



ACADEMIC CHALLENGE FOR  
**ACES**  
ENGINEERING AND SCIENCE



EASTERN ILLINOIS UNIVERSITY

## 2022 Academic Challenge

# REGIONAL MATHEMATICS EXAM

**Mathematics Test Production Team**

Douglas Brandt, Eastern Illinois University

### GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

**Time: 40 Minutes**

**Number of Questions: 30**

**DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!**

©2022 Eastern Illinois University  
All rights reserved



**2022 Academic Challenge  
Regional Mathematics Exam**

1. Determine the period of the function  $f(t) = 4\cot(2t)$ 
  - a)  $1/2\pi$
  - b)  $2/\pi$
  - c)  $\pi/2$
  - d)  $\pi$
  - e)  $2\pi$
  
2. How many non-real complex roots there are for the function  $f(x) = x^4 + 4x^3 + 2x^2 + 4x + 2$ ?
  - a) 0
  - b) 1
  - c) 2
  - d) 3
  - e) 4
  
3. Given the equation,  $(y + x)(2y + 2) = (2y + 4)$ , solve for  $y$ .
  - a)  $y = -1, -2, -x$
  - b)  $y = 1, 2, -x$
  - c)  $y = \frac{-x \pm \sqrt{x^2 + 8}}{2}$
  - d)  $y = \frac{-x \pm \sqrt{x^2 - 4x - 8}}{2}$
  - e)  $y = \frac{-x \pm \sqrt{x^2 - 4x + 8}}{2}$
  
4. A, B and C toss a fair, 6-sided die in order, beginning with A, and continuing in order until a "1" or a "2" is tossed. What is the probability that A tosses the first "1" or "2"?
  - a)  $1/3$
  - b)  $6/21$
  - c)  $8/21$
  - d)  $9/19$
  - e)  $2/3$
  
5. Determine  $x$  if  $\log_{16}(x) + \frac{3}{4} = 2$ .
  - a) 4
  - b) 16
  - c) 32
  - d) 64
  - e) 128
  
6. What is the product of the roots of the equation  $2x^2 + 12x + 36 = 0$ ?
  - a) 12
  - b) 18
  - c) 24
  - d) 48
  - e) 72
  
7. How many unique license plates can be made if a plate must consist of any two letters (Latin alphabet of English language) that may be repeated, followed by three single-digit numbers that may not be repeated?
  - a) 1950
  - b) 82750
  - c) 125750
  - d) 360360
  - e) 486720
  
8. If  $\det \begin{bmatrix} 1 & 0 & 2 \\ -3 & 3 & 0 \\ 0 & -1 & x \end{bmatrix} = 4$ , determine  $x$ .
  - a)  $-2/3$
  - b)  $2/3$
  - c)  $-4/3$
  - d)  $4/3$
  - e) 0
  
9. Which of the following logic expressions is equivalent to  $\overline{(\bar{A} + B)} \cdot C$ ?
  - a)  $\overline{(A \cdot \bar{B})} + \bar{C}$
  - b)  $(A \cdot \bar{B}) + C$
  - c)  $(A \cdot \bar{B}) + \bar{C}$
  - d)  $(A + \bar{B}) \cdot C$
  - e)  $(A \cdot C) + (\bar{B} \cdot C)$

10. Consider the infinite series

$$\sum_{n=1}^{\infty} 5^{n+3} \cdot 20^{-(n+1)}$$

Determine the sum of the first 25 terms of the series to four significant figures.

- a) 2.083      b) 2.999      c) 3.677      d) 4.122      e) 38.74

11. You place \$1000 into a savings account that earns interest at a constant rate  $r$  compounded monthly. At the end of four years, the account has a balance \$1200. What is the interest rate  $r$  to three significant figures?

- a) 0.388%      b) 1.04%      c) 4.00%      d) 4.567%      e) 4.663%

12. A triangle has sides of 12, 8 and 16. What is the area of this triangle?

- a)  $4\sqrt{30}$       b)  $8\sqrt{15}$       c)  $10\sqrt{24}$       d)  $12\sqrt{15}$       e)  $14\sqrt{15}$

13. Angles A and B form a linear pair. If  $m\angle A = (3x - 50)^\circ$  and  $m\angle B = (2x + 31)^\circ$  what is  $m\angle A$ ?

- a)  $16.6^\circ$       b)  $39.8^\circ$       c)  $44.2^\circ$       d)  $69.4^\circ$       e)  $102.2^\circ$

14. For what value of  $k$  does  $kx^2 + 5x + 2$  have only one real solution?

- a) 1      b) 2      c)  $5/2$       d)  $13/4$       e)  $25/8$

15. A satellite tracking station is located in Tucson, AZ. If a satellite following a circular orbit passes over Tucson 175 miles above earth, how far along the earth's surface can the station track the satellite before it moves below the horizon? (Assume the earth is a sphere with radius 3960 miles.)

- a) 1156 miles      b) 1178 miles      c) 1190 miles  
d) 1207 miles      e) 1442 miles

16. The equation of an ellipse is:

$$4x^2 + 8x + y^2 - 6y - 3 = 0$$

At what point on the ellipse is the maximum  $y$  value located?

- a) (-1, 7)      b) (-1, 16)      c) (-1, 5)      d) (-1, 12)      e) none of the above

17. The trajectory of a particle can be given by the parametric form  $(2 \cos t, 3 \sin t)$ . Determine the distance between the particle's location at  $t = 0$  and its location at  $t = \pi/2$ .

- a) 0      b) 1      c)  $\sqrt{5}$       d) 5      e)  $\sqrt{13}$

18. Give all solutions to the inequality  $x^2 - 4x > -2x + 3$
- a)  $-3 < x < 1$       b)  $-1 < x < 3$       c)  $3 < x < 9$   
d)  $x < -1$  or  $x > 3$       e)  $x < 3$  or  $x > 9$
19. What is the dot product of  $-2\mathbf{i} + 2\mathbf{j}$  and  $3\mathbf{i} - 2\mathbf{j}$ ?
- a)  $\mathbf{i}$     b) -10    c) 1      d) 10      e) These vectors do not have a dot product
20. Determine the area of a region bounded by  $y = 2 + 2x$  and  $y = 2 + x^2$  between  $x = 0$  and  $x = 2$ .
- a) 0      b) 1      c)  $4/3$       d)  $3/2$       e) 2
21. A rectangle has perimeter 33.8 and area 35.6. About what is the difference of its length and width?
- a) 9.6      b) 11.9      c) 14.4      d) 16.9      e) 23.2
22. What are the polar coordinates of a position with Cartesian coordinates  $x = -2$  and  $y = 4$  in the form  $(r, \theta)$ ?
- a)  $(4.47, 116.6^\circ)$       b)  $(4.47, 63.4^\circ)$       c)  $(4.47, 243.4^\circ)$   
d)  $(4.47, 296.6^\circ)$       e)  $(-4.47, 243.4.6^\circ)$
23. What is the quotient of the division of  $4 + 2i$  by  $2 + 3i$ ?
- a)  $\frac{6-2i}{3}$       b)  $\frac{2-2i}{3}$       c)  $\frac{4-2i}{5}$       d)  $\frac{4+2i}{5}$       e)  $\frac{14-8i}{13}$
24. A material is subject to exponential decay. If the material has a half-life 20.0s, approximately what fraction of the original material will remain after 75 s pass?
- a) 0.267      b) 0.1887      c) 0.1335      d) 0.0743      e) 0.0235
25. The probability that Jack and Jill will go up a hill together is 19%. The probability that Jack will go up the hill is 37% while the probability that Jill will go up the hill is 21%. What is the probability that neither of the two will go up the hill?
- a) 18%      b) 23%      c) 42%      d) 61%      e) none of the above
26. Two times the reciprocal of a real number is three less than twice the number. What is the positive number?
- a)  $\frac{1}{2}$       b) 1      c) 2      d) 4      e) 6

27. A large cafeteria sink can be drained in 10 minutes. It takes 15 minutes to fill the sink with cold water only and it takes 20 minutes to fill the sink with hot water only. Beginning with an empty, stopped-up sink, turn on the cold water and leave it on. 5 minutes later turn on the hot water and leave it on and 5 minutes later open up the drain. How long from the beginning will it take for the sink to fill?

- a) 11 minutes      b) 15 minutes      c) 17 minutes  
d) 19 minutes      e) 21 minutes

28. What regular polygon inscribed in a circle has sides equal in length to the radius of the circle?

- a) Triangle    b) Pentagon    c) Hexagon    d) Heptagon    e) None. It is not possible.

29. One leg of a right triangle is 50 inches. A line parallel to the OTHER leg and at a distance of 20 inches from that leg cuts off a right trapezoid of area 520 square inches. Find the lengths of the bases of the trapezoid.

- a) 9.2 in, 15.3 in      b) 9.9 in, 24.8 in      c) 10.8 in, 17.9 in  
d) 19.5 in, 32.5 in      e) 34.7 in, 57.8 in

30. Billy picked some apples at the orchard. On his way home he visited three friends. To the first Billy gave  $\frac{1}{3}$  of his apples, to the second  $\frac{1}{4}$  of the remaining apples, and to the third  $\frac{1}{5}$  of the last apples. In all, he gave away 36 apples. What was the original number of apples picked by Billy?

- a) 6 apples    b) 22 apples    c) 26 apples    d) 40 apples    e) 60 apples



