

2022 Academic Challenge
Sectional Mathematics Exam

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1. The cost of producing widgets is \$0.75 for each widget, with a fixed expense of \$13,500. Each widget is sold for \$1.85. Assuming you sell every widget you produce, how many widgets must be sold to be closest to breaking even?
a. 5039 b. 7297 c. 12273 d. 18000 e. none of the above

2. A chord of length 7 inches is 12 inches from the center of a circle. What is the diameter of the circle rounded to the nearest tenth?
a. 12.5 in. b. 13.9 in. c. 25.0 in. d. 27.8 in. e. none of the above

3. The lengths of all sides of a convex polygon are equal. If the sum of the interior angles is 1260° , how many sides are there?
a. 5 b. 6 c. 7 d. 8 e. 9

4. In tomorrow's weather forecast there is a 35% chance of snow only and a 20% chance of sleet only. There is a 10% chance that it will snow and sleet. What is the probability of snow or sleet?
a. 0.15 b. 0.45 c. 0.55 d. 0.65 e. none of the above

5. The square of the sum of two numbers is 9 and the square of the difference of the two numbers is 121. What is the product of the two numbers?
a. 41 b. 39 c. 31 d. 28 e. -21

6. The game of 801 is played as follows: Throw a fair pair of 6-sided dice. If the sum of the faces is prime, then you win \$150. If you throw a sum which is a multiple of 4, then you lose \$75. Other outcomes have no payoff. What is the expected value of this game for the player?
a. \$12.75 b. \$22.85 c. \$39.25 d. \$41.50 e. \$43.75

7. What is the determinant of matrix **A** below?

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 5 & 9 \\ 3 & 8 & 7 \end{bmatrix}$$

- a. -154 b. -91 c. -80 d. -71 e. -55

8. For the vectors, $\mathbf{a} = \langle 1, 2, 4 \rangle$, $\mathbf{b} = \langle -3, 0, 2 \rangle$ and $\mathbf{c} = \langle 5, -1, 3 \rangle$, find $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$.
- a. 52 b. -52 c. 39 d. -39 e. $\langle 4, -1, 6 \rangle$
9. What is the number 456_7 expressed in base ten?
- a. 216 b. 237 c. 321 d. 334 e. 382
10. For an arbitrary scalene triangle, M is the common point of the perpendicular bisectors of the sides, N is the common point of the altitudes, P is the common point of the interior angle bisectors, and Q is the common point of the centroids. Three of these points lie on a line that does not contain the other point. Which point is not on the line of the other three?
- a. M b. N c. P d. Q e. none of the above
11. Which of the following equations is true for all values of x?
- a. $\cos 4x + \sin 4x = 1$
 b. $\sin 4x = 4 \cos x \sin x$
 c. $\cos (x + 1) = \cos x + \cos 1$
 d. $\sin 2x = \cos^2 x - \sin^2 x$
 e. $\cos 3x = \cos 2x \cos x - \sin 2x \sin x$
12. What is the area that is bounded by the x-axis, the y-axis and the line $y = -(1/17)x + 2$?
- a. 8.5 b. 17 c. 25.5 d. 34 e. 68
13. The inverse of the number of hobgoblinspookadelics (H) is equal to the sum of the inverses of quints (I) and quants (J). If there are 2 hobgoblinspookadelics and 6 quints, how many quants are there?
- a. 1 b. 2 c. 3 d. 4 e. None of the above.
14. A paved path that leads from a cabin to the outhouse heads east for 100 feet, then turns 45 degrees to the north and heads northeast for 100 feet until it reaches the outhouse. An anxious camper can jog this path at fifteen feet per second. A truly desperate camper can head straight to the outhouse from the cabin, but this path goes through the grass (and possibly poison ivy) and slows the camper to fourteen feet per second. Which path takes less time?
- a. Paved path by 0.13 seconds b. Paved path by 0.95 seconds
 c. Grassy path by 0.13 seconds d. Grassy path by 1.00 seconds
 e. Grassy Path by 3.20 seconds
15. Identify the following conic section: $-9x^2 - 16y^2 + 18x - 32y - 6 = 0$
- a. Parabola b. Ellipse c. Hyperbola d. Circle e. Line

16. What is the domain of the function:

$$\frac{\sqrt{x-4}}{x^2-6x+5}$$

- a. $[4, \infty)$ b. $[4, 5) \cup (5, \infty)$ c. $(-\infty, 5) \cup (5, \infty)$
d. $(-\infty, 1) \cup (1, 5) \cup (5, \infty)$
e. None of the above.

17. Solve the equation:

$$\log 3x - \log(x-3) = 1$$

- a. $x = 10/3$ b. $x = 20/6$ c. $x = 30/7$ d. $x = 40/8$ e. None of the above.

18. Factor completely the following:

$$(x+5)^3 - (x+5)$$

- a. $(x+3)(x+4)(x+5)$
b. $(x+4)(x+5)(x+6)$
c. $(x+5)(x+6)(x+7)$
d. $(x+6)(x+7)(x+8)$
e. None of the above

19. Solve the equation below for x :

$$\left(\frac{1}{2}\right)^{-4x} = 2\sqrt{2}$$

- a. $-3/8$ b. $-1/8$ c. $1/8$ d. $3/8$ e. None of the above.

20. A committee of three is to be chosen from ten eligible representatives. Six of the representatives are from the hard-working party, and 4 are from the do-nothing party. How many possible committees have a majority of the members from the hard-working party?

- a. 40 b. 60 c. 80 d. 100 e. 120

21. Robbers leave a town speeding at 70 mph. 12 minutes later the sheriff leaves town at 84 mph. How many hours will it take for the sheriff to catch up with the robbers?

- a. 0.75 hours b. 1.0 hours c. 1.25 hours d. 1.5 hours e. 1.75 hours

22. Which of these vectors is perpendicular to the vector sum of $\mathbf{i} + 2\mathbf{j}$ and $2\mathbf{i} - \mathbf{j}$?

- a. $-3\mathbf{i} - \mathbf{j}$ b. $-\mathbf{i} + 3\mathbf{j}$ c. $\mathbf{i} + 3\mathbf{j}$ d. $\mathbf{i} + \mathbf{j}$ e. these vectors cannot be added together

23. Which of these matrices has no inverse?

a. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ c. $\begin{bmatrix} 2 & 1 \\ -1 & -2 \end{bmatrix}$ d. $\begin{bmatrix} 2 & \frac{1}{2} \\ \frac{1}{2} & 2 \end{bmatrix}$ e. $\begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$

24. What is the sum of the numbers from 123 to 321 inclusive?

- a. 44,055 b. 44,178 c. 44,252 d. 44,471 e. None of the above

25. A metal frame with a triangular profile has sides six feet in length, an eight foot wide base, and is sitting on level ground. One of the sides of the base is then jacked up a foot. The other side of the base remains on the ground, and the shape of the frame remains the same. How high off the ground is the highest point of the frame now?

- a. 4.47 feet b. 4.94 feet c. 4.97 feet d. 5.08 feet e. 5.47 feet

26. How many distinct ways can the letters in the word *pineapple* be re-arranged using all nine letters?

- a. 12 b. 3024 c. 30240 d. 181440 e. 362880

27. Convert the polar coordinate $(r, \theta) = (1.5, 5)$ to rectangular coordinates (x, y) .

- a. $(0.43, -1.44)$ b. $(4.99, 0.35)$ c. $(0.13, 1.49)$ d. $(-1.24, -2.51)$ e. none of these

28. A 6 sided dice is loaded in such a way that getting a "6" is 6 times as likely as any other face. What is the probability of getting a prime number?

- a. $1/11$ b. $3/11$ c. $1/8$ d. $3/8$ e. None of the above

29. Two ships leave port at 8 A.M. One has a bearing of 36 degrees North of East at 13 miles per hour. Another ship has a bearing of 47 degrees South of East at 18 miles per hour. Approximately how far apart are the two ships after 3 and half hours?

- a. 73 miles b. 21 miles c. 68 miles d. 81 miles e. 44 miles

30. One half of the inverse of one less than a number is equal to the number. Rounded to the nearest 0.01, what is one possible value of the number?

- a. -2.34 b. 1.37 c. 2.34 d. 3.33 e. None of the above