2020 Academic Challenge
## State Mathematics Exam

1. A company increases production at a rate of 250 units per week. The weekly demand function is modeled by $p = 100 - 0.001x$ where p is the price per unit and x is the number of units produced in a week. Find the rate of change of the revenue with respect to time when the weekly production is 1500 units.

   a) $24,250/week   b) $12,050/week   c) $19,720/week
   d) $17,880/week   e) $20,000/week

2. Determine the volume of the solid generated by rotating the portion of $y = 6 - 2x$ bounded by x = 0 and x = 3 around the x-axis.

   a) 18π b) 36π c) 54π d) 108π e) 144π

3. Find $\int \dfrac{\cos x}{3 + 2\sin x}\, dx$.

   a) $2\ln|3 + 2\sin x| + C$ b) $\ln|3 + 2\sin x| + C$ c) $\dfrac{1}{2}\ln|3 + 2\sin x| + C$

   d) $\dfrac{1}{5}\ln|3 + \cos x| + C$ e) $2\ln|3 + 2\cos x| + C$

4. A semi-Norman window is formed by setting an equilateral triangle on top of a rectangle so that their sides line up. If the area is 108 sq. ft. and the perimeter is minimized, what is the length of a side of the equilateral triangle? Round your answer to the nearest tenth of a foot.

   a) 8.7   b) 10.1   c) 21.6   d) 75.4   e) It cannot be minimized.

5. Given $f(x) = 2x^2 - x$ and $g(x) = 3x - 4$, determine $(g \circ f)(x)$.

   a) $(g \circ f)(x) = 6x^2 - 3x - 4$
   b) $(g \circ f)(x) = 6x^3 - 3x^2 - 4x$
   c) $(g \circ f)(x) = 6x^3 - 11x^2 + 4x$
   d) $(g \circ f)(x) = 6x^4 - 11x^3 + 4x^2$
   e) $(g \circ f)(x) = 18x^2 - 51x + 20$

6. Kate will only go bike riding on a day that is neither cloudy nor windy. The probability that it is cloudy on a day is 0.3. The probability that it is windy on a day is 0.2. The probability that it is windy and cloudy on a day is 0.1. What is the probability that Kate will go bike riding on a day? Round to two decimal places.

a) 0.99   b) 0.60   c) 0.55   d) 0.40   e) 0.15

7.   How many nonreal zeroes does $P(x) = 4x^6 + 1$ have?

     a) 0   b) 1   c) 2   d) 4   e) 6

8.   What is the magnitude of the quotient of $(12 + 5i)$ and $(3i - 4)$? Round to the nearest tenth if necessary.

     a) 2.4   b) 2.6   c) 5.8   d) 17.0   e) Nonreal answer

9.   A person given a flu shot has a probability of 0.8 of becoming immune to that flu strain. If 60 people are given the flu shot, find the probability that at most 44 people develop immunity to the flu strain. Round to two decimal places.

     a) 0.98   b) 0.88   c) 0.60   d) 0.27   e) 0.13

10.   Sam can paint 1 square foot in 10 seconds with a brush. Kay can paint 1 square foot in 2 seconds with a roller. They need to paint a room whose walls are a total of 600 square feet. Sam starts painting with her brush at 7:00 AM. At 7:10 (i.e., ten minutes later), Kay joins in painting with her roller. The two work together until the room is finished. Determine at what time they finish painting. Round your answer to the nearest whole minute.

     a) 7:15 AM   b) 7:25 AM   c) 7:27 AM   d) 7:42 AM   e) 7:57 AM

11.   A child starting from the top of a metal slide, pushes a doll with an initial rate of 25 cm/s. At this start, the doll then accelerates at 35cm/s during each second. After how many seconds is the velocity 305 cm/s? Round to the nearest tenth of a second.

     a) 4.5 sec   b) 6.0 sec   c) 8.0 sec   d) 8.5 sec   e) 9.0 sec

12.   How many of the following measures can be the angle measure of a regular polygon: 60°, 90°, 120°, and 180°?

     a) 0   b) 1   c) 2   d) 3   e) 4

13.   A building site is 60 miles away from a supply depot. A construction truck makes one round trip in 7 hours total. The average speed from the depot to the building site is 5 mi/hr less than the return trip from site to depot because of the weight of the load. What is the average speed of the truck when it returns from the building site to the depot?

     a) 20 mph   b) 17 mph   c) 15 mph   d) 12 mph   e) 10 mph

14. What is the sum of the real x-values for which the graph of $y = x^3 + 1$ intersects the graph of $y = 3x^2 + 17$?

   a) 3   b) 3.5   c) 4   d) 4.5   e) 5

15. How many edges does a regular dodecahedron have?

   a) 12   b) 20   c) 24   d) 30   e) 60

16. Pilot Daffy is flying a plane travelling at a speed of 300km/hr. Daffy wants to fly directly to a destination that is $43.2°$ south of east, but there is a tail wind blowing from the west at a speed of 40 km/hr. In order to end up flying the correct direction, what heading should pilot Daffy direct the plane? Round to the nearest degree.

   a) $32° \text{NE}$ b) $42° \text{NW}$ c) $43° \text{SE}$ d) $44° \text{SW}$ e) $48° \text{SE}$

17. If $x = t - 1$ and $y = 3t^2 + 3t - 2$, what is the x-coordinate of the vertex of the parabola formed?

   a) −1.5   b) −1   c) −0.5   d) 1   e) 2

18. A storm front moving east at 22 km/h and south at 12.5 km/h. Find the resultant velocity. Round degrees and velocity to 1 decimal place.

   a) $25.3° \text{SE}$; 29.3 km/h   b) $60.4° \text{SE}$; 29.3 km/h   c) $29.3° \text{SE}$; 60.4 km/h
   d) $29.6° \text{SE}$; 25.3 km/h   e) $60.4° \text{SE}$; 25.3 km/h

19. What is the shape of the graph of $r = 5\cos(6\theta)$?

   a) Rose   b) Lemniscate   c) Limaçon
   d) Cardioid   e) None of these

20. In the figure below, triangles ACD and BCE are equilateral and congruent, and the area of triangle ABC is twice that of triangle CDE. If the area of the trapezoid ABED is 20 square inches, determine the trapezoid's perimeter. Round your answer to the nearest tenth of an inch.
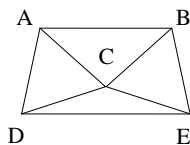
a) 17.9 inches　　　　　b) 18.8 inches　　　　　c) 19.0 inches
d) 19.5 inches　　　　　e) Insufficient information

21.　Write an equation of the line that is perpendicular to $y-2=\dfrac{1}{2}x+4$ and passes through $(5,2)$.

a) $y=2x-4$　　　　　b) $y=\dfrac{1}{2}x-\dfrac{1}{2}$　　　　　c) $y=-\dfrac{1}{2}x+\dfrac{9}{2}$

d) $y=-2x+12$　　　　　e) $y=-2x+4$

22.　The number of minutes, t, that it takes to charge a battery to a level C is $t=-200\log\left(1-\dfrac{C}{C_0}\right)$ where C_0 is its maximum charge. How long will it take to charge the battery to 75% of its maximum charge? Round to the nearest minute.

a) 25　　　　b) 58　　　　c) 60　　　　d) 120　　　　e) 277

23.　What is the shape of the graph of $y^2=6x-8y-x^2+24$?

a) line　　　　b) circle　　　　c) cardioid　　　d) hyperbola　e) parabola

24.　An urn contains the whole numbers from 1 through 100. Two numbers are drawn without replacement. What is the probability that both are multiples of either 3 or 7? Round your answer to three decimal places.

a) 0.182　　　　b) 0.185　　　　c) 0.218　　　　d) 0.221　　　　e) 0.224

25.　In the figure below, each of the three circles have an area of 1 square inch. Assuming the three circles are tangent, determine the area of the shaded region between them. Round your answer to the nearest hundredth of a square inch.
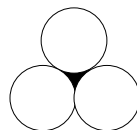
a) 0.05 in^2　　　b) 0.08 in^2　　　c) 0.10 in^2　　　d) 0.12 in^2　　　e) 0.15 in^2

26. Find k when the determinant of matrix $\begin{bmatrix} 1 & 3 & 2 \\ -2 & k & -1 \\ 2 & 4 & 0 \end{bmatrix}$ is 2.

    a) $k = -1$　　b) $k = 3$　　c) $k = -5$　　d) $k = 0$　　e) $k = 1$

27. Determine how many solutions for $\sin^3 x = \sin x$ exist on $[0, 2\pi)$.

    a) 1　　　b) 2　　　c) 3　　　d) 4　　　e) 5

28. Write $\left(81^{\frac{1}{3}} m^{\frac{5}{3}} n^{\frac{6}{3}} \right)\left(9^{\frac{1}{3}} m^{\frac{1}{3}} n^{\frac{10}{3}} \right)$ in simplest radical form:

    a) $9m^2 n^5 \sqrt[3]{n}$　　　　　　b) $9m^2 n^4 \sqrt[3]{n}$　　　　　　c) $6m^2 n^6 \sqrt[3]{mn}$

    d) $12m^2 n^4 \sqrt[3]{m^2 n}$　　　　e) $6m^2 n^4 \sqrt[3]{m^2 n}$

29. What is $\lim\limits_{x \to 0} \dfrac{x}{\csc x}$?

    a) 0　　　b) 1　　　c) −1　　　d) ∞　　　e) Does not exist

30. Four biology students, Amy (f), Bill (m), Charles (m), and Dana (f), recently gave presentations on animals. The presentations were at 9:00, 9:10, 9:20, and 9:30. The following facts are also true:

    i.   Dana went right after one boy, and right before the other.
    ii. The presentation on mammals was right after Bill's presentation
    iii. A boy presented on birds right before a girl presented on fish.
    iv. Amy went right after the presentation on reptiles

    Based on these facts, who presented third (at 9:20), and what was their topic?

    a) Bill, reptiles
    b) Charles, reptiles
    c) Charles, birds
    d) Dana, fish
    e) Dana, mammals