2022 Academic Challenge
State Mathematics Exam

1. The difference between two numbers is 6 and the difference between their squares is 108. What is a possible quotient of the two numbers?

   a) 1/6   b) 1/4   c) 1/2   d) 1   e) none of the above.

2. Four children have adopted a total of ten pets. Four dogs were adopted, five cats were adopted, and one rabbit was adopted. The following six statements describe some facts about who adopted what.

   i. Each family was careful to adopt only pets that got along with each other.
   ii. Buddy adopted exactly twice as many pets as his sister Debbie.
   iii. Antoine adopted more pets than his sister Cherise.
   iv. Buddy's dogs didn't like either of the other dogs.
   v. Antoine's cat didn't like any of the other cats.
   vi. Cherise's dog was the last pet adopted.

   Who adopted the rabbit?

   a) Antoine   b) Buddy   c) Cherise   d) Debbie
   e) Not enough information given to determine who did.

3. The sum of A, B and C is 35. C is one more than twice A. B is one less than twice A. What is 2A − 3B + C?

   a) −10   b) 1   c) 62   d) 84   e) none of the above

4. What binary number is the result of subtracting the binary number 11001011 from the binary number 100100101?

   a) 1010100   b) 1011010   c) 1110001   d) 11011010   e) 111110000

5. What is the directrix of the parabola below:

$$y = \tfrac{2}{3}x^2 - 2$$

   a) $y = -2$ b) $y = -\dfrac{2}{3}$ c) $y = -\dfrac{19}{8}$ d) $y = -\dfrac{17}{6}$ e) $y = -\dfrac{19}{6}$

6. A hyperbola has foci on the points (-4, 2) and (6, 2), and passes through the point (4, 2). What is the general form equation of the hyperbola?

   a) $-16x^2 - 32x + 9y^2 - 36y - 124 = 0$ b) $-4x^2 + 8x + 3y^2 - 12y - 4 = 0$
   c) $4x^2 - 8x - 3y^2 + 12y - 20 = 0$ d) $16x^2 - 32x - 9y^2 + 36y - 164 = 0$
   e) $25x^2 - 50x - 9y^2 + 36y - 236 = 0$

7. What is the area contained between the x-axis, the y-axis and the line $y + 45x + 15 = 0$?

   a) 1   b) 1.25   c) 2.5   d) 3.75   e) 5

8. What is the area inside the figure bounded by the polar equation

$$9r^2 \cos^2 \theta + 16r^2 - 400 = 0$$

   a) 12π b) 15π c) 20π d) 25π e) This equation doesn't create a closed figure.

9. Which response is equivalent to the expression

$$3\ln(x-1) + 2\ln(y+1) - \ln(xy-1)$$

   a) 0   b) $(x-1)^3 + (y+1)^2 - (xy-1)$ c) $\dfrac{(x-1)^3 (y+1)^2}{(xy-1)}$

   d) $\ln \dfrac{(x-1)^3 (y+1)^2}{(xy-1)}$ e) $\dfrac{\ln(x-1)^3 \ln(y+1)^2}{\ln(xy-1)}$

10. A fourth degree polynomial with real coefficients has complex roots at $x = 2 - 3i$ and $x = -3 + 2i$. What is the polynomial?

    a) $x^4 - 2x^3 - 24x^2 + 50x - 25 = 0$
    b) $x^4 - 2x^3 + 2x^2 - 26x + 169 = 0$
    c) $x^4 + 2x^3 - 24x^2 - 50x - 25 = 0$
    d) $x^4 + 2x^3 + 2x^2 + 26x + 169 = 0$
    e) Not enough roots were given to determine the equation.

11. You are given **A** = 2**i** + **j** + 3**k** and **B** = **i** + 3**j** + 2**k**. Find vector **C** such that the cross products **A** x **B** and **A** x **C** are of equal length, but point in opposite directions.

    a) **C** = **i** – 2**j** + **k**   b) **C** = –**i** – 3**j** – 2**k**   c) **C** = 3**i** – **j** + 4**k**
    d) All of the above
    e) No such vectors exist

12. A computer is depreciated using the straight-line method. Three years after the purchase, its value is $12,500. Seven years after the purchase, its value is $9,000. What is the computer's value after 13.5 years?

    a) $3,312.50   b) $4,625.00   c) $4,812.50   d) $5,025.00   e) None of the above

13. The research department of a company has spent 65% of its budget. The advertising department has spent 72% of its allotted budget. The quality control department has spent 80% of its budget. Each department has the same dollar amount of money left over. What percentage of the total budget for the three departments has been spent so far? (Round your answer to the nearest percent.)

a) 70%   b) 72%   c) 74%   d) 76%   e) Not enough information given

14. A biker and jogger leave the same building at the same time. The jogger has a rate of 3.25 mph. After 2.5 hours, the biker is 25.625 miles ahead of the jogger. If both are moving at constant rate, what is the biker's rate?

a) 10 mph   b) 12.25 mph   c) 13.5 mph   d) 14.75 mph   e) None of the above

15. Circles A, B, and C are drawn so that they are concentric, with the radius of C being three times that of A, and the radius of B being twice the radius of A. If we consider Ring I to be the area between C and B, and Ring II to be the area between B and A, what is the ratio of Ring I to Ring II?

a) 2 to 1   b) 3 to 2   c) 5 to 3   d) 9 to 4   e) Not enough information given

16. Simplify the expression:

$$\frac{1}{\sin q \cos q} - \frac{\cos q}{\sin q}$$

a) $\sin q$ b) $-\cos q$ c) $\tan q$ d) $-\sec q$ e) None of the these

17. y varies linearly with respect to the square root of x. $y = 1$ when $x = 4$ and $y = -2$ when $x = 9$. What is y when $x = 8$?

a) $-7/5$ b) $-\sqrt{2}$ c) $-6\sqrt{2}+7$ d) $\left(-6\sqrt{2}\right)/5+17/5$ e) None of the above

18. What is the period of the function $\frac{2}{3}\sin\left(\frac{3}{4}x\right)+\frac{3}{4}\cos\left(\frac{2}{3}x\right)$

a) $\frac{1}{2}\pi$ b) 4π c) 8π d) 12π e) 24π

19. Neal and Bob toss a coin and the first to toss a head wins. If Bob gets to toss the coin first, what is the probability that he wins?

a) 3/7
b) 2/3
c) 6/7
d) 3/5
e) None of the above

20. All seven of the circles shown below are tangent and have a radius of 10 inches. The points A, B, and C are centers of their respective circles. What is the area of the triangle ABC? (Round your answer to the nearest square inch.)

a) 314 square inches
b) 450 square inches
c) 471 square inches
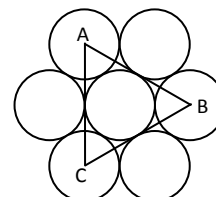
d) 520 square inches
e) Not enough information given

21. Determine the area that is bounded by the x axis, $x = 1$, $x = 3$ and $y = 12x^2 + 3$.

   a) 96   b) 110   c) 117   d) 124   e) None of the above

22. An ellipse is defined by the set of parametric equations

$$x = \sqrt{3}\cos t \quad \text{and} \quad y = 2\sin t$$

   What is the slope of the line tangent to the ellipse at the point on the ellipse where $y = 1$ and $x > 0$?

   a) −1   b) −2   c) −5/2   d) −3   e) −7/2

23. Determine the inverse of the matrix

$$\begin{bmatrix} -\frac{1}{2} & 3 \\ 1 & 2 \end{bmatrix}$$

   a) $\begin{bmatrix} +2 & -3 \\ -1 & -2 \end{bmatrix}$ b) $\begin{bmatrix} -2 & \frac{1}{3} \\ 1 & \frac{1}{2} \end{bmatrix}$ c) $\begin{bmatrix} \frac{1}{2} & 1 \\ \frac{1}{3} & -2 \end{bmatrix}$ d) $\begin{bmatrix} -\frac{1}{2} & \frac{3}{4} \\ \frac{1}{4} & \frac{1}{8} \end{bmatrix}$ e) The matrix has no inverse.

24. Two children are standing on opposite sides of a canyon. The boy on the east edge has to look down 72 degrees below horizontal to see the center of the river at the bottom while the girl on the western edge has to look down only 61 degrees below horizontal to see the center of the river. A sign says that the top edges of the canyon are 325 m apart. How deep, approximately, is the canyon?

   a) 247 m   b) 369 m   c) 406 m   d) 507 m   e) 1,290 m

25. Determine the positive value of x for which

$$3^{x^2-2x} = 6561$$

   a) 2   b) 4   c) 6   d) 8   e) 10

26. Box A contains 4 green and 3 blue chips. Box B contains 3 green and 6 blue chips. Box C contains 4 green and 2 blue chips. A box is selected at random and two chips are drawn without replacement. What is the probability of selecting at least one green chip? (Round your answer to the nearest thousandth.)

   a) .023   b) .203   c) .417   d) .791   e) .884

27. Which equation is equivalent to the following logical function table?

A	B	C	D
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

a) $D = \overline{AC} + CA$ b) $D = \overline{AC} + \overline{AB} + A\overline{C}$ c) $D = \overline{\overline{AC} + \overline{AB} + A\overline{C}}$ d) $D = \overline{\overline{AC} + \overline{AB}}$ e) $D = \overline{\overline{AB}}$
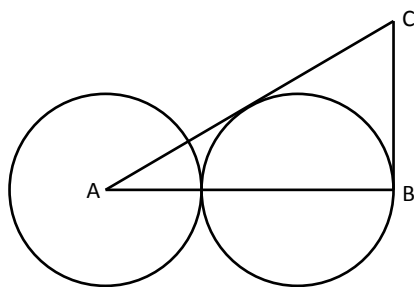
28. The probability that yesterday's lifeguard locked a fence around a pool is 75%. There is a jar of similar looking keys, but only one will unlock the fence. The probability that today's lifeguard can get into the pool without needing more than one randomly chosen key is 43.75%. How many keys are in the jar?

a) 2   b) 3   c) 4   d) 5   e) None of the above

29. A harmonic sequence begins with 1, 3, … What is the third number in the sequence?

a) –3   b) 5   c) 6   d) 7   e) 9

30. Two circles of radius R are tangent to each other. Point A is the center of the left circle and segment AB passes through the point of tangency and point B is on the right circle. Segment BC is tangent to the right circle at point B. What is the length of segment AC, which is tangent to the right circle?

a) $\dfrac{10}{3}R$ b) $\dfrac{7}{2}R$ c) $\dfrac{5\sqrt{2}}{2}R$ d) $\left(1+\sqrt{6}\right)R$ e) $2\sqrt{3}R$