Academic Challenge

Physics Solutions (Regional) – 2021

1. Correct Answer: C

$$2.30 \times 10^5 \frac{m}{s} \times \frac{1inch}{2.54\,cm} \times \frac{100cm}{1m} \times \frac{1ft}{12inches} \times \frac{1\,mi}{5280\,ft} \times \frac{60s}{1min} \times \frac{60min}{1hr} = 5.14 \times 10^5 \frac{mi}{hr}$$

2. Correct Answer: A

We need to break each vector down to components. Consider north to be +y and east to be +x

Vector 1	$1_x = 0$	$1_y = +1.33\ km$
Vector 2	$2_x = -2.25\ km$	$2_y = 0$
Vector 3	$3_x = 1.82km \times \cos(180-40)$	$3_y = 1.82km \times \sin(180-40)$
	$3_x = 1.82km \times (-.766) = -1.39km$	$3_y = 1.82km \times (.643) = +1.17km$
Vector 4	$4_x = 0$	$4_y = -1.11km$
Sum of x	$T_x = -3.64$	$T_y = +1.39$

Reconstruct the vector using the Pythagorean Theorem and the arc Tan. This gives a magnitude for T of 3.90 km and the angle T makes with the negative x axis is 20.9˚

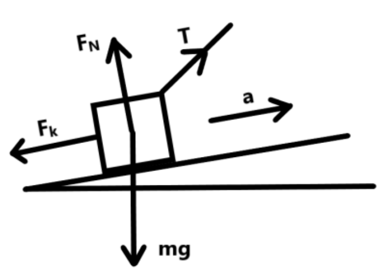
3. Correct Answer: D

$$\rho = {}^{M}/_{V}$$

Volume of a sphere: $V = \frac{4}{3}\pi R^3$

So the mass is: $M = \rho V = \rho \frac{4}{3}\pi R^3 = 8.95\ kg$

4. Correct Answer: D



From this force diagram it is possible to write a few equations:

$$Tcos(25) - F_k - mgsin(10) = ma$$

$$F_N + Tsin(25) - mgcos(10) = 0$$

And the definition of kinetic friction:

$$F_k = \mu_k F_N$$

Using the second to find F_N and plugging that into the third to get F_k and plugging that into the first it is possible to solve for T, the tension in the rope.

$$T = \frac{ma + \mu_k mgcos(10) + mgsin(10)}{\cos(25) + \mu_k \sin(25)} = 23.96N$$

Now we apply the definition of work to get:

$$W = \boldsymbol{F} \cdot \boldsymbol{s} = Fscos(\theta) = 23.96N \times 2.79m \times \cos(25) = 60.59\,J$$

5. **Correct Response: C**

$$0 = \Delta KE + \Delta PE = \frac{1}{2}mv^2 - 0 + mg2R - mgh$$

In order to stay on the track at the top of the loop we must have the acceleration at the top of the loop equal to zero so gravity is equal to centripetal acceleration.

$$\frac{v^2}{R} = g \qquad \text{so} \qquad \frac{1}{2}mv^2 = \frac{1}{2}mgR$$

Putting these equations together we get:

$$0 = \frac{1}{2}mgR + mg2R - mgh$$

Which can be solved for R in terms of h. $R = \frac{2}{5}h$

6. **Correct Answer: C**

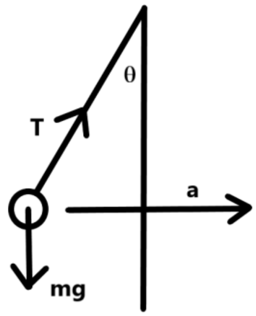
This is a work-energy problem.

$$W = \Delta KE + \Delta PE$$

$$-x\mu_k mgcos(\theta) = 0 - \frac{1}{2}mv_o^2 + mgxsin(\theta)$$

Where the coefficient of friction, the mass, the angle, and the initial velocity are all given. So it is simply a matter of solving for x and that gives x=0.901 m.

7. **Correct Answer: C**



From the force diagram we can write:

$$Tcos(\theta) - mg = 0$$
$$Tsin(\theta) = ma$$

If we move the mg in the first equation to the other side we can divide the bottom equation by the top equation and see:

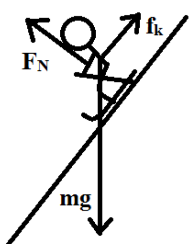
$$\frac{Tsin(\theta)}{Tcos(\theta)} = \frac{ma}{mg}$$

Which gives us:

$$\tan(\theta) = \frac{a}{g}$$

Plugging in the numbers and solving for θ gives us θ = 7.26°

8. Correct Response: E



From the force diagram we can write:

$$mgsin(\theta) - f_k = ma$$

And

$$F_N - mgcos(\theta) = 0$$

And we know the force of friction is $f_k = \mu_k F_N$

Solving for F_N in the second and using it to find f_k in the third we obtain

$$mgsin(\theta) - \mu_k mgcos(\theta) = ma$$

Solving for μ_k we get

$$\mu_k = \frac{gsin(\theta) - a}{gcos(\theta)} = 0.668$$

9. Correct Response: C

In a coordinate system with y going positive down the hill we see that the equations of motion are simply:

$$y = y_0 + v_{0y}t + \frac{1}{2}at^2$$
$$v = v_{0y} + at$$

Plugging in the numbers for the first and solving for t we get:

$$80 = \frac{1}{2}6.12\ t^2$$

And solving for t gives 5.113s

Plugging that into the second equation of motion yields v = 31.29 m/s.

10. Correct Response: C

The equation for range is $R = \frac{v_o^2 \sin(2\theta)}{g}$

This is maximized when $\theta = 45^\circ$ so that the sine is 1. Solving for v_o gives

$$v_o = \sqrt{Rg} = 193.3\ \frac{m}{s}$$

11. Correct Response: B

Angular momentum is given by $L = I\omega$ where w is the angular velocity and I is the moment of inertia. For a sphere the moment of inertia is $I = \frac{2}{5}MR^2$ so plugging in the numbers for the Earth we get 9.7365x10^{37} kg m^2. For the angular velocity we know that it takes 24 hours to make one revolution. Converting this to radians per second:

$$1\frac{rev}{24\ hr} \times \frac{2\pi}{1\ rev} \times \frac{1\ hr}{60\ min} \times \frac{1\ min}{60\ s} = 7.2722 \times 10^{-5}\frac{rad}{s}$$

This gives us the angular momentum

$$L = I\omega = 7.08 \times 10^{33}\frac{kg\ m^2}{s}$$

12. Correct Answer: E

We use conservation of energy to find their speed at the bottom of the swing:

$$mgh = \frac{1}{2}mv^2$$

The grip force, F, must be enough to make the acceleration equal to uniform circular motion so the force equation is given by

$$F - mg = m\frac{v^2}{R}$$

Putting these together we get

$$F = m\frac{2gh}{R} + mg = 750.3\ N + 605.3\ N = 1355.6\ N$$

13. Correct Answer: B

The power can be found by:

$$P = F \cdot v$$

Since the lifting is at a constant velocity, the force is simply mg. The velocity is also simply calculated as v=x/t. So the answer is:

$$P = mg\frac{x}{t} = 665 \times 9.81 \times \frac{38.3}{21.0} = 11898\ W$$

14. Correct Response: B

We find the mass of the acorn from the energy lost in the problem.

$$\Delta E = mgh_2 - mgh_1$$

$$m = \frac{\Delta E}{gh_2 - gh_1} = 0.007614556\ kg$$

We use the conservation of energy to find the speed that the acorn was falling toward the rock and the speed that the acorn was rising up from the rock.

$$mgh_1 = \frac{1}{2}mv_1^2$$

And similarly for the height after the bounce. Solving for v_1 we get 21.19665 m/s. Solving for v_2 we get 6.49484 m/s. The change in momentum of the rock is equal to the change in momentum of the acorn which is (plus sign because the change gives negative but the directions of the velocities are opposite too):

$$mv_1 + mv_2 = 0.210858\ kg\ m/s$$

15. Correct Response: D

Relativistic length contraction: $L = L_o\sqrt{1-\left(v^2/c^2\right)}$

The 1% means that: $\frac{L_o - L}{L_o} \times 100 = 1$

Writing this all out gives:

$$\frac{L_o - L_o\sqrt{1-\left(v^2/c^2\right)}}{L_o} \times 100 = 1$$

Which reduces to $\sqrt{1-\left(v^2/c^2\right)} = 0.99$

Solving for v gives $v = 0.141067\ c = 4.2320 \times 10^7\ \frac{m}{s}$

16. Correct Response: D

The range of a projectile is given by

$$R = \frac{v_o^2 \sin(2\theta)}{g}$$

And this becomes a problem of plugging in to this equation:

$$R = \frac{(6.58)^2 \sin(2 \times 8.75)}{9.81} = 1.32716\, m$$

17. Correct Response: E

The first law of thermodynamics states $\Delta U = Q - W$
And an isothermal process means that $\Delta U = 0$
Which leads us to the result that $Q = W$
In isothermal expansion we have $W = nRT\, ln\left\{\frac{V_f}{V_o}\right\}$
Where n is the number of moles of the gas, R is the gas constant (8.3145 J/mole K), T is the temperature in kelvin, and V_f and V_o are the final and initial volumes.
So we can find much of these values from the problem statement:

$$n = \frac{N}{N_A} = \frac{1.06 \times 10^{24}}{6.023 \times 10^{23}} = 1.75992\, moles$$

$$T_k = T_C + 273.15 = 388.15K$$

$V_f = 4 \times V_o$ So we can also say that $\frac{V_f}{V_o} = 4.0$
Plugging in we get the work as W = 7873.8 J and the head added is the same as shown above.

18. Correct Response: A

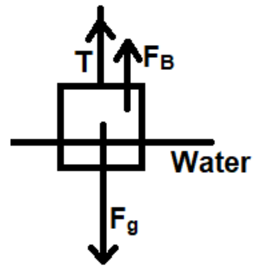
Factual recall: the path of a projectile is a parabola.

19. Correct Response: E

Torque is a vector quantity and is the cross product between the radius and the force vectors. According to the rules of calculating the cross product:

$$\hat{x}(2 \times 0 - 0 \times 7) - \hat{y}\big(3 \times 0 - 0 \times (-5)\big) + \hat{z}\big(4 \times 7 - 2 \times (-5)\big) = \hat{z}38$$

20. Correct Response: B



The mass of the cube is 2700 kg/m^3 x (0.0501 m)^3 = .33953 kg. We can see from the force diagram that the force equation is:

$$T + F_B - F_g = 0$$

Where we know that $F_g = mg$ and m is the mass of the cube, given above. We also know that the buoyant force is given by $F_B = \rho g V$ where V is the volume of the object that is submerged and ρ is the density of the fluid (water). Finally, we are given the tension in the string, T.

If we write the volume under water and the volume of the cube we see that the fraction under water is given by

$$x = \frac{V}{V_{Cube}}$$

So now the force equation can be written as follows:

$$2.96N + \rho_{water} x V_{cube} g - \rho_{Al} V_{cube} g = 0$$

We can solve this for x to get:

$$x = \frac{\rho_{Al} V_{cube} g - 2.96N}{\rho_{water} V_{cube} g} = 0.30057$$

21. Correct Response: B

The speed of a wave on a wire under tension depends upon the tension in the wire and the linear density of the material making up the wire.

$$v = \sqrt{\frac{T}{\lambda}} = \sqrt{\frac{T}{M/L}} = \sqrt{\frac{478N}{0.332\ kg/17.0\ m}} = 156.4478\ \frac{m}{s}$$

And the time it takes to travel down and back the wire is:

$$t = \frac{d}{v} = \frac{34.0m}{156.4478\ \frac{m}{s}} = 0.217325\ s$$

22. Correct Response: D

Gauss' Law relates the net flux of an electric field through a closed surface to the net charge enclosed by that surface.

23. Correct Response: D

$$\frac{R\,u}{M} = \frac{\left[\frac{kg}{s}\right]\left[\frac{m}{s}\right]}{[kg]} = \frac{m}{s^2} = units\ of\ acceleration$$

24. Correct Response: A

The gravitational potential energy is given by:

$$U = -G\frac{mM_E}{R} = -6.673\times10^{-11}\frac{Nm^2}{kg^2}\times\frac{38500kg\times5.97\times10^{24}kg}{2.11\times10^7m} = -7.26898\times10^{11}\,J$$

25. Correct Response: D

The energy released will be the change in energy from the kinetic energy of moving at speed with the potential energy of the higher altitude to being stopped with the potential energy of being at the surface of the Earth.

$$\Delta E = E_f - E_i = -G\frac{mM_E}{R_E} - \left\{-G\frac{mM_E}{R} + \frac{1}{2}mv^2\right\}$$

$$\Delta E = -6.673\times10^{-11}\frac{Nm^2}{kg^2}\times\frac{38500kg\times5.97\times10^{24}kg}{6.37\times10^6m} - \frac{1}{2}38500kg\times\left(27500\frac{m}{s}\right)^2 + 6.673\times10^{-11}\frac{Nm^2}{kg^2}\times\frac{38500kg\times5.97\times10^{24}kg}{2.11\times10^7m}$$

$$\Delta E = -2.40778\times10^{12}J - 1.45578\times10^{13}J + 7.26898\times10^{11}J = -1.624\times10^{13}J$$

The energy that the meteorite loses (negative above) is the energy released in the collision (change it to positive). To find out how many atomic bombs this is equivalent to we divide this number by the energy in one atomic bomb.

$$Energy\ Released = \frac{1.624\times10^{13}J}{9.20\times10^{13}\frac{J}{Atomic\ Bomb}} = 0.1765\ atomic\ bombs$$

26. Correct Response: D

The force of friction is given by:

$$f_k = \mu_k F_N$$

In this case F_N is the force of gravity = mg So the result is simply:

$$f_k = 0.295 \times 55.1\ kg \times 9.81 \frac{m}{s^2} = 159.457\ N$$

27. Correct Response: A

In order to balance the sum of the torques to the left of the string and to the right of the string must be zero. The torque on the left would be the mass of that piece times the distance from the string to the center of mass. The same would be true for the one on the right. The left side is arranged so that the center of mass would be somewhere near the apex of the fan. The right side would have the center approximately in the geometric center of that stick. The length of the stick is more than the broom part and its center would be further from the string. So, since torque is mass times distance the shorter distance must have more mass in order to have equal torque.

28. Correct Response: B

The capacitance for capacitors in parallel is given by the sum of the capacitances. For capacitors in series the inverse of the total is the sum of the inverses of the capacitors. For this case the 0.100 mF and the 0.300 mF capacitors are in parallel and their sum is in series with the 0.500 mF capacitor.

$$0.1\ mF + 0.3\ mF = 0.4\ mF$$

$$\frac{1}{C} = \frac{1}{0.4\ mF} + \frac{1}{0.5\ mF} = 4.5 \frac{1}{mF}$$

So the total capacitance, C, is 1/4.5 mF = 0.222 mF.

29. Correct Response: E

The conservation of momentum also means that the center of mass moves at a constant velocity unless acted upon by outside forces. This also means that without external forces a reference frame tied to the center of mass would also continue to move at a constant velocity throughout the collision.

30. Correct Response: E

The forces on the bucket are gravity and the tension in the rope. These forces combine to result in an acceleration of 1.04 m/s^2. Therefore the force equation is:

$$F_R - F_g = ma$$

Realizing that F_g is mg we can solve this for F_R.

$$F = \left(1.04\frac{m}{s^2} + 9.81\frac{m}{s^2}\right) \times 3.75\ kg = 40.6875\ N$$

31. Correct Response: D

The key here is to consider the number of nucleons. On the left side we add the numbers in the top row to get 236. On the right side we add 88 and 136 to get 224. We see that each neutron contributes 1 and to make the two sides equal we will then need 12 of them. So Y must be 12.

32. Correct Response: E

de Broglie is the person who first proposed wave particle duality.

33. Correct Response: E

If the angular velocity of the grinding wheel is constant, the net torque on the wheel must be zero. There are two torques acting on the wheel, the motor and the frictional torque of the metal against the wheel These must be equal in magnitude to add to zero. . (Note that the normal force of the metal against the wheel causes no torque.)

$$5.2\ Nm = \tau_{fr} = RF_{fr}sin90° = R\mu_k F_N$$

Solving for μ_k we get:

$$\mu_k = \frac{5.2\ Nm}{0.22m \times 65.0N} = .363636$$

34. Correct Response: C

The diffraction grating equation is

$$dsin\theta = m\lambda$$

Where d is the distance between grooves in the grating (which is 1/number of lines per m), m is the order, and λ is the wavelength of the light.

We find the angle of reflection for both wavelengths:

$$\theta_{red} = arcsin\left[\frac{m\lambda_{red}}{d}\right] = arcsin\left[m\lambda_{red} \times 360000\frac{lines}{m}\right] = 0.238412\,rad = 13.660°$$

$$\theta_{teal} = arcsin\left[\frac{m\lambda_{teal}}{d}\right] = arcsin\left[m\lambda_{teal} \times 360000\frac{lines}{m}\right] = 0.175865\,rad = 10.076°$$

The difference in degrees is the answer = 3.58367°

35. Correct Response: A

The electrons leaving the surface of the metal in the photoelectric effect experiment follow the equation:

$$KE_{max} = hf - \varphi$$

Where h is Planck's Constant, f is the frequency of light $f = \frac{c}{\lambda}$ and φ is the work function.

So we have:

$$\frac{hc}{\lambda} = KE_{max} + \varphi$$

And:

$$\lambda = \frac{hc}{KE_{max} + \varphi} = \frac{6.626 \times 10^{-34} Js \times 2.9979 \times 10^{8}\frac{m}{s}}{\{0.315eV + 1.95eV\} \times \frac{1.602 \times 10^{-19} J}{1eV}} = 5.4744 \times 10^{-7} m = 547.44\,nm$$