will Randy make in 18 months?
- a) \$16,410.77 b) \$15,926.36 c) \$10,405.28 d) \$1,926.36 e) \$1,410.77
20. Sam took her initial monthly check and spent 25% of it on rent. She spent 40% of the remaining amount on other bills. She spent 20% of the amount left after bills on food. If she was left with \$180 after food, how much was her original paycheck? Round to the nearest whole dollar.
- a) \$212 b) \$333 c) \$378 d) \$500 e) \$1,200

21. (Matt, transformations, easy) The graph of $y = |x|$ is transformed using the following steps in the given order: shift up 3, shift left 6, stretch vertically by a factor of 9, and finally reflected over the y-axis. Which of these is an equation for the new graph?
- a) $y = 9|-x - 6| + 3$ b) $y = 9|-x - 6| - 3$ c) $y = \frac{1}{9}|-x + 6| + \frac{1}{3}$
 d) $y = \frac{1}{9}|-x - 6| + 3$ e) $y = 9|-x + 6| + 27$
22. Let $A = \begin{bmatrix} 1 & k \\ 2 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & -3 \\ m & 1 \end{bmatrix}$. If A and B are inverses of each other, find k.
- a) $-\frac{7}{2}$ b) $\frac{2}{7}$ c) 3 d) -3 e) 0
23. Which of the following would be the rate of change of $y = e^{2t} - 8t^2 + 5\sqrt{t} + 7$ when $t = 0$?
- a) 2 b) 8 c) $7 + \frac{2}{e}$ d) $7 + \frac{1}{e}$ e) Undefined
24. Determine whether the series $\sum_{n=1}^{\infty} \frac{6}{2n^2 + 6n}$ converges or diverges. Either state that the series diverges, or if it converges, find the sum.
- a) $\frac{11}{6}$ b) $\frac{25}{12}$ c) 3 d) $\frac{49}{21}$ e) The series diverges
25. A tank has two pumps, A and B, and a drain. If only pump A is active, it can fill the tank from empty in 40 minutes. If only pump B is active, it can fill the tank from empty in 30 minutes. If only the drain is active, it can empty the tank from full in 20 minutes. Suppose we start with the tank full. We open the drain at 1:00, turn on pump A at 1:10, and pump B at 1:20. Does the tank end up empty or full after this, and when does this happen?
- a) Empties at 3:20 b) Fills at 2:30 c) Fills at 2:50
 d) Fills at 3:00 e) Fills at 3:20
26. Which of the following expressions is equal to $\log_8(x^2 - 3x - 4)$ over its entire domain?
- I. $\log_8(x - 4) + \log_8(x + 1)$
 II. $\log_8(x - 4)\log_8(x + 1)$
 III. $\frac{\ln(x^2 - 3x - 4)}{\ln 8}$
- a) I, III b) I, II, III c) I only d) II only e) III only

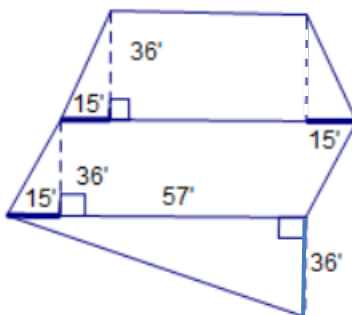
27. Find a polar coordinate to represent the Cartesian coordinate $(-2, 2)$.

- a) $\left(2\sqrt{2}, -\frac{7\pi}{4}\right)$ b) $\left(\sqrt{2}, -\frac{3\pi}{4}\right)$ c) $\left(2\sqrt{2}, -\frac{3\pi}{4}\right)$
 d) $\left(2\sqrt{2}, \frac{3\pi}{4}\right)$ e) $\left(\sqrt{2}, \frac{7\pi}{4}\right)$

28. What is the period of $\sin 3x + \cos 6x$?

- a) $\frac{\pi}{18}$ b) $\frac{\pi}{6}$ c) $\frac{2\pi}{9}$ d) $\frac{4\pi}{9}$ e) $\frac{2\pi}{3}$

29. The diagram below represents a parcel of land that is composed of an isosceles trapezoid, a parallelogram, and a right triangle. The owner of the parcel want to install a decorative border around the perimeter of this parcel. If the border comes in 5-foot flexible sections, how many sections should be purchased if the landowner wants to have 20 feet more than what is required? Note that the diagram is not to scale, and that the width of the parallelogram is $15 + 57 = 72$ feet.



- a) 335 b) 288 c) 150 d) 78 e) 67
30. Amy, Brad, Cindy, David, Eric, Felicia, and Greg are lined up in a row. They were originally in alphabetical order (Amy first). This put Brad between his two sisters, and put Felicia between her two brothers. Because they were all fighting, everyone moved to a new place in line. Now, no one is standing next to a person they had been standing next to before, and no one is standing next to a person they are related to. There is a boy standing at each end. Every boy is standing next to at least one girl, and every girl is standing next to at least one boy. David isn't related to anyone else in line, and he is once again standing between two people who aren't related to each other. Who is standing in the middle?
- a) Amy b) Brad c) Cindy d) Felicia e) Greg

