ACES – Academic Challenge
Physics Solutions (Regional) – 2022

1. **Correct Response: C**

We begin by converting the polar coordinates point to Cartesian coordinates:

$$P_{2x} = 5.1 \times \cos(11.3) = 5.00$$

$$P_{2y} = 5.1\sin(11.3) = 1.00$$

If we subtract these two position vectors we get the vector between them. And the question asks for the magnitude of that vector.

$$T_x = P_{2x} - P_{1x} = 5.00 - 3.00 = 2.00$$

$$T_y = P_{2y} - P_{1y} = 1.00 - 2.00 = -1.00$$

The magnitude of the difference vector, T, is given by

$$|T| = \sqrt{(T_x)^2 + \left(T_y\right)^2} = \sqrt{4.00 + 1.00} = 2.236$$

This answer rounds to 2.24.

2. **Correct Response: C**

A typical giraffe is certainly over 10 ft tall and not 30 ft tall. The only answer in this list that falls in that range of distances is 5 m.

3. **Correct Response: B**

$$\rho = \frac{M}{V}$$
$$V_{sphere} = \frac{4}{3}\pi R^3$$

So we need to solve for R

$$R = \sqrt[3]{\frac{3}{4\pi}\frac{M}{\rho}} = 2.461 \times 10^7 m$$

4. **Correct Response: C**

We break this down into two trips. The first is the one into the wind. Flying 60.0 m/s into a 15.0 m/s headwind gives you a ground speed of 45.0 m/s. Traveling 500 km (=500000m) at that speed will take (500000m / 45.0 m/s) equals 11,111 sec. Then the plane turns around and flies with the wind which gives it a ground speed of 75.0 m/s. The same distance is covered and this time the trip takes (500000m / 75.0 m/s) equals 6,666 sec. The total trip time is 17,778 sec. Converting this to minutes gives (17778 sec* 1min/ 60 sec) equals 296.3 min.

5. Correct Response: E

Breaking these vectors down into Cartesian coordinates gives:

$$A_x = 3.71\cos(151) = -3.2448$$
$$A_y = 3.71\sin(151) = 1.7986$$
$$B_x = 4.27\cos(247) = -1.6684$$
$$B_y = 4.27\sin(247) = -3.9306$$

To find **B-A** we subtract the components.

$$C_x = B_x - A_x = -1.6684 - (-3.2448) = 1.5764$$
$$C_y = B_y - A_y = -3.9306 - 1.7986 = -5.7292$$

Reconstructing the C vector gives

$$|C| = \sqrt{C_x^2 + C_y^2} = \sqrt{(1.5764)^2 + (-5.7292)^2} = 5.942$$
$$\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{-5.7292}{1.5764} = -74.61$$

Now we need to consider what calculators do with calculating the arctan. Usually they calculate from -180˚ to 180˚ but we are considering this in the range 0˚ to 360˚. To convert in this case we just subtract 74.61˚ from 360˚ to get 285.39˚ so the final vector is 5.94 at an angle of 285˚

6. Correct Response: B

Newton's Law gives $F = ma$ so we can write $F = 0.460\ kg\ \times 65.0\ \frac{m}{s^2} = 29.9\ N$

7. Correct Response: B

The initial velocity for the projectile motion comes from the acceleration while in contact with the foot.

$$v = v_o + at = 0.00 + 65\frac{m}{s^2} \times 0.125s = 8.125\frac{m}{s}$$

This velocity becomes the initial velocity for the hang time calculation. The y component of this velocity is given by:

$$v_{iy} = v_i sin\theta = 8.125\frac{m}{s} \times sin(57.1) = 6.822\frac{m}{s}$$

The ball will go up until its vertical velocity is zero at which point it will come back down. Since both trips take the same time we only need to find the time to the top and multiply by 2. The velocity equation in the y direction looks like:

$$v_y = v_{iy} - gt$$

And we need to find the time with v_y is zero. That occurs when:

$$t = \frac{v_{iy}}{g} = \frac{6.822m/s}{9.8\ m/s^2} = \ .6961\ s$$

Since this is half of the total trip we need to multiply by two to get 1.392 s.

8. Correct Response: A

At the top of the swing the force of gravity must supply the force that is keeping the water moving in a circle so that

$$m\frac{v^2}{r} = mg$$

Solving for v gives:

$$v = \ \sqrt{rg} = \ \sqrt{\frac{d}{2}g} = 2.978\frac{m}{s}$$

9. Correct Response: B

The height above the bottom of the swing where the mass starts is given by

$$h = L - Lcos(\theta)$$

where L is the length of the swing and θ is the angle that the swing makes with the vertical. The conservation of energy states:

$$mgh + \frac{1}{2}m{v_i}^2 = 0 + \frac{1}{2}m{v_f}^2$$

Where m is the mass of the swing with riders and v_i is the initial speed of the swing and v_f is the final speed of the swing. Solving for v_f gives:

$$v_f = \ \sqrt{2gh + {v_i}^2} = 10.556\frac{m}{s}$$

10. Correct Response: A

The vertical speed is:

$$v_y = \ v_i sin(\theta) = 32.2\frac{m}{s}sin(12.2) = 6.805\frac{m}{s}$$

And the rate of change in potential energy depends on the vertical velocity:

$$\frac{dPE}{dt} = \frac{d(mgh)}{dt} = mg\frac{dh}{dt} = mgv_y = 1850kg \times 9.8\frac{m}{s^2} \times 6.805\frac{m}{s} = 1.233 \times 10^5\frac{J}{s}$$

11. Correct Response: B

Conservation of energy (larger mass is m_1 and pulley is m_p):

$$m_1gh = m_2gh + \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2$$

But $I = \frac{1}{2}m_pR^2$ for a disk such as the pulley and $\omega = \frac{v}{R}$ for a string in contact with the pulley at the radius R and moving as speed v. Putting these together and solving for v gives:

$$v = \sqrt{\frac{2(m_1 - m_2)gh}{\left(m_1 + m_2 + \frac{1}{2}m_p\right)}} = 2.272\frac{m}{s}$$

12. Correct Response: C

If we call the tension in the string on the m_1 side of the pulley T_1 and on the other side T_2 then a force diagram for each mass would show:

$$T_1 - m_1g = -m_1a$$

$$T_2 - m_2g = m_2a$$

Where we consider a the magnitude of the acceleration and take care of the direction with the sign in front of the term. Now with α being the angular acceleration of the pulley we see that for the pulley we have the torque equation:

$$RT_1 - RT_2 = I\alpha$$

So realizing that a and α are related by $\alpha = \frac{a}{R}$ we have

$$T_1 - T_2 = I\frac{a}{R^2}$$

Putting this all together we find:

$$I\frac{a}{R^2} = -\{m_1 + m_2\}a + [m_1 - m_2]g$$

Using the moment of inertia from the previous problem we see that

$$a = \frac{(m_1 - m_2)g}{\left(m_1 + m_2 + \frac{1}{2}m_p\right)} = 2.133\frac{m}{s^2}$$

The system started from rest and accelerated for the given time so the speed of the masses at that time is: $v = 0 + 2.133\frac{m}{s^2} \times 0.250s = 0.5333\frac{m}{s}$ we now change that to the angular velocity of the pulley through: $\omega = \frac{v}{R} = \frac{0.5333\frac{m}{s}}{0.15m} = 3.555\frac{rad}{s}$

13. Correct Response: B

The moment of inertia for this system about the axis is:

$$I = \sum m_i r_i^2 = 3ml^2 + 2ml^2 + m(3l)^2 + m(2l)^2 = 18ml^2$$

The rotational kinetic energy is given by:

$$KE_R = \frac{1}{2} I\omega^2 = 43\frac{ml^2}{T^2}$$

Plugging the moment of inertia in and solving for ω gives:

$$\omega = \sqrt{\frac{86}{18T^2}} = \frac{2.1858}{T}$$

14. Correct Response: B

This is a static equilibrium problem. We write the torque equation for the situation with the chain:

$$(11.0cm)m_s g - (50cm - 11cm)m_c g = 0$$

Solving this for the requested ratio results in:

$$\frac{m_c}{m_s} = \frac{11.0\ cm}{39.0\ cm} = 0.28205$$

15. Correct Response: E

The integral under the curve will give us the change in momentum:

$$\int F dt = \int m a dt = \int m\frac{dv}{dt}dt = m\int_{v_i}^{v_f} dv = m(v_f - v_i)$$

Where we know that v_i is zero because it starts from rest. The area under the curve in the graph can be divided up into two triangles. The first goes from the point (0,0) to (4,60) to (6,0) and has an area of

$$½\ \text{bh} = ½\ (6\text{s})(60\text{N}) = 180\frac{kg\ m}{s}$$

The second triangle goes from point (0,0) to the point (0,30) to the point (1,15) and has an area of

$$½\ \text{bh} = ½\ (30\text{N})(1\text{s}) = 15\frac{kg\ m}{s}$$

So the total area under the curve is 195 kg m/s. But we need the speed and this is the momentum so we need to divide by the mass to get the speed. $v_f = \frac{195\ kg\ m/s}{12\ kg} = 16.25\frac{m}{s}$

16. Correct Response: C

The slope of the line segment is the rise over the run which is (we consider all constants to have the appropriate units)

$$\frac{0-60}{6-4} = -30$$

The y intercept would be found from the point slope formula with m below being the slope:

$$(y-y_1) = m(x-x_1)$$
$$(y-60) = -30(x-4)$$

Rearranging this gives:

$$y = 180 - 30x$$

Now we realize that the y axis is F and the x axis is t. but F=ma so we divide by the mass to get the acceleration equation:

$$\frac{F}{m} = \frac{180-30t}{m}$$

so

$$a = 15 - 2.5t$$

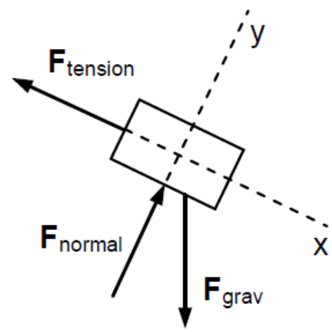
17. Correct Response: B

The free-body diagram of the block is shown. Using the coordinate system shown, the sum of the x components of the force must sum to zero.

$$F_{tension} = F_{grav}\sin(30°)$$

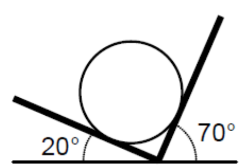
So, since F_{grav} is the weight, we get

$$F_{tension} = w\sin(30°)$$



18. Correct Response: D

As the surfaces of the V are frictionless, the forces of those surfaces are in a direction normal to the surface. Using toward the right as the positive x direction and upward as the positive y-direction,



$$\sum \bar{F} = m\bar{a} = 0 \quad \Rightarrow \quad F_{20x} + F_{70x} = 0 \quad \Rightarrow \quad F_{20}\cos 70° - F_{70}\cos 20° = 0 \quad \Rightarrow \quad F_{70} = \frac{\cos 70.0°}{\cos 20.0°}F_{20}$$

and $F_{20y} + F_{70y} - mg = 0 \quad \Rightarrow$

$$\Rightarrow \quad F_{20}\sin 70° + F_{70}\sin 20° = F_{20}\sin 70° + \frac{\cos 70.0°}{\cos 20.0°}F_{20}\sin 20°0 - mg = 0$$

$$F_{20} = \frac{mg}{\sin(70°) + \cos(70°)\tan(20°)} = \frac{24.5\ N}{0.93969 + 0.12449} = 23.02N$$

19. Correct Response: B

In an inelastic collision Mechanical Energy is not conserved. But in any collision momentum is conserved. These concepts are what answer B says.

20. Correct Response: A

$$\frac{e^2 h}{\varepsilon_o G} \Rightarrow \frac{C^2 Js}{\frac{C^2}{Nm^2}\frac{Nm^2}{kg^2}} = \frac{Js}{\frac{1}{kg^2}} = kg^2 Js = kg^2 \frac{kg\ m^2}{s^2} s = \frac{kg^3 m^2}{s}$$

21. Correct Response: E

Sound intensity goes as (amplitude*frequency)^2 from the equation:

$$I = \frac{1}{2}\sqrt{\rho B}\omega^2 A^2$$

so the ratio of the sound intensities for various frequencies and amplitudes would be (subscript H means high and subscript L means low):

$$\frac{I_L}{I_H} = 1.33 = \frac{(A_L f_L)^2}{\{A_H f_H\}^2}$$

Note that ω is directly proportional to f so we can say that. Solving this for (A_L/A_H) gives

$$\frac{A_L}{A_H} = \sqrt{1.33}\frac{f_H}{f_L} = 10.979$$

22. Correct Response: C

On a velocity vs time graph the slope of the curve is the acceleration. The point with the steepest slope and the largest magnitude acceleration and is point C.

23. Correct Response: A

The velocity of the center of mass of any system of discrete masses is given by:

$$\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{\sum m_i}$$

Since the fifth mass is the only one moving the top of that fraction is just the velocity of the 5th mass times the 5th mass ($m_5 v_5$) and the bottom is the sum of all of the masses.

$$\vec{v}_{cm} = \frac{m_5 v_5 \vec{\boldsymbol{j}}}{m_1 + m_2 + m_3 + m_4 + m_5} = \frac{\frac{5}{2} m v \vec{\boldsymbol{j}}}{\frac{15}{2} m} = \frac{1}{3} v \vec{\boldsymbol{j}}$$

24. Correct Response: E

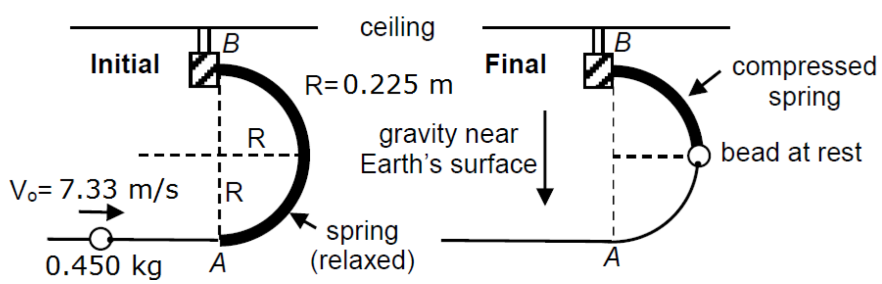
Pascal's Principle (also known as Pascal's Law) is stated as:

Pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and to the walls of the containing vessel.

25. Correct Response: D

The gravitational attraction will pull as if to move the lead object back towards the other but that will reduce its orbital energy and drop it to a lower orbit. The lower altitude orbit is a smaller period and takes less time for an orbit. The following object gains energy making it go to a higher orbit and that means a longer period. Thus, relative to each other, the objects will move apart with the lead one moving away in the forward direction and the following one moving away in the backwards direction.

26. Correct Response: C



Using conservation of energy:

$$E_{final} = E_{initial}$$

$$\frac{1}{2}kx_{final}^2 + mgh_{final} + \frac{1}{2}mv_{final}^2 = \frac{1}{2}kx_{init}^2 + mgh_{init} + \frac{1}{2}mv_{init}^2$$

$$\frac{1}{2}kx_{final}^2 + mgR + \frac{1}{2}m(0)^2 = \frac{1}{2}k(0)^2 + mg(0) + \frac{1}{2}mv_{init}^2$$

$$\frac{1}{2}kx_{final}^2 + mgR = \frac{1}{2}mv_{init}^2 \qquad \text{(} \leftarrow \text{With practice, one would probably start here.)}$$

$$k = \frac{mv_{init}^2 - 2mgR}{x_{final}^2} = \frac{m\left(v_{init}^2 - 2gR\right)}{\left(\frac{2\pi R}{4}\right)^2}$$

$$k = \frac{24.178 - 1.9845}{.12491} = 177.67\frac{N}{m}$$

27. Correct Response: C

The 2C and 6C capacitors are in series so we can determine their effective capacitance first (and call it C_1).

$$\frac{1}{C_1} = \frac{1}{2C} + \frac{1}{6C}$$

Which can be solved to get $C_1 = 1.5C$.

Then C_1 is in parallel with the 3C capacitor. We get the effective capacitance (C_2) of this part of the system by adding the capacitances.

$$C_2 = 1.5C + 3C = 4.5C$$

Finally, we see that C_2 is in series with the C capacitor. Now we can determine the capacitance of the system of capacitors, C_{eff}:

$$\frac{1}{C_{eff}} = \frac{1}{C} + \frac{1}{4.5C}$$

Solving gives $C_{eff} = \frac{9}{11}C$

28. Correct Response: B

$$m = \gamma m_o = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

If we want m = 2.13×10^{-27}kg and m_o = 1.68×10^{-27}kg then we must have $\gamma = 1.2679$ we can now solve for v.

$$v = c\sqrt{1 - \frac{1}{\gamma^2}} = 0.61474$$

29. Correct Response: B

For any reversible process $\Delta S = 0$.

30. Correct Response: D

It does not matter that the charge is not in the center of the sphere. The total charge on the interior surface must be equal and opposite of the charge inside so that there is no electric field within the conductor. Therefore on the interior surface there is +2q. That is where 2q of the 3q added to the exterior of the shell has gone. That leaves only +1q on the outer surface of the shell.

31. Correct Response: E

The problem states that the energy in the photon is enough to raise the oscillator by 2 energy levels. That means:

$$hf = h\frac{c}{\lambda} = 3.7365 \times 10^{-19} J = 2\hbar\omega$$

where the last term is the energy of 2 quantum levels for a harmonic oscillator. The ground state of a harmonic oscillator is given by

$$\frac{1}{2}\hbar\omega = \frac{1}{4}2\hbar\omega = \frac{1}{4} \times 3.7365 \times 10^{-19} J = 9.341 \times 10^{-20} J$$

32. Correct Response: C

The equation for mean free path is:

$$\lambda = \frac{V}{4\pi\sqrt{2}r^2 N}$$

Where V is the total volume of the system, r is the radius of the interacting particles, and N is the total number of particles in the area. It depends on the number of particles per unit of volume and also the cross sectional area of the particles. We have been given all of the variables on the right side so plugging in we get:

$$\lambda = \frac{0.250m^3}{4\pi\sqrt{2}(0.0015m)^2 3250} = 1.9238\, m = 192.38\, cm$$

33. Correct Response: A

The atomic mass of Helium is 4.0 amu, the atomic mass of Carbon is 12.0 amu and the atomic mass of Oxygen is 16.0 amu. Oxygen is the most massive per atom and so it will take fewer atoms to produce the same total mass.

34. Correct Response: C

The resolution of a telescope (or any diffraction limited system) is given by

$$\theta_{min} = 1.22\frac{\lambda}{D}$$

Where θ_{min} is the minimum angular separation between sources, λ is the wavelength of the light, and D is the diameter of the aperture (telescope tube). We realize that θ_{min} is given by the star separation (we will call X) divided by the distance from Earth to the stars (we will denote as Y). So we can now solve

$$X = 1.22\frac{\lambda Y}{D} = 1.22\frac{5.62 \times 10^{-7} m \times 47.5\ ly \times 9.46 \times 10^{15}\frac{m}{ly}}{1.85} = \ 1.66536 \times 10^{11}\ m$$

Which, by the way, is not too different than the distance from the Sun to the Earth.

35. Correct Response: C

The kinetic energy of the alpha particle is the initial energy of the system. The potential energy that it acquires in approaching the silver atom will be the energy of the system when the alpha particle comes to rest. Setting these equal gives:

$$\frac{1}{2}m_{alpha}v^2 = k\frac{q_{alpha}q_{silver}}{d}$$

We first must determine the energy of the alpha in mks units.

$$\frac{1}{2}m_{alpha}v^2 = 0.3\ MeV \times \frac{1.602 \times 10^{-19}J}{1.00\ eV} \times \frac{1.00 \times 10^6 eV}{1.00\ MeV} = 4.806 \times 10^{-14}J$$

So we can now solve for d:

$$d = k\frac{2e \times 47e}{4.806 \times 10^{-14}J} = 8.988 \times 10^9\frac{Nm^2}{C^2} \times \frac{94 \times (1.602 \times 10^{-19}C)^2}{4.806 \times 10^{-14}J} = 4.512 \times 10^{-13}m$$