2020 Academic Challenge

MATHEMATICS TEST – REGIONAL

Physics Test Production Team

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as ⬤ , not ⊙ , ◍ , ◐ , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

Time: 40 Minutes   Number of Questions: 30

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

1. The vertical velocity v (in ft/sec) in terms of time t (in seconds) after a missile is launched is given by $v = 1000 - 32t$. At what values of t is the missile moving upward?

   a) $0 \le t < 31.25$ b) $t < 31.25$ c) $t = 31.25$

   d) $t > 31.25$ e) $t \ge 0$

2. A triangle has sides of length 8 and 10 with an angle of 40 degrees between them. What is the area of this triangle? Round your answer to the nearest square unit.

   a) 20   b) 26   c) 30   d) 31   e) 40

3. City A and City B are 300 miles apart. A car leaves City A at noon and drives toward City B at 50 mph. A second car leaves City B at 12:30 and heads toward City A at 60 mph. Determine whether or not the two cars pass each other and, if so, at what time. Round your answer to the nearest minute.

   a) Yes, they pass each other at 12:49.
   b) Yes, they pass each other at 2:27.
   c) Yes, they pass each other at 3:00.
   d) Yes, they pass each other at 3:30.
   e) The cars never pass each other.

4. What is the overall sum of the series whose first terms are 8, 4, 2, 1, … ?

   a) 15   b) 16   c) 128   d) 256   e) The series is divergent.

5. A circle with a circumference of 5 inches is inscribed within an equilateral triangle as shown below. Determine the perimeter of the triangle. Round your answer to the nearest tenth of an inch.
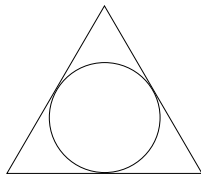
   a) 6.8 in   b) 7.9 in   c) 8.3 in   d) 9.9 in   e) 10.4 in

6. Find the values of x for which $\sqrt{\dfrac{x-4}{x+4}}$ represents a real number.

   a) $x > -4$ b) $x < -4$ c) $x < -4$ or $x > 4$ d) $-4 < x < 4$ e) $x < -4$ or $x \ge 4$

7. The walls and ceiling of a whispering gallery form a portion of an ellipse. The middle of the ceiling is 30 feet above the ellipse's center, which gives us a semi-minor axis length of 30 feet. The points where two friends can whisper to each other from across the room and be heard are the focal points of the ellipse. These focal points are each 10 feet from the walls, which means the direct distance from each focus to the nearest vertex of the major axis is 10 feet. How far are the foci from each other? Round your answer to the nearest tenth of a foot.

   a) 40.0   b) 56.6   c) 60.0   d) 63.2   e) 80.0

8. Find the area of a triangular parcel of land with sides 205 ft., 322 ft., and 415 ft. Round your answer to the nearest hundred square feet.

   a) 30,100 ft^2 b) 31,200 ft^2 c) 32,300 ft^2
   d) 32,600 ft^2 e) 33,100 ft^2

9. A convex polyhedron has 38 faces and 24 vertices. How many edges does it have?

   a) 12   b) 14   c) 16   d) 60   e) 64

10. A radioactive substance has a half-life of 100 days. If we start with 1000 grams of the substance, determine how many days it will take until we have only 150 grams left. Round your answer to the nearest day.

   a) 125 days   b) 190 days   c) 271 days   d) 274 days   e) 354 days

11. In a circle, an arc subtends a central angle measuring 160°. If the length of the arc measures 8π, what is the circumference of the circle?

   a) 9π b) 9   c) 18π d) 18   e) 6π

12. If log (x + 9) + log x = 2, which of these is a solution rounded to two decimal places?

   a) 0.22   b) 6.47   c) 9.22   d) 15.47   e) More than one of these

13. Let $f(x) = 3x^2 + 2x - 1$ and $g(x) = 2x^2 - 4$. Determine $f(g(x))$ and simplify.

   a) $f(g(x)) = 10x^2 + 39$
   b) $f(g(x)) = 12x^4 - 20x^2 - 57$
   c) $f(g(x)) = 18x^4 + 24x^3 - 12x^2 - 2$
   d) $f(g(x)) = 12x^4 - 44x^2 + 39$
   e) $f(g(x)) = 9x^4 + 6x - 5$

14. What is $-\sum_{k=1}^{258} i^k$?

   a) i   b) –i   c) 64   d) –1 + i   e) 1 – i

15. The path of a particle can be modeled by the parametric expression $(4+3t, 5-2t)$. Determine the total distance traveled by the particle from t = 0 to t = 2. Round your answer to the nearest tenth of a unit.

    a) 4.5   b) 5.0   c) 7.2   d) 9.1   e) Insufficient information

16. Find the polar form of the complex number $3+4i$. Answers are given with degrees rounded to the nearest tenth.

    a) $5\left(\cos(36.9°)+i\sin(36.9°)\right)$
    b) $5\left(\cos(53.1°)+i\sin(53.1°)\right)$
    c) $\sqrt{7}\left(\cos(36.9°)+i\sin(36.9°)\right)$
    d) $\sqrt{7}\left(\cos(46.9°)-i\sin(46.9°)\right)$
    e) $\sqrt{5}\left(\cos(36.9°)-i\sin(36.9°)\right)$

17. Matrix A and B are both square matrices with the same dimensions. Matrix A has a determinant of 7, and Matrix B has a determinant of 5. What is the determinant of $2A^{T}B$?

    a) –35   b) –5/7   c) 10/7   d) 70   e) 140

18. Write the $\log_6\left(\frac{1}{36}\right)=-2$ in exponential form.

    a) $\frac{1}{36}=6^{-2}$ b) $-2=6^{\frac{1}{36}}$ c) $6^{-2}=36$ d) $6^2=-36$ e) $\frac{1}{36}=-2^6$

19. What is the period of $\tan\left(\frac{2}{7}t-6\right)$? Round your answer to the nearest tenth.

    a) 0.6   b) 11.0   c) 21.0   d) 22.0   e) Undefined

20. A swimming pool has two pumps that can fill it. Pump A can fill the pool from empty to full in 5 hours, and pump B can fill it in 6 hours. Suppose the pool starts empty. At 6:00 AM, pump B is turned on. An hour later (i.e., 7:00 AM) pump A is turned on, and both pumps are left on until the pool fills. Determine what time the pool is filled. Round your answer to the nearest minute.

    a) 9:00 AM   b) 9:11 AM   c) 9:12 AM   d) 9:16 AM   e) 9:18 AM

21. Find the arithmetic mean of the observed values listed in the given frequency distribution. Round your answer to the nearest tenth.

Value	1	2	3	4	5	6	7	8	9	11
Frequency	1	2	1	3	2	1	1	1	2	1

    a) 4.0   b) 4.6   c) 5.3   d) 5.6   e) 6.0

22. What is the domain of $\log(\log(\log(x)))$?

    a) $(0,\infty)$ b) $(-\infty,\infty)$ c) $(100,\infty)$ d) $(10,100)$ e) $(10,\infty)$

23. How many distinguishable ways can we arrange the letters in MISSISSIPPI?

    a) 16,632   b) 25,788   c) 30,219   d) 34,650   e) 207,900

24. The population of a bacterial colony is given by $P(t) = 200 \cdot 2^{t/5}$ bacteria after t hours. How long will it take for the population to quintuple? Round your answer to the nearest tenth of an hour.

    a) 2.2   b) 4.0   c) 4.6   d) 11.6   e) 12.5

25. The path of a rock launched from a catapult can be modeled by the function $H(d) = -0.002d^2 + 1.6d + 10$, where d is the distance in feet downfield from the catapult and H is how high the rock is above the ground, also in feet. Determine how far away from the catapult the rock lands. Round your answer to the nearest whole foot.

    a) 330 ft   b) 400 ft   c) 660 ft   d) 800 ft   e) 806 ft

26. Simplify $\dfrac{\sec^2\theta}{\cot\theta} - \tan^3\theta$ to a basic trig function.

    a) $\cos\theta$ b) $\sin\theta$ c) $\tan\theta$ d) $\csc\theta$ e) $\sec\theta$

27. For the equation $\dfrac{2s-5x}{4y+6z} = \dfrac{4s+3x}{5y+3z}$, solve for s and leave your answer as a simplified fraction. For this solution, how many terms are in the denominator?

    a) 0   b) 1   c) 2   d) 3   e) 4

28. Find $\displaystyle\lim_{x\to\infty}\dfrac{12x^2-1}{24x^2+1}$.

    a) $-\infty$ b) $-\dfrac{1}{2}$ c) 0   d) $\dfrac{1}{2}$ e) Undefined

29. Give all solutions to $\dfrac{x^2-3x-4}{x^2-4} \le 0$.

    a) $[0,\infty)$ b) $[-1,4]$ c) $(-\infty,-1]\cup[4,\infty)$
    d) $[-2,-1]\cup[2,4]$ e) $(-2,-1]\cup(2,4]$

30. On a Friday night at a local pizzeria, the four delivery drivers made a total of 30 deliveries. On each delivery, the driver received a tip amount that was a whole number from 1 to 9. The following statements are also true:

   I. No delivery driver received the same tip amount twice.
   II. Each delivery driver went on a different number of deliveries.
   III. Anne's largest tip amount was four times bigger than her smallest tip amount.
   IV. Although Bill went on two more deliveries than Chris, he made $3 less in total tips.
   V. Denny made the most deliveries.

   What was the combined total amount of tips made by the four delivery drivers?

   a) $155
   b) $158
   c) $160
   d) $162
   e) Insufficient information