WYSE – Academic Challenge
Physics Exam Solutions (State) – 2019

1. Correct Response: A

Apply kinematics equation for one-dimensional motion with constant acceleration:

$x = x_0 + v_{0x}t + (1/2)a_x t^2$

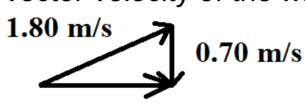
In the above equation, the time t refers to the time elapsed since the initial time $t = 0$, and x_0 and v_{0x} are the position and velocity at that time, respectively. In this problem, consider the initial time at $t = 1.00$ s and the time elapsed since that time is $t - 1.00$ s. So,

$x = 1.00 \text{ m} + (5.00 \text{ m/s})(t - 1.00 \text{ s}) + (1/2)(2.00 \text{ m/s}^2)(t - 1.00 \text{ s})^2$

Evaluation the above equation at $t = 2.00$ s, we have $x = 7.00$ m.

2. Correct Response: D

This is a vector problem. The speed that the swimmer moves must be added to the vector velocity of the water to create a sum that is directly across the river.



and because this is a right triangle we can use the Pythagorean theorem to determine the sum of the first two vectors. $x = \sqrt{1.80^2 - 0.70^2} = 1.66\frac{m}{s}$.
Using this as the speed that the swimmer can move across the river we find the time:

$$t = \frac{d}{v} = \frac{54.9}{1.66} = 33.1\ s$$

3. Correct Response: A

Vector addition:

$A_x = 3.47\cos 132° = -2.32$ $\qquad$ $B_x = 1.80$
$A_y = 3.47\sin 132° = 2.58$ $\qquad$ $B_y = -0.80$

$$C_x = A_x + B_x = -2.32 + 1.8 = -0.52$$
$$C_y = A_y + B_y = 2.58 - 0.80 = 1.78$$

$C = \sqrt{C_x^2 + C_y^2} = 1.85$ $\qquad$ $\theta = \arctan\frac{C_y}{C_x} = arctan\frac{1.78}{-0.52} = -73.71$

Realizing that we are in the second quadrant we must add 180 to this to get the correct angle. -73.71+180=106.3˚.

4. Correct Response: C

Apply kinematics equation for one-dimensional motion with constant acceleration:

$y = y_0 + v_{0y}t + (1/2)a_y t^2$

In the coordinate system chosen for this problem, $y_0 = 0$, $v_{0y} = -v_0$, and $a_y = g$. So,

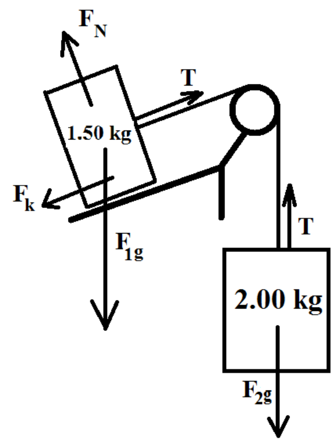
$y = -v_0 t + (1/2)gt^2$.

5. **Correct Response: D**

The position is given by the integral of the velocity or the area under the curve in the figure. The first incline is a triangle with area $(1/2)b_1h_1$. The second part is the constant velocity section which has an area of b_2h_2. The third part is a triangle sitting atop a rectangle. These parts add up: 4.5+12+3+1=20.5 m.

6. **Correct Response: C**

The Force Diagram looks like this:



The frictional force is kinetic friction because tension will overcome static friction and gravity. The force equations for the 1.50 kg block are:

$$F_N - mg\cos 20° = 0$$
$$-F_k - mg\sin 20° + T = m_1 a$$

We know that the frictional force is given by $F_k = \mu_k F_N$

The force equation that can be written for the 2.00 kg block is:

$$m_2 g - T = m_2 a$$

All of these equations can be combined to solve for a. Use the first to find F_N and use the definition of kinetic friction to get F_k and then solve the third equation for T and plug all of these into the second equation. So $a = \frac{g(m_2 - \mu_k m_1 \cos 20° - m_1 \sin 20°)}{(m_1 + m_2)} = 3.69\,\frac{m}{s^2}$

7. **Correct Response: C**

In the process of solving the previous we can look at the equation for the hanging mass and get $T = m_2(g - a) = 12.22\ N$

8. **Correct Response: B**

The extension of the spring depends on the tension in the string. From a force diagram for each of the two masses we can immediately write the force equations:

$$T = m_1 a$$
$$m_2 g - T = m_2 a$$

Two equations and two unknowns can be solved. The result is that $a = \frac{m_2}{m_1+m_2} g$ and so we can solve for T to get $T = m_1 a = \frac{m_1 m_2}{m_1+m_2} g = kx$ where we have added kx because that is the force of the spring (neglecting signs). Solving for x gives 0.100 m or 10.0 cm.

9. Correct Response: D

The trick to this problem is to convert miles per hour to meters per second.

$183 \frac{mi}{hr} \times \frac{5280\, ft}{1mi} \times \frac{12in}{1\, ft} \times \frac{2.54cm}{1in} \times \frac{1m}{100cm} \times \frac{1hr}{60min} \times \frac{1min}{60s} = 81.8\ \frac{m}{s}$

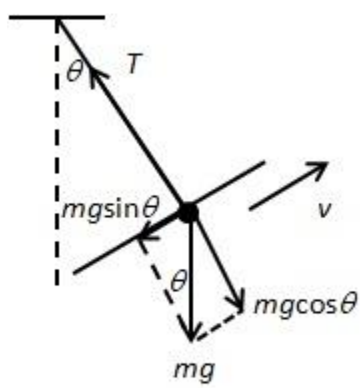
With this speed we can cover the distance in a time of $t = \frac{d}{v} = \frac{200m}{81.8\frac{m}{s}} = 2.44s$

10. Correct Response: A

We use the equation $P = \vec{F} \cdot \vec{v}$ and solve for force. The result is 1450/81.8=17.7N a very low drag force indeed.

11. Correct Response: B

A free-body diagram for the bob of the pendulum is shown below.



The tangential component of the bob's acceleration is equal to the rate of change of its linear speed, that is, $a_t = dv/dt$. The tangential component of the net force is $-mg\sin\theta$. The negative sign is due to the fact that this force component is opposite to the direction of motion. So,

$a_t = dv/dt = -mg\sin\theta/m = -g\sin\theta = -(9.80\text{ m/s}^2)\sin 10.0° = -1.70\text{ m/s}^2$.

12. Correct Response: E

Refer to the free-body diagram in Question #3. The radial component of the net force is $F_{rad} = T - mg\cos\theta$ and the radial component of the acceleration is $a_{rad} = v^2/L$. Apply Newton's second law:

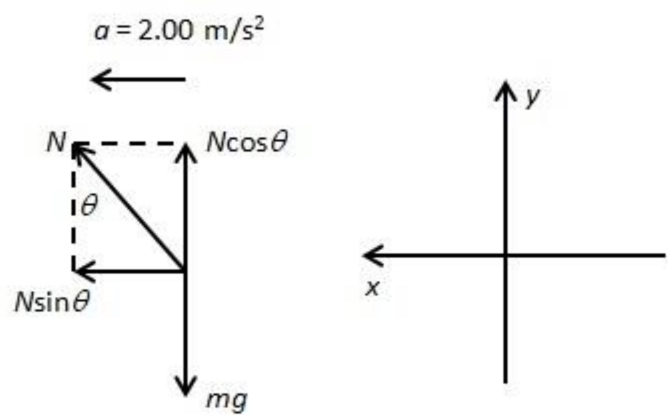
$F_{rad} = ma_{rad} \rightarrow T - mg\cos\theta = mv^2/L$

Solving for v, we have

$v = [L(T - mg\cos\theta)/m]^{1/2}$ = {(1.00 m)[6.00 N – (0.500 kg)(9.80 m/s^2)cos 10.0°]/(0.500 kg)}$^{/1/2}$ = 1.53 m/s.

13. Correct Response: B

Since the object is at rest relative to the inclined plane, the object has the same acceleration as the inclined plane, that is, the object has an acceleration of 2.00 m/s^2 directed to the left relative to the ground. Two forces are acting on the block: normal force directed upward perpendicularly to the inclined plane and gravity directed downward. A free-body diagram for the block is shown below.



Apply Newton's second law in both the x and y directions:

$N\sin\theta = ma$ and $N\cos\theta - mg = 0$

Solving for θ, we have $\tan\theta = a/g$ and $\theta = \arctan(a/g)$ = arctan (2.00 m/s^2/9.80 m/s^2) = 11.5°.

14. Correct Response: A

The magnitude of angular momentum L is defined as $L = I\omega$, where I is the moment of inertia and ω is the angular speed. Since $[I] = ML^2$, $[\omega] = 1/T$, $[L] = ML^2/T$.

15. Correct Response: C

Total momentum before collision: (2.00 kg)(1.00 m/s) = 2.00 kg·m/s

Total momentum after collision: (2.00 kg)(0.550 m/s) + (1.00 kg)(0.900 m/s) = 2.00 kg·m/s
So, total momentum is conserved as expected for a collision.

Total kinetic energy before collision: (1/2)(2.00 kg)(1.00 m/s)^2 = 1.00 J
Total kinetic energy after collision: (1/2)(2.00 kg)(0.550 m/s)^2 + (1/2)(1.00 kg)(0.900 m/s)^2 = 0.708 J

There is a loss in kinetic energy during collision. The collision is inelastic. However, because the two objects do not have the same final velocity, the collision is not completely inelastic.

16. Correct Response: E

Escape velocity is the velocity such that the total energy of the object is zero and thus the kinetic energy (which is certainly positive) must be equal to the opposite of the gravitational potential energy (gravitational potential energy is negative).

17. Correct Response: C

This is a conservation of momentum problem. The cars are totally inelastic because their bumpers lock together and the final object is one with a mass that is the sum of the two masses.

$p_{1ox} = m_1 v_{1ox} = 1550kg \times 30.0\frac{m}{s} = 46500\frac{kg\ m}{s}$ $\qquad p_{1oy} = 0$

$$p_{2ox} = m_2 v_{2ox} = 1270kg \times 22.0\frac{m}{s} \times \cos 55° = 16025\frac{kg\ m}{s}$$

$$p_{2oy} = m_2 v_{2oy} = 1270kg \times 22.0\frac{m}{s} \times \sin 55° = 22887\frac{kg\ m}{s}$$

So now the final momentum must equal the initial momentum. The two cars are locked together so that their velocities are the same and we can treat the final system as a single mass (total mass of 2820 kg).

$$p_{fx} = 46500\frac{kg\ m}{s} + 16025\frac{kg\ m}{s} = 62525\frac{kg\ m}{s} = Mv_{fx}$$

Solving for v_{fx} gives us 22.2m/s. Similarly for y we see that $v_{fy} = \frac{22887\frac{kg\ m}{s}}{2820kg} = 8.12\frac{m}{s}$

Reconstructing the vector from the x and y components yields a magnitude of 23.6 m/s and a direction of 20.1˚.

18. Correct Response: B

The energy lost can be calculated from the kinetic energies before and after the collision.

$$\frac{1}{2}m_1 v_{1o}^2 + \frac{1}{2}m_2 v_{2o}^2 = 1004840J$$

And after $\frac{1}{2}Mv_f^2 = \frac{1}{2}(2820kg)\left(23.6\frac{m}{s}\right)^2 = 785314J$

So that the loss of mechanical energy is the difference or 2.20x10^5 J.

19. Correct Response: E

This problem has three objects that are rotating on an axis through the center. The first is a rod of length *l* and mass *m* rotating on its center. The moment of inertia for this object is $\frac{1}{12}ml^2$. The other two objects are rods of length *l* and mass *m* that are rotating about an axis parallel to their center axis and offset by a distance of 2*l* from that axis. The parallel axis theorem gives us

$$I = I_{cm} + mD^2$$

Where D is the distance between the axis of rotation and the axis through the center of mass of the object. For each of the two massive parts left we get:

$$I = \frac{1}{12}ml^2 + m(2l)^2$$

And since there are two of these items (one at each end) we get a total moment of inertia that is the sum of them all:

$$I = \frac{1}{12}ml^2 + 2\left\{\frac{1}{12}ml^2 + m(2l)^2\right\} = \frac{3}{12}ml^2 + 2m(2l)^2$$

20. Correct Response: D

First it is necessary to convert 10.3 rev/s into radians/s. $10.3\frac{rev}{s} \times \frac{2\pi\, rad}{1\, rev} = 64.7\frac{rad}{s}$

At this point it is a rotational kinematics with constant angular acceleration problem. We have the two equations of motion:

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

Where we have ω_o is zero, ω is 64.72 rad/s, and α is 34.1 rad/s^2. We can solve the second for t and plug it into the first.

$$t = \frac{\omega}{\alpha} = 1.898\, s$$

$$\theta = \frac{1}{2}\alpha t^2 = 61.4\, rad$$

21. Correct Response: C

We must start by converting rev/s to rad/s for the angular velocity of the wheel.

$$\omega = 1.67\frac{rev}{s} \times \frac{2\pi\, rad}{1\, rev} = 10.49\frac{rad}{s}$$

The relationship between linear speed and angular velocity is

$$v = \omega r$$

Which we can now solve for r to get: $r = \frac{v}{\omega} = 0.0748\, m$

22. Correct Response: D

Apply thin-lens equation: $1/s + 1/s' = 1/f$

$s + s' = 100$ cm → $s' = 100$ cm – s

Substituting s′ in the thin-lens equation with the above expression, we have
$1/s + 1/(100 \text{ cm} - s) = 1/20$ cm

Rearranging the variables, we obtain a quadratic equation:

$s^2 - (100 \text{ cm})s + (100 \text{ cm})(20 \text{ cm}) = 0$

Solving the above quadratic equation, we have s = 72.4 cm or 27.6 cm. The shorter object distance, s = 27.6 cm will result in an enlarged image on the screen. (Since $|M| = |s'/s|$, $|M|>1$ implies $s < 50$ cm.)

23. Correct Response: C

Apply conservation of angular momentum to find the final moment of inertia:

$I_1\omega_1 = I_2\omega_2$ → $I_2 = I_1\omega_1/\omega_2 = (30.0 \text{ kg}\cdot\text{m}^2)(5.00 \text{ rad/s})/(8.00 \text{ rad/s}) = 18.75 \text{ kg}\cdot\text{m}^2$ (keeping an extra significant figure in this intermediate result)

Now, apply work-energy theorem:

$W_{tot} = \Delta K = (1/2)I_2\omega_2^2 - (1/2)I_1\omega_1^2 = (1/2)(18.75 \text{ kg}\cdot\text{m}^2)(8.00 \text{ rad/s})^2 - (1/2)(30.0 \text{ kg}\cdot\text{m}^2)(5.00 \text{ rad/s})^2 = 225$ J

24. Correct Response: B

This problem is about resonance: when the driving frequency is close to the natural frequency of an oscillating system, the system has a large response (amplitude). The natural (angular) frequency of a simple pendulum is $\omega_0 = (g/L)^{1/2}$. So, ω should be close to $(g/L)^{1/2}$.
Alternatively, note that the torque for a simple pendulum is given by:

$$\tau = -mgL\sin(\theta)$$

For the small angle approximation and realizing what the torque is we can rewrite this equation as

$$I\frac{d^2\theta}{dt^2} = -mgL\theta$$

The moment of inertia for the pendulum mass is simply mL^2 so that we get (after cancelling like terms:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

Recognizing that solutions to this equation include sine and cosine with the frequency being:

$$\omega_o = \sqrt{\frac{g}{L}}$$

This gives the natural frequency of the system and so if we drive at that frequency we obtain resonance.

25. Correct Response: C

When a wave passes through an opening, it spreads out. This spreading of the wave is called diffraction.

26. Correct Response: A

The two speakers (sources) are π radians out of phase, which corresponds to an equivalent path-length difference of $\lambda/2$. The total path-length difference between the two sound waves is

Δ_p = 4.00 m – 2.00 m + $\lambda/2$

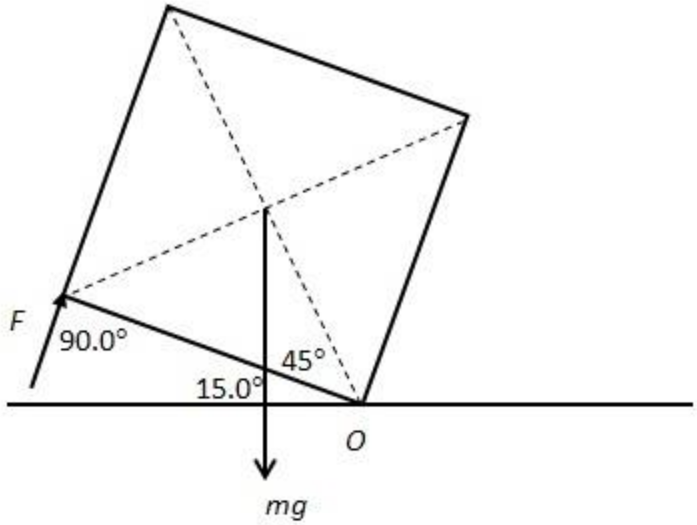
For constructive interference, $\Delta_p = n\lambda$, where n is a whole number. So,

4.00 m – 2.00 m + $\lambda/2 = n\lambda \rightarrow \lambda$ = (2.00 m)/(n – ½).

When n = 1, λ_{max} = 4.00 m and $f_{min} = v/\lambda_{max}$ = (344 m/s)/(4.00 m) = 86.0 Hz.

27. Correct Response: D

Since the object is in static equilibrium, the net torque about point O must be zero.



The object has a square shape and its half diagonal length is $L\sin 45°$, where L is the side length of the object. The lever arm of gravity about point O is then $L\sin 45°\cos 60°$. So,

$mgL\sin 45°\cos 60° - FL = 0$

Solving for F, we have $F = mgL\sin 45°\cos 60°/L = mg\sin 45°\cos 60°$ = (5.00 kg)(9.80 m/s^2)sin45°cos60° = 17.3 N.

28. Correct Response: E

According to Faraday's Law of Induction, the induced electromotive force is proportional to the rate of change of the magnetic flux through the ring. Since the magnetic field is uniform in space and remains constant with time, and the area of the ring and its orientation do not change either, the rate of change of the magnetic flux through the ring is zero, and therefore the induced current in the ring is zero.

29. Correct Response: A

Apply ideal gas law: $PV = nRT$. From 1 → 2, $P_1 = P_2$, $V_2 > V_1$, thus $T_2 > T_1$. From 2 → 3, $V_2 = V_3$, $P_3 < P_2$, thus $T_3 < T_2$. So, $T_1 < T_2$ and $T_2 > T_3$.

30. Correct Response: C

According to energy conservation, $hf = \Delta E = E_2 - E_1$, and so, $f = (E_2 - E_1)/h$.

31. Correct Response: C

Due to the plunger, the tube of air can be treated as a pipe that is half open and half closed. For two consecutive natural modes (or standing wave patterns), the change in length of the pipe is $\Delta L = \lambda/2$. So, $\lambda/2 = 0.386$ m and $\lambda = 2(0.386 \text{ m}) = 0.772$ m.

Therefore, the speed of sound in air is $v = \lambda f = (0.772 \text{ m})(440 \text{ Hz}) = 340$ m/s.

32. Correct Response: D

Time dilation gives us $t = 1.70x10^{-8}s = \gamma t' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}t' = \frac{t'}{\sqrt{1-(0.85)^2}}$

Solving for t' we see that $t' = 8.955x10^{-9}s$

Now we can take this time to the decay equation for materials with a half-life.

$$N = N_o e^{-\frac{0.693t'}{t_{1/2}}} = N_o \times ..5307$$

Thus we see that we have 53.1% of the total particles that started in the beam.

33. Correct Response: E

In a thermally isolated system, entropy never decreases. The only process for which the entropy can decrease is to exchange heat with the external environment, with the heat flowing out of the system.

34. Correct Response: C

R_2 and R_3 are in parallel with each other and their parallel equivalent is in series with R_1:

$$R = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = 200\Omega + \left(\frac{1}{300\Omega} + \frac{1}{600\Omega}\right)^{-1} = 400\Omega$$

35. Correct Response: B

This is a torque problem. Using the axis of rotation of the point where the board contacts the floor and calculating the sum of the torques (=0) we see:

$1.00m \times M_C g \sin 65 + 1.50m \times M_B g \sin 65 - 3.00m \times F_N \sin 155 = 0.00$ Solving for F_N gives: F_N= 87.6N.