



2020 Academic Challenge

MATHEMATICS TEST – SECTIONAL

Physics Test Production Team

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

Time: 40 Minutes

Number of Questions: 30

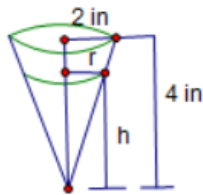
DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

Academic Challenge
2020 Sectional Mathematics Exam

1. In a game, you catch 4 of item A and 3 of item B. The probability of catching a discolored item A is 1%, the probability of catching a discolored item B is 2%, and the coloration of each item is independent of the others. What is the probability that two or more of the seven items are discolored? Round your answer to the nearest hundredth of a percent.

a) 0.24% b) 0.40% c) 6.80% d) 9.60% e) 10.00%

2. There is a snow cone with a radius of 2 inches and a height of 4 inches as shown below. Slushy mix is being pumped into the cone at a rate of $2 \text{ in}^3/\text{min}$. Find the rate at which the slushy level rises when the slushy is 3 inches deep.



a) 0.14 in/min b) 0.18 in/min c) 0.20 in/min
d) 0.22 in/min e) 0.28 in/min

3. If $2 + i$ is one of the roots of a cubic polynomial with all real coefficients, which one of the following must be one of the roots?

a) $2 - i$ b) $1 + 2i$ c) $1 - 2i$ d) 0 e) None of these

4. Aviator Bruce flew 70 miles east from Whiting Field to Coastland. From Coastland, he flew 100 miles north to Bay Watch. Find the least westward angle of turn, to the nearest degree, that must be made at Bay Watch to return to Whiting Field.

a) 35° b) 80° c) 145° d) 200° e) 210°

5. The vectors $A = \langle -4, 3 \rangle$ and $B = \langle 4, 3 \rangle$ both have a magnitude of 5 units. What is the magnitude of the vector $4A - 3B$? Round your answer to the nearest tenth of a unit.

a) 5.0 units b) 22.2 units c) 25.0 units d) 27.8 units e) 28.2 units

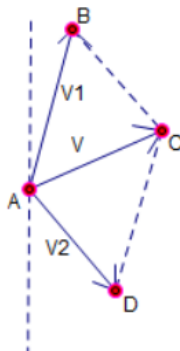
6. In a convex pentagon, what is the sum of the degree measures of the exterior angles?

a) 72° b) 108° c) 360° d) 540° e) 1260°

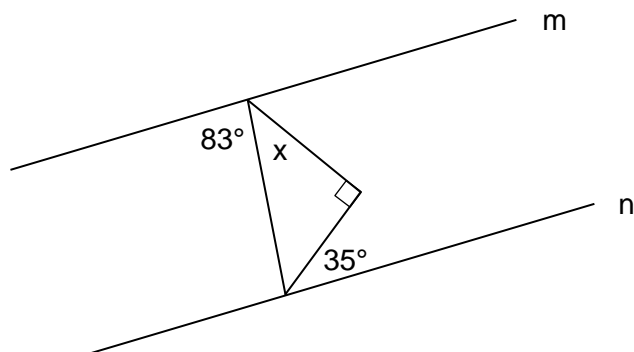
7. Find the sum of the series $\sum_{k=1}^{\infty} 4 \left(-\frac{3}{5} \right)^{k-1}$.

a) $\frac{5}{32}$ b) $\frac{2}{5}$ c) 2 d) $\frac{5}{2}$ e) $\frac{25}{8}$

8. What is the area of a regular octagon with side length 8 units? Round to one decimal.
- a) 38.6 b) 64.0 c) 154.5 d) 221.7 e) 309.0
9. A commercial jet is headed 20° east of north with an air speed of 550 miles per hour, corresponding to V_1 in the diagram below. The wind is blowing 25° east of south at a speed of 50 miles per hour, corresponding to V_2 . Find the ground speed of the jet, which would correspond to V . Note that the magnitudes shown in the diagram are not to scale. Round to the nearest mile per hour.



- a) 525 mi/hr b) 523 mi/hr c) 512 mi/hr
d) 255 mi/hr e) 200 mi/hr
10. In a population of alien animals known as fribbles, 10% of all fribbles are red, 60% of red fribbles have horns, and 20% of horned fribbles are red. What percentage of all fribbles have horns? Round your answer to the nearest whole percent.
- a) 1% b) 10% c) 30%
d) 50% e) Situation can't exist as written
11. A cone has a volume of 44 cubic meters and its radius is 5 times its height. What is its radius rounded to one decimal place?
- a) 1.4 m b) 2.1 m c) 4.1 m d) 4.4 m e) 5.9 m
12. Given that lines m and n in the diagram below are parallel, find x .



- a) 55° b) 50° c) 48° d) 42° e) 40°

13. Three times the cube of a number is equivalent to twenty-two more than that number. What is the sum of all possible values of the number?
- a) -2 b) -1 c) 0 d) 2 e) 22
14. In five years, June will be twice as old as Charlotte. Four years ago, June was three times as old as Charlotte. Find June's present age.
- a) 13 b) 15 c) 31 d) 35 e) 39
15. What is the eccentricity of the ellipse given by the equation $\frac{x^2}{36} + \frac{y^2}{100} = 1$? Round your answer to two decimal places.
- a) 0.36 b) 0.60 c) 0.64 d) 0.75 e) 0.80
16. Matrix A and Matrix B are square matrices of the same size. Matrix A has a determinant of 7, while Matrix B has a determinant of 24. What is the determinant of A + B?
- a) 17 b) 31 c) 168 d) 24/7 e) Insufficient information
17. Brenda has \$400 invested at 2% simple annual interest and \$600 at 3% simple annual interest. State the additional amount she must invest at 6% simple annual interest so that her total annual income will be 4% of her total investments.
- a) \$140 b) \$700 c) \$840 d) \$900 e) \$1000
18. If $x = 2 \cos t$ and $y = \sin t$, what shape do these make in the Cartesian plane?
- a) Circle b) Lemniscate c) Ellipse d) Cardioid e) None of these
19. Factor $1 - b^{12}$ completely.
- a) $(1-b)(1+b)(1+b+b^2)(1-b+b^2)(1+b^2)(1-b^2+b^4)$
 b) $(1-b)(1+b)(1-b+b^2)(1+b^2)(1-b^2+b^4)$
 c) $(1+b)(1+b)(1+b+b^2)(1-b+b^2)(1+b^2)(1-b^2+b^4)$
 d) $(1-b)(1-b)(1+b+b^2)(1-b+b^2)(1+b^2)(1-b^2+b^4)$
 e) $(1-b^6)(1+b^2)$
20. Determine the area enclosed by the graph of the polar function $r^2 = \frac{36}{4 + 5 \sin^2 \theta}$. Round your answer to the nearest tenth of a square unit.
- a) 12.6 square units b) 15.7 square units c) 18.8 square units
 d) 28.3 square units e) Graph does not enclose an area

21. What is the domain of $\ln e^{(x^x)}$?
- a) $(0, \infty)$ b) $(-\infty, \infty)$ c) $(-\infty, 0) \cup (0, \infty)$ d) $(1, \infty)$ e) None of these
22. How many 4-digit whole numbers exist where the last digit must be 9 and no digits are repeated?
- a) 720 b) 648 c) 512 d) 508 e) 448
23. Given $\frac{x^2 - 4}{x^2 - 2x - 3} \leq 0$, solve for x.
- a) $[-2, 2]$ b) $[-2, -1] \cup [2, 3]$ c) $(-2, -1) \cup (2, 3)$
d) $[-2, -1) \cup [2, 3)$ e) $(-\infty - 2] \cup (-1, 2] \cup (3, \infty)$
24. Simplify $\frac{\csc \alpha}{\cot \alpha + \tan \alpha}$ into a single trigonometric function.
- a) $\sin \alpha$ b) $\cos \alpha$ c) $\cot \alpha$ d) $\tan \alpha$ e) $\csc \alpha$
25. An investor takes her starting capital and invests 20% of it in utilities. She takes 25% of the remaining after utilities and invests it in construction. She takes 40% of the remaining after utilities and construction and invests it into service industries. She then takes the uninvested \$18,000 and puts it into various accounts. Determine how much the investor put into utilities.
- a) \$10,000 b) \$24,000 c) \$36,000 d) \$50,000 e) \$120,000
26. Which of the following is an antiderivative of $8xe^{3x^2} dx$?
- a) $\frac{1}{8}e^{3x^2}$ b) $\frac{4}{3}e^{3x^2}$ c) $4x^2e^{x^3}$ d) e^{3x^2} e) $4x^2e^{3x^2} + 8xe^{x^3}$
27. If you invest \$2000 in a savings account at 6% APR compounded monthly, how long will it take for your money to triple? Round to the nearest year.
- a) 12 years b) 15 years c) 18 years d) 21 years e) 24 years
28. What is $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$?
- a) 0 b) 1 c) -1 d) 3 e) ∞
29. Given $343^x = 2401^{x-2}$, solve for x.
- a) 0 b) 1 c) 2 d) 4 e) 8

30. A project team is made up of the manager Anne (f), Bill (m), Chris (f), Dave (m), Erin (f), and the assistant manager Flo (f). They are currently working on four projects. They start work at 8:00 and need to have all four projects done by 4:00. Anne starts the day by assigning Bill to project 1, Chris to project 2, Dave to project 3, and Erin to project 4. Anne assigns Flo to help on the newest project, while Anne herself starts assisting on the oldest project.
- I. At 10:00, Anne switches from the oldest project to a second that needs her help.
 - II. At 11:00, Bill and Dave stop working to go on a one-hour lunch break.
 - III. At noon, Bill returns and gets back to work on project 3. Flo stays to finish the newest project, while Dave joins everyone else on their one-hour lunch break.
 - IV. At 1:00, Flo finishes the newest project and heads off to a late one-hour lunch. Everyone else goes back to what they'd been working on before lunch, except for Erin, who switches over to the project that has only received three work-hours.
 - V. At 2:00, Flo returns from lunch and begins working on non-project office duties.
 - VI. At 3:00, the oldest project gets finished. The guy who'd been working on it starts helping Flo. The woman working with Anne assures her she can finish in an hour, so Anne switches over to help the two working on the other remaining project.
 - VII. At 4:00, the last two projects get finished, much to everyone's relief.

Which project took the most total work-hours, and how many did it require?

- a) Project 2 received the most at 11 work-hours.
- b) Project 2 received the most at 12 work-hours.
- c) Project 3 received the most at 11 work-hours.
- d) Projects 2 and 3 both received the most at 11 work-hours each.
- e) Insufficient information