2021 SECTIONAL MATHEMATICS EXAM

1. A portfolio has 10,000 shares of stock each valued at $50 per share. The number of shares in the portfolio is currently increasing by 150 shares per day, but the total value of the portfolio is decreasing by $18,500 per day. At what rate is the price of each share currently changing?

   a) Increasing by $0.50 per day
   b) Increasing by $1.10 per day
   c) Increasing by $2.50 per day
   d) Decreasing by $2.60 per day
   e) Decreasing by $3.73 per day

2. A group of firefighters place a forty-foot long ladder up against a wall. The distance from the top of the ladder to the ground is 21 feet more than the distance from the base of the ladder to the wall. What angle does the base of the ladder make with the ground? Round to the nearest degree.

   a) 23° b) 26° c) 64° d) 67° e) 69°

3. Ann's house is 100 ft from Bob's, and Bob's house is 150 ft from Candi's. The three homes are not colinear. At Bob's house, the angle between the lines of observation to the other houses is $84°$. How far is Ann's house from Candi's? Round to the nearest foot.

   a) 171 ft b) 175 ft c) 180 ft d) 189 ft e) 250 ft

4. For a parabola whose focus is at (1, 9) and whose directrix is y = 7, what is the distance from the vertex to the directrix? Round to the nearest whole unit.

   a) 1 b) 2 c) 3 d) 4 e) 5

5. What is the domain of $y = \dfrac{\sqrt{3-x}}{x+2}$?

   a) $(3,\infty)$ b) $[3,\infty)$ c) $(-\infty,-2)\cup(-2,\infty)$
   d) $[-3,2)\cup(2,\infty)$ e) $(-\infty,-2)\cup(-2,3]$

6. A hot water tank in the shape of a right circular cylinder has a height of 6.75 ft with a diameter of 2.5 ft. Based on the conversion 7.48 gal = 1 ft^3, determine how many gallons the tank holds. Round to the nearest gallon.

   a) 114 gal b) 248 gal c) 265 gal d) 285 gal e) 300 gal

7. If $x = t^2 + 3$ and $y = t^2 - 9$, what degree of polynomial is y when written in y = f(x) form?

a) 0 (constant) b) 1 (linear) c) 2 (quadratic)
d) 3 (cubic) e) Not a polynomial

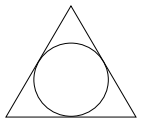
8. A struggling town sold 70 kiosk shops for $900,000. If some of the shops cost $20,000 per shop and the remainding cost $10,000 per shop, how many shops were bought at $10,000?

a) 50 b) 40 c) 30 d) 20 e) 10

9. A committee of 6 is being selected. There are 11 men and 7 women available for the committee. How many possible such committees have more women than men?

a) 1,925 b) 2,163 c) 12,789 d) 18,564 e) 23,100

10. A circle is inscribed inside an equilateral triangle as shown below. If the circle has a circumference of 1 inch, what is the perimeter of the triangle? Round to the nearest tenth of an inch.



a) 1.5 inches b) 1.7 inches c) 1.8 inches d) 2.0 inches e) 3.1 inches

11. A state park wants to have a logo painted onto an irregularly shaped cliff face near the entrance. The commissioned artist wants to estimate the surface area of the cliff face using trapezoidal rule. The artist walks along the base of the cliff and makes height estimates every 25 feet. The heights are as follows:

Horizontal distance	0	25	50	75	100	125	150	175	200
Height	0	94	92	89	74	55	77	96	22

Using the trapezoidal rule, approximate the surface area to the nearest square foot.

a) 14,430 ft^2 b) 14,980 ft^2 c) 15,000 ft^2
d) 14,700 ft^2 e) 14,500 ft^2

12. Consider the vectors <6, –5, 0>, <3, 2, –27>, <2, –1, 4>, and <0, 0, 0>. How many of these vectors are orthogonal to <2, 4, 0>?

a) 0 b) 1 c) 2 d) 3 e) 4

13. A square playground has a perimeter of 320 feet. A concrete sidewalk 3 feet wide and 4 inches deep is to be constructed around the playground. Find the total volume of the sidewalk. Round to the nearest cubic foot.

a) 325 ft^3 b) 332 ft^3 c) 344 ft^3 d) 3,900 ft^3 e) 3,984 ft^3

14. The graph of $8x^2 - y^2 < 2$ takes on what shape?

a) Interior of the branches of a hyperbola
b) Exterior of the branches of a hyperbola
c) Interior of an ellipse
d) Exterior of an ellipse
e) Interior of a parabola

15. A spherical bubble is increasing in volume at a constant rate of 3 cubic inches per second. At the moment its diameter is 3 inches, how fast is its diameter increasing? Round to the nearest tenth of an inch per second.

a) 0.1 inches per second b) 0.2 inches per second c) 0.3 inches per second
d) 0.5 inches per second e) 1.0 inch per second

16. Find the sum of the series $\sum_{k=2}^{16}(5k+2)$.

a) 1410 b) 752 c) 705 d) 652 e) 602

17. Find the sum of all real solutions to the equation $3 \cdot 49^x + 7 \cdot 7^x + 2 = 0$. Round to two decimal places.

a) –0.56 b) –0.21 c) 0.36 d) 2.33 e) No solutions

18. A cross-country car race starts at noon. At the start, car A stalls giving car B a five-minute head start. Once they both get going, car A travels at an average speed of 80 mph and car B travels at an average speed of 75 mph. When will car A overtake car B, and how far have they traveled? Round to the nearest minute and the nearest mile.

a) Car A overtakes car B at 1:20 PM. They have traveled 100 miles.
b) Car A overtakes car B at 1:25 PM. They have traveled 100 miles.
c) Car A overtakes car B at 1:20 PM. They have traveled 107 miles.
d) Car A overtakes car B at 1:25 PM. They have traveled 107 miles.
e) Car A never overtakes car B.

19. What is the coefficient of x when you multiply (x – 5 – 2i) by (x – 5 + 2i)?

a) –10 b) –6 c) –5 d) 14 e) 29

20. The production cost at a factory can be modeled by the function $C(n) = 25.6n + 6.4$, where C is the cost in thousands of dollars and n is thousands of items produced. Determine how many items were produced if the cost was $16,000. Round to the nearest whole item.

a) 375 b) 416 c) 625 d) 409,606 e) 416,000

21. (Linda, polar, med) Represent the complex number $3 + 4i$ in its polar form.

a) $5\left(\cos(53.1°) + i\sin(53.1°)\right)$

b) $5\left(\cos(36.9°) + i\sin(36.9°)\right)$

c) $5\left(i\cos(36.9°) + \sin(36.9°)\right)$

d) $5\left(\cos(53.1°) + i\right)$

e) $5i\left(\cos(53.1°) + \sin(53.1°)\right)$

22. What is the range of the function $y = -|-x - 2| + 4$?

a) $[4, \infty)$ b) $(4, \infty)$ c) $(-\infty, 4]$ d) $(-\infty, 4)$ e) $(-\infty, \infty)$

23. You have 5 red, 3 blue, 2 orange, and 7 yellow marbles. Find the probability of randomly grabbing 6 marbles with at least 2 yellow.

a) $\frac{15}{136}$ b) $\frac{141}{884}$ c) $\frac{315}{884}$ d) $\frac{569}{884}$ e) $\frac{743}{884}$

24. Which of the following is the rate of change in $y = e^{2t} + \frac{6}{t} - 5t$ when t = ln 2? Round to two decimal places.

a) –9.49 b) 0.51 c) 4.69 d) 8.69 e) 9.00

25. In a population of birds, 20% of them have gray plumage and the remaining 80% have black plumage. Of the birds with gray plumage, 40% have blue eyes and the remaining 60% have green eyes. Of the birds with black plumage, 70% have blue eyes and the remaining 30% have green eyes. If a randomly selected bird has blue eyes, what is the probability it also has gray plumage?

a) 8% b) 12.5% c) 14.3% d) 36.4% e) 40.0%

26. Given the equation $2\left(4^{x-1}\right) = 15^x$, solve for x. Round to three decimal places.

a) 3.907 b) –3.481 c) 0.424 d) –0.524 e) 0.429

27. For values of $x > 0$, how many of the following expressions are equivalent to $\log_2 x + \log_8 x$?

I. $4 \log_2 x$ II. $\log_8 x^4$ III. $\log_{16} x$ IV. $\log_{16} x^2$

a) 0 b) 1 c) 2 d) 3 e) 4

28. We invest $8000 at 7% simple interest and $11,000 in a second simple interest account such that the total interest earned from both accounts in one year is $1550. What is the simple interest rate on the $11,000? Round to the nearest whole percent.

a) 5% b) 6% c) 7% d) 8% e) 9%

29. What is the period of $\sin 3x + \cos 4x$?

a) $\frac{\pi}{12}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) π e) 2π

30. The owners of four different cars recently purchased gas. The four cars were a Chevy, a Ford, a Honda, and a BMW. The cars all have different mileage, but all are multiples of 10k with a total of 100k. The gas amounts were all different, but all were multiples of $5 with a total of $50.

I. The Ford has 30k.
II. The Chevy has fewer miles than the Honda.
III. The car of the person who bought $15 has 20k less miles than the Ford.
IV. The car of the person who bought $10 has 10k more miles than the Honda.
V. The owner of the BMW spent more than the owner of the car with 20k miles.

What is the combined mileage of the two cars whose owners spent the most on gas?

a) 40k b) 50k c) 60k d) 70k e) Insufficient information