Academic Challenge
Physics Solutions (State) – 2020

1. **Correct Response: E**
Since M and X are different quantities and dimensions they cannot be added or subtracted. So E is the only expression with only multiplication and division.

2. **Correct Response: E**
The density is the mass divided by the volume so the mass is the density times the volume. Length x depth x height is the volume. $M = 8 \times 20 \times 5 \times 1.2 = 960\ kg$

3. **Correct Response: E**
The angles between the vectors is unknown. So they can range from 0° to 180°. If it is the former the sum would be the algebraic addition or 208 and if it is the latter it is the difference or 34. For other angles it is possible to get every magnitude in between these two values. 218 is not between these two.

4. **Correct Response: D**

Apply kinematics equations for projectile motion. Set up the *xy* coordinate system so that the origin is at the launch site, the +*x* direction is to the right, and the +*y* direction is upward.

$x = x_0 + v_{0x}t + (1/2)a_x t^2 \rightarrow x = (v_0 \cos\theta)t$
$y = y_0 + v_{0y}t + (1/2)a_y t^2 \rightarrow y = (v_0 \sin\theta)t - (1/2)gt^2$

When the ball lands on the ground, $y = 0$. So, $0 = (v_0 \sin\theta)t - (1/2)gt^2$.

Solving for the time of flight t, we have $t = 2v_0 \sin\theta/g$.

So, the range R is $R = (v_0 \cos\theta)(2v_0 \sin\theta/g) = v_0^2 \sin 2\theta/g$.

For the same range, we have $v_{01}^2 \sin 2\theta_1 = v_{02}^2 \sin 2\theta_2$. Solving for v_{02}, we have

$v_{02} = v_{01}(\sin 2\theta_1/\sin 2\theta_2)^{1/2} = (10.0\ \text{m/s})(\sin 30.0°/\sin 60.0°)^{1/2} = 7.60\ \text{m/s}$

5. **Correct Response: C**
This is the definition of a conservative force.

6. **Correct Response: A**

Set up the coordinate system so that the origin is at the position of the hand and the +*y* direction is downward. Considering the ball's motion in the elevator's frame, we have

$v_{0y} = 0$, $a_y = 9.80\ \text{m/s}^2 - 0.600\ \text{m/s}^2 = 9.20\ \text{m/s}^2$, $y_0 = 0$, and $y = 1.20$ m.

Apply kinematics equation for one-dimensional motion.

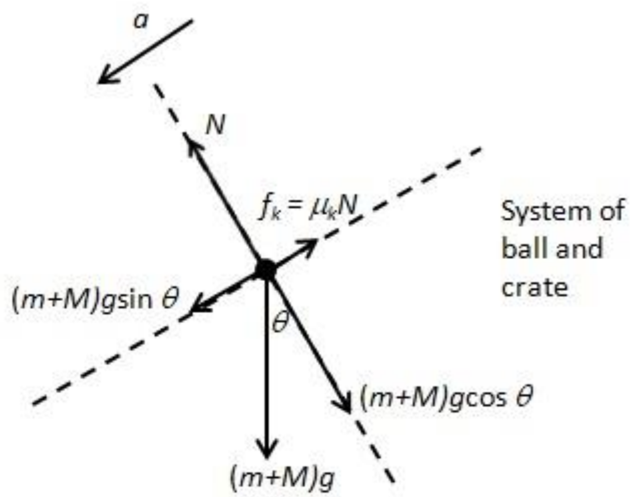
$y = y_0 + v_{0y}t + (1/2)a_y t^2 \rightarrow y = (1/2)a_y t^2$

Solving for t, we have

$t = (2y/a_y)^{1/2} = [2(1.20\ \text{m})/(9.20\ \text{m/s}^2)]^{1/2} = 0.511$ s

7. **Correct Response: C**

First, consider the ball and the crate as a system and find the common acceleration of the ball and crate. Draw a free-body diagram for the system and include all external forces acting on the system.



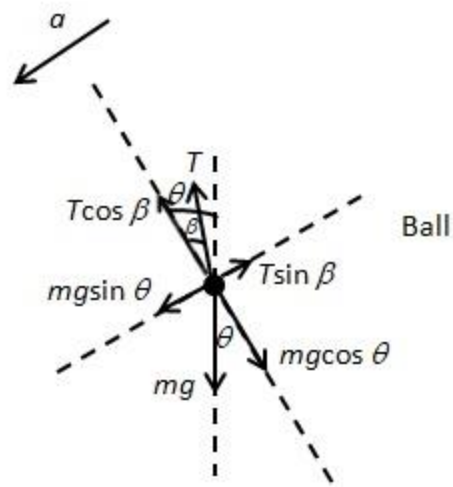
Apply Newton's 2nd Law to the system of the ball and crate.

$(m+M)g\sin\theta - \mu_k N = (m+M)a$, and
$N = (m+M)g\cos\theta$, where m and M are the masses of the ball and crate, respectively.

So, $(m+M)g\sin\theta - \mu_k(m+M)g\cos\theta = (m+M)a$.

$a = g(\sin\theta - \mu_k\cos\theta)$

Draw a separate free-body diagram for the ball.



Apply Newton's 2nd Law to the ball.

$mg\sin\theta - T\sin\beta = ma$, and $T\cos\beta = mg\cos\theta$. So,

$\tan\beta = (g\sin\theta - a)/g\cos\theta = [g\sin\theta - g(\sin\theta - \mu_k\cos\theta)]/[g\cos\theta] = \mu_k$

$\beta = \arctan(\mu_k) = \arctan(0.2000) = 11.30°$ and $\alpha = \theta - \beta = 30.0° - 11.30° = 18.7°$

8. Correct Response: E

If there is no friction between the crate and the inclined plane, $a = g\sin\theta$ and $\tan\beta = 0$. Thus, $\beta = 0$ and $\alpha = \theta = 30.0°$.

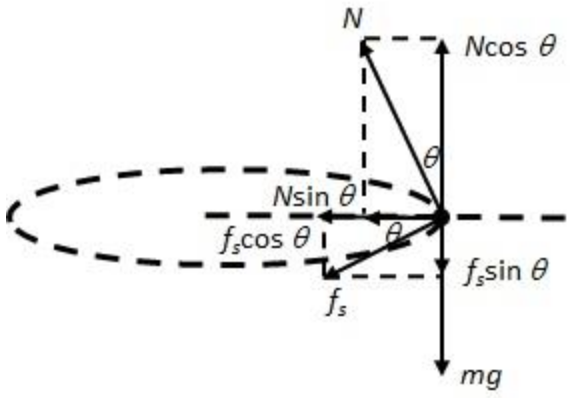
9. Correct Response: D

The change in the velocity is the area under the acceleration as a function of time curve.

$$v = \frac{1}{2}(2 \times 6) + (2 \times 4.5) + \frac{1}{2}(2 \times 1.5) = 16.5\ \frac{m}{s}$$

10. Correct Response: B

Draw a free-body diagram for the block, as shown below.



The maximum speed of rotation occurs when the block is on the verge of sliding upward along the inclined plane. Thus, the static frictional force is directed downward along the inclined plane and $f_s = f_{s,\,max} = \mu_s N$.

Applying Newton's 2nd Law for circular motion, $F_C = ma_c$, and realizing $a_c = v^2/R = (R\omega)^2/R = R\omega^2$, we have

$$N\sin\theta + \mu_s N\cos\theta = mR\omega^2 \rightarrow \omega = \sqrt{\frac{N(\sin\theta + \mu_s\cos\theta)}{mR}} \qquad (1)$$

In the vertical direction,

$$N\cos\theta = \mu_s N\sin\theta + mg \rightarrow N = mg/(\cos\theta - \mu_s\sin\theta) \qquad (2)$$

Substituting N with Eq. (2) in Eq. (1) and rearranging the terms, we have

$$\omega = \sqrt{\frac{g(\sin\theta + \mu_s\cos\theta)}{R(\cos\theta - \mu_s\sin\theta)}}$$

11. Correct Response: A

Impulse is defined as force times time. So, [I] = [F][T] = {[M][L]/$[T]^2$}[T] = [M][L]/[T].

12. Correct Response: E

The acceleration is given by the second derivative of the position:

$$a(t) = \frac{d^2x}{dt^2} = -7.46\frac{m}{s}$$

And the force is the mass times the acceleration =1.62 (-7.46)= -12.085 N.

13. Correct Response: A

The two blocks are at rest relative to each other (even though they are on the verge of sliding relative to each other) and thus have the same acceleration. Since the surface is frictionless, the net force acting on the two-block system has a magnitude F and is directed to the left. Apply Newton's Second Law in the horizontal direction:

$F = (m_1 + m_2)a \Rightarrow a = F/(m_1 + m_2) = 12.0 \text{ N}/(3.10 \text{ kg} + 2.20 \text{ kg}) = 2.26 \text{ m/s}^2$

14. Correct Response: C

Launching the wrench backwards means that it has less total energy and therefore, will begin an elliptical orbit with apogee equal to the original radius. The ellipse has a smaller semimajor axis, so by Kepler's 3rd Law, it will also have a shorter period. That means that it will, in fact, pass the space station over the next half period and will also be between the station and the Earth.

15. Correct Response: E

Apply Conservation of Mechanical Energy to find the speed v of the first object just before it collides with the second object.

$mgh = (1/2)mv^2 \rightarrow v = (2gh)^{1/2}$.

Apply Conservation of Momentum to find the speed v' of the two objects (stuck together) right after the collision.

$mv = 2mv' \rightarrow v' = v/2 = (gh/2)^{1/2}$.

Apply Conservation of Mechanical Energy to find the height h' that the two objects can reach on the second inclined plane.

$(1/2)(2m)v'^2 = (2m)gh' \rightarrow h' = h/4$

16. Correct Response: A

Apply thin-lens equation to lens #2 (diverging lens): $1/s_2 + 1/s_2' = 1/f_2$.

Here, $s_2' = 250$ cm – 65.0 cm = 185 cm and $f_2 = -85.0$ cm. Solving for s_2, we have $s_2 = -58.24$ cm. Since $s_2 < 0$, the object for lens #2 is virtual and is at x = 65 cm + 58.2 cm = 123.2 cm, behind Lens 1. Thus, the image of lens #1, which serves as the object for lens #2, is a real image.

Apply thin-lens equation to lens #1 (converging lens): $1/s_1 + 1/s_1' = 1/f_1$.

Here, $s_1' = 65.0$ cm – 15.0 cm + 58.24 cm = 108.24 cm and $f_1 = 45.0$ cm. Solving for s_1, we have $s_1 = 77.02$ cm. Since $s_1 > 0$, the object of lens #1 is real and is located on the front side (to the left) of lens #1. So, x = 15.0 cm – 77.02 cm = –62.0 cm. The source is located at x = –62.0 cm.

17. Correct Response: D

At normal incidence the apparent depth of an object in a material with index of refraction, n_w, is given by:

$$d = n_w d_{app}$$

Where d is the actual depth and n_w is the index of the water and d_{app} is the apparent depth. Solving for the depth gives us 1.34x2.79m = 3.74 m.

18. Correct Response: D

The first issue is to determine the speed of the eagle when it gets to the water. Based on the height that it starts its fall we find that (without any air resistance) it would be

$$mgh = \frac{1}{2}mv^2$$

Solving for v there would result in 40.9 m/s which is considerably higher than the terminal speed. So the speed at the water is 23.6 m/s. The net force of gravity and water stops the eagle in the distance of 2.79 m. This is a work energy problem:

$$\frac{1}{2}mv^2 - Fd_{app} = 0$$

Solving this for F gives 374 N.

19. Correct Response: E

Apply work-energy theorem: $W_{tot} = \Delta K$.

$(mg\sin\theta - \mu_k N)s = (1/2)mv_2^2 - (1/2)mv_1^2$, where $N = mg\cos\theta$. So,

$mg(\sin\theta - \mu_k\cos\theta)s = m(v_2^2 - v_1^2)/2$. Solving for μ_k, we have

$$\mu_k = \tan\theta - \frac{v_2^2 - v_1^2}{2gs\cos\theta} = \tan 40.0° - \frac{(3.50 \text{ m/s})^2 - (1.50 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(2.00 \text{ m})\cos 40.0°} = 0.506$$

20. Correct Response: B

Power is related to the force and the speed according to the equation:

$$P = Fv$$

We are given the mass and the coefficient of friction which allows us to calculate the force of friction which is what the power is overcoming.

$$F_k = \mu_k F_N = \mu_k mg = 0.55 \times 865kg \times 9.81\frac{m}{s^2} = 4667.1N$$

So the power is simply this times the speed, or 10500 W. However, the answers are in horsepower so we need to convert by dividing by 746. The total power in the correct units is 14.07 hp.

21. Correct Response: B

Once the rod starts moving, there is an induced current I flowing through the rod.

Apply Faraday's Law of Induction.

$|\varepsilon| = |d\Phi_B/dt| = B|dA/dt| = BLv$ and $I = |\varepsilon|/R = BLv/R$.

According to Lenz's Law, the direction of current flow in the closed loop is such that the resulting magnetic force acting on the rod opposes the motion, so is directed upward parallel to the inclined plane and the magnitude of the magnetic force is

$F_B = ILB = B^2L^2v/R$.

Apply Newton's 2nd Law to the rod in the direction parallel to the inclined plane: $F_{net} = ma$.

$mg\sin\theta - F_B = ma \rightarrow mg\sin\theta - B^2L^2v/R = ma \rightarrow a = g\sin\theta - B^2L^2v/mR$.

22. Correct Response: C

We start with the parallel axis theorem to determine the moment of inertia of the disk rotating around the axis.

$$I = I_c + md^2 = \frac{1}{2}mr^2 + md^2 = 0.0599kg\,m^2$$

Then the torque is given by

$$\tau = I\alpha = 9.46 \times 10^{-3} kg\frac{m^2}{s^2}$$

23. Correct Response: A

Rotational kinematics gives us:

$$\omega = \omega_o + \alpha t = 0 + .158\frac{rad}{s^2}14.6s = 2.3068\frac{rad}{s}$$

And then we can calculate the angular momentum (though we must determine the moment of inertia based on an axis through the center now.

$$L = I\omega = \frac{1}{2}mr^2\omega = .0733\,kg\,\frac{m^2}{s}$$

24. Correct Response: C

Realizing that the initial velocity should be resolved into x and y components we can write the kinematic equations for position of a projectile under the force of gravity.

$$y = y_o + v_{o\,y} t - \frac{1}{2} g t^2$$
$$x = x_o + v_{o\,x} t$$

Since we know the angle of launch we can solve the second for t and plug it into the first to solve for v_o.

$$t = \frac{345\ m}{v_o cos45}$$

And

$$0.00 = 5.75\ m + \ v_o (sin45) \frac{345\ m}{v_o cos45} - \frac{1}{2} g \left\{ \frac{345\ m}{v_o cos45} \right\}^2$$

This can be solved for v_o to get 57.7 m/s.

25. Correct Response: D

Isochoric means the same volume.

26. Correct Response: D

Proper length is a length that is measured in the frame that is at rest relative to the object.

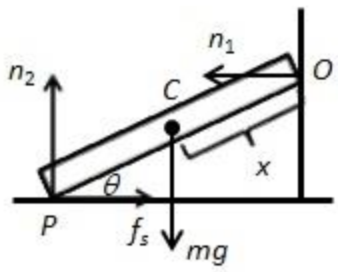
27. Correct Response: A

Suppose that at some instant of time the source emits a crest. The crest travels towards the listener at the speed of sound *v* in air. After a time interval of *T*, where *T* is the period of the source, the source emits another crest. By this time, the previous crest has traveled a distance *vT* and the source has traveled a distance *uT*. So, in the region in front of the source, the wavelength (distance between adjacent crests) of the sound wave as measured by the listener is

$\lambda' = vT - uT = (v - u)T = (v - u)/f$.

28. Correct Response: D

Draw a free-body diagram of the object, as shown below.



Assume that the center of mass is a distance x from point O.

Apply condition for static equilibrium, net force = 0, in the vertical direction.

$n_2 = mg$. Therefore, the normal force exerted by the floor on the object does not change when the center of mass changes.

Choose point O as the point of rotation. Apply condition for static equilibrium, net torque = 0, about point O.

$n_2 L\cos\theta = mgx\cos\theta + f_s L\sin\theta \rightarrow mgL\cos\theta = mgx\cos\theta + f_s L\sin\theta$
$\rightarrow$ $mg\cos\theta(L - x) = f_s L\sin\theta \rightarrow \tan\theta = mg(L - x)/f_s L$.

The maximum value for the static frictional force is $f_{s,\,max} = \mu_s n_2 = \mu_s mg$. So, for a given x,

$\tan\theta_{min} = mg(L - x)/\mu_s mgL = (L - x)/\mu_s L$. When C shifts downward to C', x increases, and thus θ_{min} decreases.

29. Correct Response: A

This is partially a conversion problem because we need to go from km/hr to m/s and g to kg and ms to s. The latter two are simply division by 1000. But for the first we have:

$$v = 263\frac{km}{hr} \times \frac{1000m}{1km} \times \frac{1\ hr}{60\ min} \times \frac{1min}{60s} = 73.056\frac{m}{s}$$

Acceleration is the change in velocity divided by the change in time. The force required is the mass times the acceleration. So we have

$$F = ma = m\frac{\Delta v}{\Delta t} = 0.0577kg \times \frac{73.056\frac{m}{s}}{0.0298s} = 141.45N$$

30. Correct Response: E

Rayleigh's Criterion: $\theta_{min} = 1.22\lambda/D$, where λ is the wavelength of light and D is the diameter of the aperture. So, the shorter the wavelength λ and the larger the diameter D of the aperture, the smaller the minimum angular separation.

31. Correct Response: C

The gravitational force from a uniform sphere depends on the mass within the sphere defined by the radius of the measurement. So we only have to worry about the mass within the radius r. For a mass m at a radius r we realize that the density of the sphere must play in to get the amount of mass within the sphere of radius r.

$$\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3}$$

And the mass within the sphere of radius r is given by:

$$M = \rho \times \frac{4}{3}\pi r^3$$

So the total gravitational force is

$$F = G\frac{mM}{r^2}$$

Now we can plug in the value for r to get:

$$F = G\frac{m \times \frac{M_E}{\frac{4}{3}\pi R_E^3} \times \frac{4}{3}\pi\left(\frac{R_E}{2}\right)^3}{\left[\frac{R_E}{2}\right]^2} = G\frac{mM_E}{2R_E^2}$$

32. Correct Response: A

The parallel resistors have a combined effective resistance of:

$$\frac{1}{R_{eff}} = \frac{1}{3\Omega} + \frac{1}{6\Omega}$$

Solving for the effective resistance we get $R_{eff} = 2\Omega$

so the total resistance of the circuit is 7Ω (5Ω+2Ω)which leads us to calculate the current through the 5Ω resistor to be 6/7 A using Ohm's Law. That current is divided between the two branches of the parallel part of the circuit in a 2:1 ratio since the resistances are in that ratio. Therefore the current through the 6Ω resistor is 2/7 A which is 286 mA.

33. Correct Response: A

Energy per photon, $E = hf = hc/\lambda$.

$0.8P = nE = nhc/\lambda$, where n is the number of photons emitted per second.

$n = 0.8P\lambda/hc = 4P\lambda/5hc$

34. Correct Response: B

The Rutherford scattering experiments established that the positive charge is not distributed uniformly throughout the volume of the atom; instead, the atom has a nucleus that is positively charged and occupies a very small space in the atom.

35. Correct Response: D

If we define the amount of Technetium as N_T and the amount of Polonium as N_P and we have the usual exponential decay as a function of time we see:

$$N_T(t) = N_o e^{-\frac{.693t}{18.0}}$$

And

$$N_P(t) = N_o e^{-\frac{.693t}{3.05}}$$

We want to find the time when $N_P = ½\ N_T$. So we can write:

$$N_P(t) = N_o e^{-\frac{.693t}{3.05}} = \frac{1}{2} N_T(t) = \frac{1}{2} N_o e^{-\frac{.693t}{18.0}}$$

Now we have an equation that we can solve for t.

$$N_o e^{-\frac{.693t}{3.05}} = \frac{1}{2} N_o e^{-\frac{.693t}{18.0}}$$

The N_o terms cancel and we can solve for t by taking the natural log of both sides:

$$-\frac{.693t}{3.05} = -ln2.00 - \frac{.693t}{18.0}$$

A little algebra gives:

$$t\left\{\frac{1}{3.05} - \frac{1}{18.0}\right\} = 1$$

This gives us t=3.6722 min.