Academic Challenge
2019 State Mathematics Exam

1. I place a 25-foot-long ladder up against a wall so that its base is 15 feet away from the wall. I then push the base of the ladder towards the wall enough so that the top of the ladder has moved 2 feet up the wall. How much has the angle that the ladder makes with the ground increased? Round to the nearest degree.

   a) 5° b) 6° c) 7° d) 9° e) 10°

2. Determine the oblique asymptote of $y = \frac{4x^3 - 6x^2 + 16x - 4}{2x^2 - 2x + 4}$.

   a) $y = 2x - 1$
   b) $y = 2x + 1$
   c) $y = 3x$
   d) $y = 5x - 4$
   e) The graph of this equation does not have an oblique asymptote.

3. For a parabola with a focus at (0, 7) and a directrix of y = 2x, what is the distance from the focus to the vertex? Round to two decimal places.

   a) 1.57 b) 3.13 c) 3.50 d) 4.20 e) 4.90

4. Solve the equation $3\cos\left(\frac{x}{2}\right) = 3 + 3\cos x$ where $0 \le x \le 2\pi.$. Which of the following values are solutions to the equation:

   i) $x = \frac{\pi}{3}$ ii) $x = \frac{2\pi}{3}$ iii) $x = \pi$

   a) i b) i, ii c) i, iii d) ii, iii e) i, ii, and iii

5. For the function $F(x) = \int_0^x (3 - 2t)\,dt$, determine what values of x make the function positive.

   a) $x < \frac{3}{2}$ b) $x > \frac{3}{2}$ c) $0 < x < 3$
   d) $x < 0$ or $x > 3$ e) This function is never positive

6. Determine the value of $\lim_{x \to 0^-} \frac{1 - e^{-x}}{x}$.

   a) –1 b) 0 c) 1 d) ∞ e) Undefined

7. A steel sculpture is in the shape of a right triangular prism with right triangles for bases. The legs of the triangles are 5 feet and 12 feet. The prism stands on the bottom triangular base and is 10 feet tall. The bottom face is left unpainted. The three side faces and the top face are to be painted red. Find the total area that is to be painted. Round to the nearest square foot.

a) 221 ft^2 b) 300 ft^2 c) 330 ft^2 d) 425 ft^2 e) 600 ft^2

8. Which of the following can replace f(x) so that $\int f(x)e^{3x^2+8x+1}dx$ can be solved using integration by substitution?

i. $6x + 8$ ii. $x^3 + 4x^2 + x$ iii. $3x + 4$

a) i, ii, iii b) i, ii c) ii, iii d) i, iii e) i only

9. Solve the exponential equation $\frac{4^x - 3 \cdot 4^{-x}}{2} = 1$.

a) $\frac{\log 3 - \log 4}{\log 3}$ b) $\log 3 - \log 4$ c) $\log 4 - \log 3$

d) $\frac{\log 4}{\log 3}$ e) $\frac{\log 3}{\log 4}$

10. Angles A and B are complementary, angles B and C are supplementary, and the measure of angle C is four times that of angle A. What is the measure of angle B?

a) 30 degrees b) 36 degrees c) 45 degrees
d) 60 degrees e) 120 degrees

11. Which of the following distributions works best to find the probability that it will take 16 rolls of a die before you get your first result of "1"?

a) hypergeometric b) geometric c) binomial
d) poisson e) normal

12. A researcher studied three categories of cats to determine frequency of a particular genetic trait. In the sample, 1/3 of the cats were feral, 5/12 were house cats, and the rest were cats from shelters. Of the feral cats, 2/3 had the genetic trait. Of the housecats, 3/4 had the trait, and of the cats from shelters, 3/5 did <u>not</u> have the trait. If a randomly selected cat has the genetic trait, what is the probability that it is a house cat? Round to three decimal places.

a) 0.507 b) 0.492 c) 0.429 d) 0.350 e) 0.158

13. What is the range of k(x) if $\cos^4 x = \cos 2x + k(x)$?

a) All reals b) [–1, 1] c) [0, 1] d) [–1, 0] e) [–2, 0]

14. Given the matrix equation $\begin{bmatrix} 0 & K & M \\ 4 & 11 & M \end{bmatrix}\begin{bmatrix} 3 & K \\ 1 & M \\ -6K & 1 \end{bmatrix}=\begin{bmatrix} 11 & 0 \\ 35 & 10M \end{bmatrix}$, solve for M.

a) $M=2$ b) $M=1$ c) $M=0$ d) $M=-1$ e) $M=-2$

15. If a sphere has a surface area of 100 square meters, what is its volume? Round to the nearest cubic meter.

a) 17 cubic meters b) 53 cubic meters c) 68 cubic meters
d) 94 cubic meters e) 167 cubic meters

16. We are given $y=ax^3+bx^2+cx+d$, where a, b, c, and d are all non-zero integers. How many of the following four statements must be true about this polynomial?

- We have a comprehensive yet finite list of possible rational zeroes.
- It must have at least one real zero.
- x = –d must be a zero.
- It must have three rational zeroes.

a) 0 b) 1 c) 2 d) 3 e) 4

17. Given the polar equation $r=6\sin 2\theta$, which of the following rectangular equations creates the same graph?

a) $\sqrt{x^2+y^2}=36x^2y^2$ b) $\left(x^2+y^2\right)^3=144x^2y$ c) $x^2\left(x^2+y^2\right)=24y^2$
d) $\left(x^2+y^2\right)^{3/2}=6y^2$ e) $\sqrt{x^2+y^2}=72x^2y^2$

18. What is the sum of the real solutions to $2\log(x-5)-2\log(x-4)=1$? Round to two decimal places.

a) 3.54 b) 4.24 c) 4.50 d) 7.78 e) No real solutions

19. If the fourth term in the expansion of $(x+2)^{12}$ is ax^9, determine the value of a.

a) 495 b) 972 c) 1,760 d) 7,920 e) 53,144

20. An oval track is 400 meters around with the starting line on the northern end. At noon, Carol starts at the starting line of the track and begins jogging clockwise at a speed of 15 kilometers per hour. Ten minutes later at 12:10, Dale gets on the track at the starting line and begins jogging counterclockwise at a speed of 12 kilometers per hour. From then on, Carol and Dale give each other a high-five each time they meet. At 12:40 when both of them meet at the finish line, they give each other one last high-five and then stop. How many high-fives did they give each other total?

a) 28 b) 29 c) 30 d) 33 e) 34

21. If (a + bi)(c + di) is a real product, which of the following must be true?

a) $ac - bd = 0$ b) $ad + bc = 0$ c) $c + d = 0$
d) $a - b = 0$ e) $ad - bc = 0$

22. Given $x = -3 + 2\sec t ; y = 2 + 3\tan t$, turn this into a Cartesian equation for the graph by eliminating the parameter t.

a) $(x+3)^2 - (y-2)^2 = 36$ b) $\frac{x^2}{4} + \frac{y^2}{9} = 1$ c) $y - 3 = \frac{(x+2)^2}{9}$
d) $\left(\frac{x+2}{3}\right)^2 + \left(\frac{y-3}{2}\right)^2 = 13$ e) $\frac{(x+3)^2}{4} - \frac{(y-2)^2}{9} = 1$

23. Given $A = \langle 3,2,3 \rangle$, $B = \langle 6,1,7 \rangle$, and $C = \langle 4,0,4 \rangle$, determine $A \bullet (B \times C)$, which is the scalar triple product of these vectors.

a) 0 b) 8 c) 328 d) 6,048 e) 12,240

24. For $y = \frac{2}{3}\tan(2x + 6\pi) - 3$, list the horizontal shift (hs), vertical shift (vs), and period (p).

a) hs : right 6π, vs : down 2 / 3, p : π
b) hs : left 3, vs : up 2 / 3, p : π
c) hs : left 3π, vs : down 3, p : $\pi / 2$
d) hs : right 6π, vs : down 2 / 3, p : π
e) hs : right 6π, vs : down 2 / 3, p : π

25. For the equation $\frac{x+m}{x-1} = mx + 1$, determine how many different real values of m make it so there is exactly one real solution for x.

a) 0 b) 1 c) 2 d) 3 e) infinitely many

26. How many distinguishable ways can the letters of “DAVID S. PUMPKINS” be rearranged so that each rearrangement includes an S as the first letter and an M as the final one?

a) 14,968,800 b) 26,195,400 c) 59,875,200
d) 479,001,600 e) 798,336,600

27. A reservoir is a cone with radius 60 meters at the top and a depth of 5 meters. Water is being pumped from the reservoir at a rate of 65 cubic meters per minute. How fast is the water level falling when the water is 3 meters deep? Round to three decimal places.

a) 0.214 m/min b) 0.016 m/min c) 0.119 m/min
d) 0.024 m/min e) 0.211 m/min

28. Matt can write a test in 3 hours, and his cat can delete a test in 4 hours. If both work against each other for the entire time, how long will it take for Matt to write one test? Round to two decimal places.

a) 1.71 b) 3.14 c) 3.43 d) 7.00 e) 12.00

29. Find the total amount of enclosed area that lies between the cubic function $f(x) = \frac{x^3}{2} + \frac{x^2}{2} - 3x$ and the x-axis. Round to the nearest whole unit.

a) 9 b) 10 c) 11 d) 12 e) 13

30. Claire's family recently went on a camping trip. On the way there, Claire rode in the minivan with her mother, her younger sister Danielle, and her baby brother Eric. Claire's older brother Andrew rode with their father in the truck pulling the fifth-wheel trailer. Claire's older sister Beth rode with their Uncle Joe in his car. Because Claire and Danielle fought the whole way there, their parents switched out who rode with whom on the way back. Claire swapped places with Andrew, and Danielle swapped places with Beth. Everyone else rode in the same vehicle both ways.

I. The ages of all eight people are unique natural numbers.
II. Uncle Joe is four times as old as Beth and five times as old as Claire.
III. Claire's father is three times as old as Andrew.
IV. Claire's age is double the sum of her two younger siblings.
V. Claire's mother's age is equal to the sum of the ages of her five children.
VI. The age total of the minivan went up by 11, and the car total went down by 7.

Which of Claire's parents is older and by how much?

a) Her father is older by 1 year.
b) Her father is older by 2 years.
c) Her father is older by 4 years.
d) Her mother is older by 1 year.
e) Insufficient information was given.