ACES – Academic Challenge
Physics Solutions (Sectional) – 2021

1. Correct Response: B

The decrease in mechanical energy is due to work done by kinetic friction.

$\Delta E = W_f = -f_k \Delta x$, where $f_k = \mu_k N$ and $N = mg$.

Thus, $\Delta E = -\mu_k mg \Delta x$ and $\Delta E/\Delta x = -\mu_k mg$.

-0.700 J/m $= -\mu_k(0.200 \text{ kg})(9.80 \text{ m/s}^2)$

$\mu_k = 0.357$

2. Correct Response: D

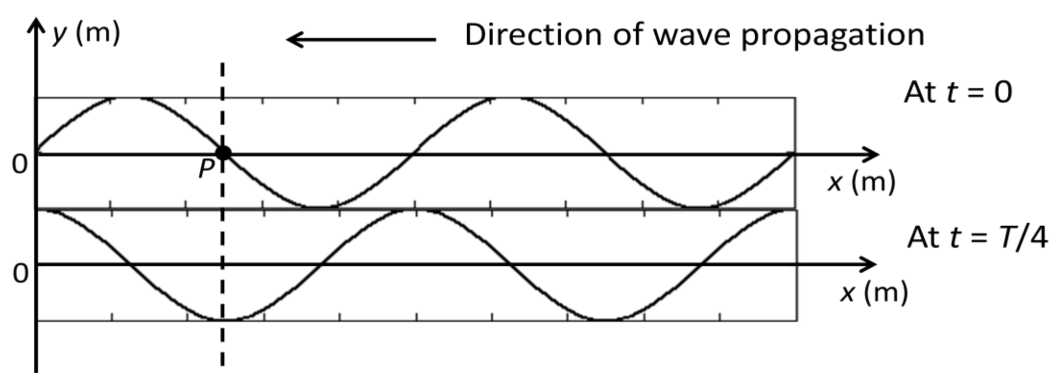
The *v*-*t* diagram shows the terminal velocity of the bungee jumper is 50.0 m/s. The terminal velocity is reached when the force of gravity is equal in magnitude to the air drag.

$mg = f_{\text{air}}$, where $f_{\text{air}} = bv_t$ and v_t is the terminal velocity.

Thus, $mg = bv_t$ and $b = mg/v_t = (70.0 \text{ kg})(9.80 \text{ m/s}^2)/(50.0 \text{ m/s}) = 13.7$ kg/s.

3. Correct Response: E

Since the wave travels in the –*x* direction, at $t = T/4$, the waveform is translated to the left by ¼ of the wavelength, as shown in the diagram below.



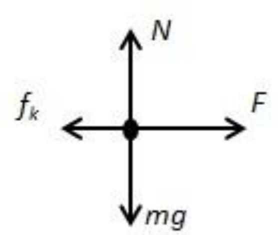
It can be seen that at $T/4$, point *P* is at the maximum displacement from the equilibrium in the –*y* direction. The restoring force is then in the +*y* direction, and thus, the acceleration is also in the $+y$ direction. Finally, since the particle is at the maximum displacement from the equilibrium, its velocity is zero.

4. **Correct Response: B**

The photoelectric effect demonstrates that light is absorbed as photons (particle nature of light). Compton's X-ray scattering from an electron also shows the particle (photon) nature of the X-ray. Davisson-Germer's experiment on electron diffraction by crystals demonstrates the wave nature of the electron. Röntgen's experiment on the production of X-rays shows the emission of X-rays as photons. Young's double-slit experiment demonstrates the wave nature of light, that is, light can interfere.

5. **Correct Response: A**

A free-body diagram for the cat and box is shown in the diagram below.



According to the definition for instantaneous power, the power delivered by Katie to the box (with the cat inside) is $P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} = Fv$, where P is the power, F is the magnitude of the horizontal force applied to the box, and v is the speed.

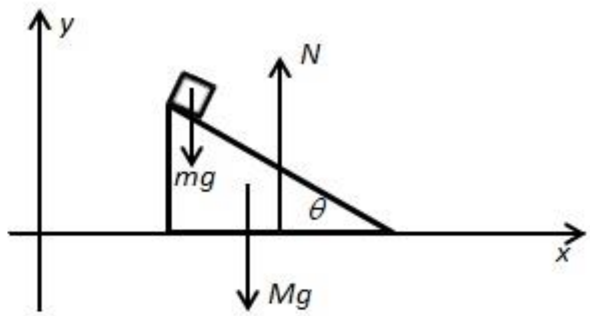
Note that while the power P remains constant, F is not constant. At first, the speed increases and F decreases; when F is equal to the magnitude of the kinetic frictional force f_k, the box reaches a maximum speed v_{max}; after that, the box moves at a constant velocity.

Setting $F = f_k = \mu_k N = \mu_k mg$ and $v = v_{max}$, we have

$P = \mu_k mgv_{\text{max}} = (0.200)(6.00 \text{ kg})(9.80 \text{ m/s}^2)(2.00 \text{ m/s}) = 23.5 \text{ W}$

6. **Correct Response: E**

Consider the block-object system. The total external force acting on the system is the gravity on the block and object and the normal force on the block by the surface, as shown in the diagram below.



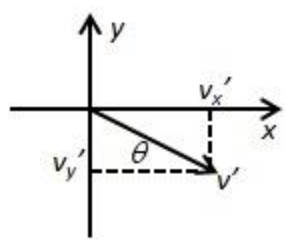
According to the Impulse-Momentum theorem, $\mathbf{I}_{ext} = \Delta\mathbf{P}$, where $\mathbf{I}_{ext}$ is the external impulse delivered to the system and $\Delta\mathbf{P}$ is the change in the total momentum of the system. Since the total external force acting on the system is only in the *y* direction, there is no change in the total momentum in the *x* direction. So, $\Delta P_x = P_{f,x} - P_{i,x} = 0$, where $P_{f,x}$ and $P_{i,x}$ are the final and initial total momentum in the *x* direction, respectively. Since $P_{i,x} = 0$, $P_{f,x} = 0$.

So, $P_{f,x} = MV_x + mv_x = 0$, where V_x and v_x are the final velocity of the block and object in the *x* direction, respectively.

So, $v_x = -MV_x/m$ = -(0.400 kg)(-0.500 m/s)/(0.200 kg) = 1.00 m/s. Note that this is the *x*-velocity of the object relative to the surface.

7. Correct Response: B

Relative to the block, at the bottom of the inclined plane, the object has a velocity **v**' directed downhill parallel to the inclined plane, as shown in the diagram below.



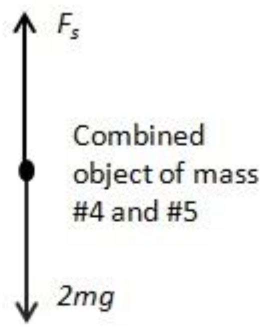
The *x*-velocity is $v_x' = v_x - V_x$ = 1.00 m/s – (-0.500 m/s) = 1.50 m/s, while the *y*-velocity is $v_y' = -v_x' \tan\theta$ = -(1.50 m/s) tan (30.0°) = -0.866 m/s.

So, relative to the block, the speed *v*' of the object is given by

$v' = [(v_x')^2 + (v_y')^2]^{1/2} = [(1.50\text{ m/s})^2 + (-0.866\text{ m/s})^2]^{1/2} = 1.73$ m/s

8. Correct Response: C

Consider mass #4 and #5 as a combined object. A free-body diagram of the combined object is shown below. The external forces acting on the combined object are the total weight 2*mg* and the spring force F_s exerted by the spring between mass #3 and #4. Since the object is at rest, the two external forces are equal in magnitude, that is, F_s = 2*mg*.



Applying Hooke's Law, $F_s = kx$, where x is the amount of stretching of the spring, we have $x = F_s/k = 2mg/k$ = 2(0.0500 kg)(9.80 m/s^2)/(20 N/m) = 0.0490 m = 4.90 cm

So, the length of the spring between mass #3 and #4 is $L = L_0 + x$ = 10.0 cm + 4.90 cm = 14.9 cm

9. Correct Response: B

It can be seen that the pendulum oscillates but the amplitude of oscillation decreases with time. So, it is an under-damped oscillation.

10. Correct Response: A

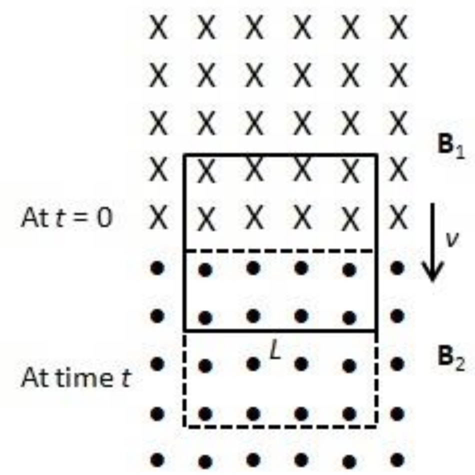
It can be seen that in the circuit, R_2 and R_3 are connected in parallel. This parallel combination is then connected in series with R_1. Suppose R is the equivalent resistance of the parallel combination and R_{eq} is the equivalent resistance of the entire circuit.

$1/R = 1/R_2 + 1/R_3 \rightarrow R = R_2R_3/(R_3 + R_2) \rightarrow R_{eq} = R_1 + R = R_1 + R_2R_3/(R_3 + R_2)$ = 6.00 Ω +(5.00 Ω)(2.00 Ω)/(5.00 Ω + 2.00 Ω) = 7.429 Ω

Applying Ohm's Law, we have $I = V/R_{eq}$ = (15.0 V)/(7.429 Ω) = 2.02 A

11. Correct Response: D

As can be seen in the diagram below, at t = 0, the total magnetic flux through the coil is zero. When the coil moves downward, more magnetic field lines are piercing the area of the coil and pointing out of the page.



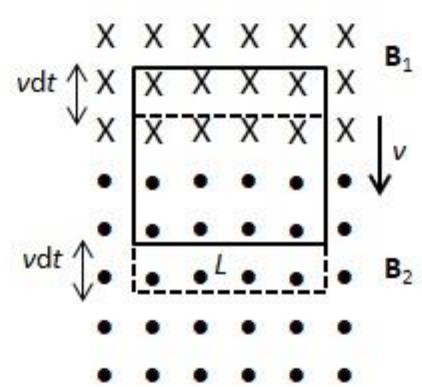
According to Lenz's Law, the induced current opposes such change by producing magnetic field lines pointing into the page within the loop. By the right-hand-rule, the induced current is in the clockwise direction.

12. Correct Response: C

According to Faraday's Law of Induction, $|\varepsilon| = N(\mathrm{d}\Phi/\mathrm{d}t)$, where $|\varepsilon|$ is the magnitude of the electromotive force, N is the number of loops in the coil, Φ is the magnetic flux, and t is time. Let's set the coil's normal to be directed out of the page. The total magnetic flux through the coil is given by

$\Phi = \mathbf{B}_1 \cdot \mathbf{A}_1 + \mathbf{B}_2 \cdot \mathbf{A}_2 = -BA_1 + BA_2$, where $\mathbf{A}_1$ and $\mathbf{A}_2$ are the respective area vectors of the coil in the two magnetic fields.

As can be seen in the diagram below, $\mathrm{d}A_1 = -Lv\mathrm{d}t$ and $\mathrm{d}A_2 = -Lv\mathrm{d}t$.



$\mathrm{d}\Phi/\mathrm{d}t = -B(\mathrm{d}A_1/\mathrm{d}t) + B(\mathrm{d}A_2/\mathrm{d}t) = -B(-Lv\mathrm{d}t/\mathrm{d}t) + B(Lv\mathrm{d}t/\mathrm{d}t) = 2BLv$

Since $N = 1$, $|\varepsilon| = 2BLv$ and $I = |\varepsilon|/R = 2BLv/R$ = 2(2.00 T)(0.300 m)(5.00 m/s)/(5.00 Ω) = 1.20 A.

13. Correct Response: B

Let t be the time of flight before the ball lands on the ground. The ball is in free fall with zero initial velocity in the vertical direction.

$h = (1/2)gt^2 \rightarrow t = (2h/g)^{1/2}$.

In the horizontal direction, the ball is in uniform motion.

$R = v_0t = v_0(2h/g)^{1/2} \rightarrow R^2 = v_0^2(2h/g)$.

So, R^2 changes linearly with h and the slope represents $2v_0^2/g$.

14. Correct Response: D

Slope = $2v_0^2/g \rightarrow v_0 = [(\text{slope})g/2]^{1/2}$ = [(3.27 m)(9.80 m/s^2)/2]$^{1/2}$ = 4.00 m/s

15. Correct Response: B

LASER stands for Light Amplification by Stimulated Emission of Radiation.

16. Correct Response: E

The pivot point P should be placed below the center of mass (CM) of the artwork. Let's set the x axis along the length of the rod with the positive direction to the right and the origin at the position of the 20.0-g sphere. x_1 = 0 m, x_2 = 0.200 m, and x_3 = 0.750 m.

x_{CM} = [(20.0 g)(0 m) + (30.0 g)(0.200 m) + (40.0 g)(0.750 m)]/(20.0 g + 30.0 g + 40.0 g) = 0.400 m

Point P should be placed below the rod at a point 0.400 m to the right of the 20.0-g mass.

17. Correct Response: A

$N = N_0e^{-\lambda t} = N_0e^{-t/\tau}$, where λ is the decay constant and the mean lifetime $\tau = 1/\lambda$.

When the number of isotope A atoms is equal to that of isotope B atoms,

$(6.00 \times 10^{16})\, e^{-t/(30.0\text{ s})} = (2.00 \times 10^{16})\, e^{-t/(50.0\text{ s})} \rightarrow \frac{e^{-t/(50.0\text{ s})}}{e^{-t/(30.0\text{ s})}} = 3.00 \rightarrow e^{t\left(\frac{1}{30.0\,s}-\frac{1}{50.0\,s}\right)} = 3.00 \rightarrow$
$t\left(\frac{1}{30.0\,s} - \frac{1}{50.0\,s}\right) = \ln(3.00) \rightarrow t = 82.4\text{ s}$

18. Correct Response: C

Apply the definition for magnification: $m = h'/h = -s'/s$, where h and h' are the object and image height, respectively, and s and s' are the object and image distance, respectively. (Note: pay attention to the sign conventions.)

Here, h= 20.0 cm, h' = 1.00 cm (positive because the image is upright), and s = 25.0 cm. So, $s' = -(h'/h)s$ = -(1.00/20.0)(25.0 cm) = -1.25 cm (negative because it is a virtual iamge).

Apply the Mirror Equation: $1/s + 1/s' = 1/f$, where f is the focal length.

$f = ss'/(s' + s)$ = (25.0 cm)(-1.25 cm)/(-1.25 cm + 25.0 cm) = -1.32 cm.

19. Correct Response: B

Apply Rotational Dynamics: $\tau = I\alpha$, where τ is torque, I is moment of inertia, and α is angular acceleration.

$\tau = 2(0.00200\ \text{kg}\cdot\text{m}^2)(2.00\ \text{rad/s}^2) = 0.00800\ \text{N}\cdot\text{m}$.

20. Correct Response: A

Apply the Ideal-Gas Law: $PV = nRT$, where P is the absolute pressure, V is the volume, n is the number of moles, R is the Gas Constant, and T is the absolute temperature.

$P\ V = nRT \rightarrow P_2V_2/P_1V_1 = (n_2T_2)/(n_1T_1) \rightarrow P_2/P_1 = (n_2T_2)/(n_1T_1)$, since $V_2 = V_1$.

$n_2/n_1 = P_2T_1/(P_1T_2) \rightarrow n_2 = n_1P_2T_1/(P_1T_2)$

n_2 = (4.00 mol)(8.00 atm)(303 K)/[(3.00 atm)(323 K)] = 10.0 mol

$n_2 - n_1$ = 10.0 mol – 4.00 mol = 6.00 mol

21. Correct Response: D

Apply Conservation of Energy: $(1/2)k_1x_1^2 = (1/2)k_2x_2^2$, where x_1 and x_2 are the maximum compressions of the first and second springs, respectively.

$x_2 = x_1(k_1/k_2)^{1/2} = (10.0\ \text{cm})(10/20)^{1/2} = 7.07$ cm

22. Correct Response: C

Apply Conservation of Momentum: $\mathbf{P}_i = \mathbf{P}_f$, where $\mathbf{P}_i$ is the total momentum just before the collision and $\mathbf{P}_f$ is that just after the collision.

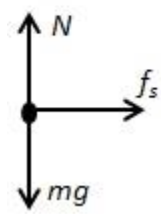
P_{ix} = (0.500 kg)(2.00 m/s)cos (-30°) + (0.400 kg)(3.00 m/s)cos 30° = 1.91 kg·m/s

P_{iy} = (0.500 kg)(2.00 m/s)sin (-30°) + (0.400 kg)(3.00 m/s)sin 30° = 0.100 kg·m/s

So, $\mathbf{P}_i = \mathbf{P}_f = (1.91\hat{\mathbf{i}} + 0.100\hat{\mathbf{j}})$ kg·m/s

23. Correct Response: D

A free-body diagram of the suitcase is shown below.



When the suitcase is on the verge of sliding relative to the conveyor belt, the static frictional force f_s is at the maximum, $f_{s,\,max.}$ So, $f_s = f_{s,\,max} = \mu_s N = \mu_s mg$.

According to Newton's 2nd Law, $f_s = ma \rightarrow \mu_s mg = ma \rightarrow a = \mu_s g$ = (0.200)(9.80 m/s^2) = 1.96 m/s^2. Note: The 15.0-kg mass is not needed.

24. Correct Response: B

Since a short wavelength corresponds to a high frequency, and the ultraviolet light has a higher frequency than the visible light, the failure is historically referred to as the ultraviolet catastrophe.

25. Correct Response: E

$P = I^2R \rightarrow Q = Pt = I^2Rt = cm\Delta T \rightarrow \Delta T = I^2Rt/cm$ = (0.600 A)^2(500 Ω)(300 s)/[(4186 J/kg·°C)(0.600 kg)] = 21.5 C°

26. Correct Response: A

Apply Newton's 2nd Law for circular motion, $F_c = mv^2/r = m\omega^2 r$, where F_c is the centripetal force, to object 1 and object 2, separately.

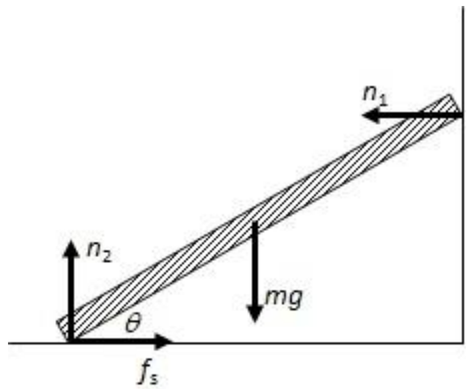
For object 2, $T_2 = m\omega^2 R_2 = 2m\omega^2 R_1$.

For object 1, $T_1 - T_2 = m\omega^2 R_1 \rightarrow T_1 = T_2 + m\omega^2 R_1 = 2m\omega^2 R_1 + m\omega^2 R_1 = 3m\omega^2 R_1$.

$T_1/T_2 = 3/2$

27. Correct Response: D

The ladder is in static equilibrium, so both the total force and the total toque are zero. A free-body diagram of the ladder is shown below.



In the vertical direction, choose the positive direction to be upward. Total force equals zero: $n_2 - mg = 0 \Rightarrow n_2 = mg$

Choose the point where the ladder is in contact with the wall as the axis of rotation. Total torque equals zero: $mg(L/2)\cos\theta + f_s L \sin\theta - n_2 L \cos\theta = 0$

$f_s = [n_2 L \cos\theta - mg(L/2)\cos\theta] / (L \sin\theta) = [mgL\cos\theta - mg(L/2)\cos\theta] / (L\sin\theta)$
$= mg\cos\theta/(2\sin\theta)$

Since $f_s \leq f_{s,\,max}$ and $f_{s,\,max} = \mu_s n_2 = \mu_s mg$, $mg\cos\theta/(2\sin\theta) \leq \mu_s mg$

$\mu_s \geq \cos\theta/(2\sin\theta) = \cos 30.0°/(2\sin 30.0°) = 0.866$

28. Correct Response: C

The atomic number is defined as the number of protons in the nucleus.

29. Correct Response: E

$\mathbf{A} + 2\mathbf{B} = 2.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}} - 8.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}} = -6.00\hat{\mathbf{i}} - 7.00\hat{\mathbf{j}}$

$|\mathbf{A} + 2\mathbf{B}| = [(-6.00)^2 + (-7.00)^2]^{1/2} = 9.22$

$\tan\theta = (-7.00)/(-6.00) = 7/6 \rightarrow \tan^{-1}(7/6) = 49.4°$. Since the vector lies in quadrant III, $\theta = 49.4° + 180° = 229°$, counterclockwise from the $+x$ axis.

30. Correct Response: B

$h = (1/2)gt^2 \rightarrow t = (2h/g)^{1/2} \rightarrow R = v_0 t = v_0(2h/g)^{1/2} \rightarrow R_2/R_1 = (v_{02}/v_{01})(h_2/h_1)^{1/2} = (1/2)2 = 1$

31. Correct Response: D

$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t = (1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}})$ m/s + $[(2.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})$ m/s$^2](5.00$ s$) = (11.0\hat{\mathbf{i}} - 17.0\hat{\mathbf{j}})$ m/s.

32. Correct Response: A

Apply Newton's Law of Gravitation: $F_G = GMm/R^2$, where M is the mass of the planet, and m is the mass of a test object at the surface.

So, $GMm/R^2 = mg \rightarrow g = GM/R^2 = G\rho(4\pi R^3)/(3R^2) = 4\pi G\rho R/3$

33. Correct Response: C

Apply the Continuity Equation for an incompressible fluid: $A_1v_1 = A_2v_2$.

$v_1/v_2 = A_2/A_1 = ½$

34. Correct Response: E

$[X] = [L]^3[M]^{-1}[T]^{-2}[L][M][L]^{-3} = [\mathrm{L}][\mathrm{T}]^{-2}$.

35. Correct Response: D

Apply Work-Energy relation: $W_{\text{ext}} + W_f = \Delta E$.

Since the object moves at a constant speed on a horizontal surface, the change in mechanical energy is zero, that is, $\Delta E = 0$.

So, $W_{\text{ext}} = -W_f = -3(-f\cdot 2\pi R) = 6\pi fR = 6\pi\,(0.300\text{ N})(0.500\text{ m}) = 2.83$ J