

2021 REGIONAL MATHEMATICS EXAM

1. A toy bank holds 80 coins consisting of nickels, dimes, and quarters. There are twice as many quarters as nickels. The total value of the coins is \$13. How many quarters are in the collection?

a) 10 b) 15 c) 20 d) 35 e) 40
2. An event has 12 teams, the top 3 of whom will each win a different trophy. In how many different ways can the trophies be awarded?

a) 33 b) 220 c) 1,320
d) 79,833,600 e) 479,001,600
3. Two cars, 240 miles apart, begin driving toward each other on a long straight highway. One car travels 50 mph and the other 70 mph. At the same time, a falcon, starting on one car, flies back and forth between the two cars as they approach each other. If the falcon flies at 80 mph and turns around instantly at each car, how far has the falcon flown when the cars meet?

a) 160 miles b) 300 miles c) 450 miles d) 600 miles e) 750 miles
4. If the two roots of a quadratic function with real coefficients are $3 - i$ and $3 + i$, what is the constant term of the expanded and simplified quadratic function?

a) $8 - 6i$ b) $8 + 6i$ c) 6 d) 10 e) Insufficient information
5. When painting a room, Tom takes 10 seconds to paint a square foot using a brush. Bill is using a roller and only takes 2 seconds per square foot. The two of them need to paint the walls of a room, covering a total of 600 square feet. Tom starts at 8:00 AM and begins painting by himself using the brush. Bill joins in at 8:10 AM, and the two of them work together to do a single coat of paint. Determine when they finish. Round to the nearest whole minute on the clock.

a) 8:15 AM b) 8:17 AM c) 8:25 AM d) 8:27 AM e) 8:52 AM
6. Five years ago, Bob was twice as old as Sue was then. Now, the difference of their ages is 25 years. How old is Bob now?

a) 20 b) 25 c) 30 d) 40 e) 55
7. Square A has sides of length s . Square B has sides of length s^2 . Now consider the area of the two squares. Which of the following is a possible percentage **change** in area from square A to square B?

- a) 0% b) 75% c) -75% d) 300% e) All of these
8. The back window of a 2-door sports car is a rectangle with semicircles on each of the sides. The window has a height of 2 feet top to bottom and a width of 6 feet at its widest point. You want to trim the boundary of the window in gold. How much trim will be needed for the job?
- a) $\pi + 8$ ft b) 8π ft c) $2\pi + 8$ ft d) $4\pi + 8$ ft e) 32π ft
9. What is the area of a regular nonagon with apothem 8? Round to one decimal place.
- a) 38.6 b) 64.0 c) 154.5 d) 209.6 e) 309.0
10. After receiving her monthly paycheck, Tricia immediately spends 20% of it on rent. Using what is left after rent, she spends 40% of that on other bills. Of the remaining after bills, she spends 25% of that on food. After food, she is left with \$900 to spend on other things. What was Tricia's original monthly check? Round to the nearest whole dollar.
- a) \$1,059 b) \$1,665 c) \$1,890 d) \$2,500 e) \$6,000
11. There are \$320 available to fence in a rectangular garden. Fencing alongside the road costs \$6 per foot, and the fencing for the other three sides costs \$2 per foot. Find the dimensions of the largest possible garden that can be fenced in.
- a) 40ft \times 40ft b) 20ft \times 30ft c) 20ft \times 40ft
d) 40ft \times 30ft e) 35ft \times 40ft
12. What is the area of a regular hexagon with sides of length 2? Round to the nearest tenth of a square unit.
- a) 1.7 b) 2.6 c) 4.1 d) 10.4 e) 12.0
13. Find the remainder of $\frac{2x^3 - 3x^2 + 8x - 2}{x^2 - x + 2}$.
- a) $-3x$ b) $2x - 1$ c) 3 d) $3x$ e) -3
14. What is the coefficient of the 7th term in the expanded form of $(x + 2)^{13}$? Assume that it is written in order of highest to lowest degree.
- a) 64 b) 1,716 c) 109,824 d) 219,648 e) 8,648,640

15. A radioactive substance is losing 10% of its material a year to exponential decay. Determine this material's half-life. Round to the nearest tenth of a year.
- a) 4.6 years b) 5.0 years c) 5.7 years d) 6.6 years e) 6.9 years
16. Find the volume of a right circular cone with a base diameter of 23.8 cm and a slant height of 15.8 cm. Round volume to the nearest cubic centimeter.
- a) 9372 cm^3 b) 2343 cm^3 c) 1541 cm^3
d) 1348 cm^3 e) 1328 cm^3
17. Matrix A and Matrix B are both square matrices with the same dimensions. Matrix A has a determinant of 7, and Matrix B has a determinant of 5. What is the determinant of $2A^T B$?
- a) -35 b) $-5/7$ c) $10/7$ d) 70 e) 140
18. In 2010, there were 1000 rabbits in Bunny Burrow. The population grows 9% every year. How many rabbits will there be in 2021? Round to the nearest rabbit.
- a) 1,990 b) 2,285 c) 2,341 d) 2,580 e) 2,691
19. What is the period of $\sin\left(\frac{2}{9}t - 4\right)$? Round to the nearest tenth.
- a) 1.4 b) 11.0 c) 14.1 d) 22.0 e) 28.3
20. Convert the rectangular coordinates $(-4, 6)$ into polar coordinates. Round to the nearest tenth of a unit and nearest tenth of a radian.
- a) $(4.5, -1.0)$ b) $(4.5, 2.2)$ c) $(7.2, -1.0)$
d) $(7.2, 1.0)$ e) $(7.2, 2.2)$
21. Given $x = t - 2$ and $y = t^2 - 4t$, determine the shape of the graph and the vertex of any parabolas or center of any circles.
- a) parabola with vertex $(0, -4)$
b) parabola with vertex $(0, -2)$
c) parabola with vertex $(0, 4)$
d) parabola with vertex $(-4, 0)$
e) circle with center $(0, -4)$
22. (Matt, logarithms, medium) What is the domain of $\log(x^2 - 2x + 3)$?
- a) $(0, \infty)$ b) $(-\infty, \infty)$ c) $(100, \infty)$ d) $(10, 100)$ e) $(10, \infty)$

23. Determine the sum of the infinite series $8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \dots$.
- a) 24 b) 18 c) 17 d) 16 e) 15
24. The population of a bacterial colony is given by $P(t) = 200 \cdot 2^{t/5}$ bacteria after t hours. How long will it take for the population to reach 10,000? Round to the nearest tenth of an hour.
- a) 1.1 b) 16.1 c) 28.2 d) 34.4 e) 46.0
25. The lowest a ceiling-mounted video projector can be aimed is a 20-degree angle down. The tech crew has been instructed to place the projector in such a way that it is aimed at the center of a thirteen-foot high wall. What is the least distance from the wall that the projector must be placed for it to be aimed properly? Round your answer to the nearest whole foot, and assume the ceiling is perpendicular to the wall.
- a) 2 feet b) 18 feet c) 20 feet d) 33 feet e) 36 feet
26. Starting with the general form of the ellipse $9x^2 - 36x + 16y^2 + 96y + 36 = 0$, find the length of the major axis.
- a) 9 b) 8 c) 7 d) 4 e) 3
27. For how many different real numbers a is $a^{19} = a$?
- a) 0 b) 1 c) 2 d) 3 e) 4
28. Given the matrices A and B and the product AB below, find k .
- $$A = \begin{bmatrix} 1 & 0 & 3 \\ k & -4 & 1 \\ 1 & 3 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix} \quad AB = \begin{bmatrix} 1 \\ -5 \\ 14 \end{bmatrix}$$
- a) 2 b) 0 c) -1 d) -3 e) -4
29. What is $\lim_{x \rightarrow \infty} \frac{x^2 - 3x + 5}{x^9 - 1}$?
- a) 0 b) 1 c) -5 d) ∞ e) $-\infty$
30. Four sisters are learning to play instruments for a band. The sisters' names are Ariella, Britany, Callie, and Danita. The four instruments in question are clarinet, flute,

saxophone, and trumpet. The girls' ages are 8, 10, 12, and 14. The following facts are also true:

- I. Ariella is learning the clarinet.
- II. The sister learning the trumpet is 4 years younger than Danita.
- III. Britany's age is a multiple of 4.
- IV. The 10-year-old is learning to play the flute.

How old is Callie?

- a) 8 b) 10 c) 12 d) 14 e) Insufficient information