Academic Challenge
Physics Solutions (State) – 2022

1. **Correct Response: E**
Solving for η gives:

$$\eta = \frac{Fl}{Av} = \frac{\frac{kg\ m}{s^2}\ m}{m^2\ \frac{m}{s}} = \frac{kg\ m^2\ s}{m^3\ s^2} = \frac{kg}{m\ \ s} = \frac{[m]}{[L][T]}$$

2. **Correct Response: A**
The volume of the sphere is given by the mass and the density:

$$V = \frac{m}{\rho} = \frac{1.47\ kg}{2.14 \times 10^4 \frac{kg}{m^3}} = 6.869 \times 10^{-5}\ m^3$$

The volume of a sphere is given by

$$V = \frac{4}{3}\pi R^3$$

where R is the radius of the sphere. We can now solve for R to get:

$$R = \sqrt[3]{\frac{3V}{4\pi}} = 0.0254\ m\ \times \frac{100\ cm}{1\ m} = 2.54\ cm$$

Where the conversion to centimeters is also included.

3. **Correct Response: A**
The angular velocity of the wheel as it is driven by Mollie is given by

$$\omega = \frac{v}{r} = \frac{1.87\ \frac{m}{s}}{0.0823\ m} = 22.72\ \frac{rad}{s}$$

Angular momentum is given by the moment of inertia times the angular velocity. So we must find the moment of inertia. The moment of inertia for a hoop is given by:

$$I = mr^2 = 0.0289\ kg\ (0.0823)^2 = 1.9575 \times 10^{-4} kg\ m^2$$

And now we can solve for the angular momentum:

$$L = I\omega = \ 22.72\ \frac{rad}{s} \times\ 1.9575 \times 10^{-4} kg\ m^2 = 4.45 \times 10^{-3} \frac{kg\ m^2}{s}$$

An even simpler solution is to note that every mass element of the hoop has the same radius & velocity, so

$$L = mvR = (0.0289kg)\left(1.87\ \frac{m}{s}\right)(0.0823\ m) = \ 4.45 \times 10^{-3} \frac{kg\ m^2}{s}$$

4. **Correct Response: B**

A sound wave with a frequency higher than 20,000 Hz is known as ultrasonic.

5. **Correct Response: D**

Let's set the origin at the initial launch site, the +x-axis to the right and the +y-axis upward. Apply kinematic equations:

$$x = (v_0 \cos\theta)t \text{ and } y = (v_0 \sin\theta)t - (1/2)gt^2$$

Upon landing, $y = 0$ and $t = 2v_0 \sin\theta/g$. So,

$$R = (v_0 \cos\theta)(2v_0 \sin\theta)/g = v_0^2 \sin 2\theta/g$$

For a given initial speed v_0, R_{max} occurs at $\theta = 45°$, and $R_{max} = v_0^2/g$. From the diagram, $R_{max} = 0.918$ m. So,

$$v_0 = (gR_{max})^{1/2} = [(9.80 \text{ m/s}^2)(0.918 \text{ m})]^{1/2} = 3.00 \text{ m/s}.$$

6. **Correct Response: A**

Consider the two blocks and the rope as a system. Set the +y-axis upward. Apply Newton's 2nd Law:

$$\sum F_{\text{ext, y}} = ma_y$$

$$F - (m_1 + m_2 + M)g = (m_1 + m_2 + M)a_y$$

$$a_y = F/(m_1 + m_2 + M) - g$$
$$= (7.00 \text{ N})/(0.200 \text{ kg} + 0.300 \text{ kg} + 0.100 \text{ kg}) - 9.80 \text{ m/s}^2 = 1.87 \text{ m/s}^2$$

7. **Correct Response: E**

Consider block m_1. Apply Newton's 2nd Law: $\sum F_{\text{ext, y}} = ma_y$

$F - m_1 g - T_U = m_1 a_y$, where T_U is the tension at the upper end of the rope.

$T_U = F - m_1(g + a_y) = 7.00 \text{ N} - (0.200 \text{ kg})(9.80 \text{ m/s}^2 + 1.867 \text{ m/s}^2) = 4.67 \text{ N}$
Consider block m_2. Apply Newton's 2nd Law: $\sum F_{\text{ext, y}} = ma_y$

$T_L - m_2 g = m_2 a_y$, where T_L is the tension at the lower end of the rope.

$T_L = m_2(g + a_y) = (0.300 \text{ kg})(9.80 \text{ m/s}^2 + 1.867 \text{ m/s}^2) = 3.50 \text{ N}$

8. **Correct Response: A**
A very simple solution is to realize that the distance, the time, and the initial velocity are given and that this is a constant acceleration problem. Therefore, the basic equation for constant acceleration is:

$$y_f = y_i + v_i t + \frac{1}{2} a t^2$$

In this problem we set $y_i = 0$ and also $v_i = 0$ and solve the equation for the acceleration to get:

$$a = \frac{2y_f}{t^2} = \frac{2 \times 1.65}{0.976144} = 3.3806 \frac{m}{s^2}$$

Alternatively we could approach this problem by considering the rotational motion as follows:

The moment of inertia of the sphere is given by

$$I_{sphere} = \frac{2}{5} mR^2$$

The conservation of mechanical energy would give us

$$mgl \sin\theta = \frac{1}{2} m v_f^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} \left\{ \frac{2}{5} mR^2 \right\} \left[\frac{v}{R} \right]^2 = \frac{1}{2} m v_f^2 + \frac{1}{5} m v_f^2$$

Solving for v_f we get:

$$v_f = \sqrt{\frac{gl \sin\theta}{0.7}} = 3.341 \frac{m}{s}$$

To find the acceleration invoke the equation of motion for constant acceleration:

$$v_f = v_i + at$$

We can now solve for the acceleration to get 3.38 m/s^2 which is, of course, the same as above.

9. Correct Response: B

Consider the small object and the disk as a system. During the collision, the external forces acting on the system include gravity on the object, gravity on the disk, and the force exerted by the axle on the disk. The lines of action of these forces pass through point *O*, and thus the net external torque on the system about point *O* is zero. Apply conservation of angular momentum: $L_1 = L_2$.

$$Rmv_0 = I\omega + Rmv,$$

where *R* is the radius of the disk, *m* is the mass of the small object, *I* is the moment of inertia of the disk, and *v* is the (linear) speed of the object immediately after the collision. Since the object sticks to the disk, *v* is also the tangential speed of point *P* right after the collision. So, $v = R\omega$.

$$Rmv_0 = I\omega + R^2 m\omega$$

$$\omega = Rmv_0/(I + R^2 m)$$
$$= (0.200\text{ m})(0.100\text{ kg})(5.00\text{ m/s})/[0.0500\text{ kg}\cdot\text{m}^2 + (0.200\text{ m})^2(0.100\text{ kg})]$$

$= 1.85 \text{ rad/s}$

10. Correct Response: C

Consider a segment of the rope from x to $x + \Delta x$. The tension force at the right end of the segment is $T(x + \Delta x)$ directed to the right and the tension force at the left end of the segment is $T(x)$ directed to the left. The mass of this segment is $m\Delta x/L$. The acceleration of the rope is F/m. Apply Newton's 2nd Law to this segment.

$$T(x + \Delta x) - T(x) = (m\Delta x/L)(F/m) = F\Delta x/L$$

$$\text{Slope} = [T(x + \Delta x) - T(x)]/\Delta x = F/L = (5.00\text{ N})/(1.50\text{ m}) = 3.33\text{ N/m}$$

11. Correct Response: A

The displacement of point mass 2 relative to point mass 1 is $s_2 - s_1$. Apply Hooke's Law. The spring force on point mass 2 due to its relative displacement from point mass 1 is

$$F_{12} = -k(s_2 - s_1)$$

Similarly, the spring force on point mass 2 due to its relative displacement from point mass 3 is

$$F_{32} = -k(s_2 - s_3)$$

So, the net spring force on point mass 2 $= -k(s_2 - s_1) - k(s_2 - s_3) = k(s_1 - 2s_2 + s_3)$

12. Correct Response: B

In uniform circular motion, the speed is constant. However, since the direction of velocity changes with time, velocity is not constant. Since velocity changes with time, acceleration is not zero. Since the direction of the centripetal force changes with time, centripetal force is not constant. Finally, centripetal acceleration is not constant because the direction of centripetal acceleration changes with time.

13. Correct Response: E

Power is work per unit time, where work is force times distance, force is mass times acceleration, acceleration is change in velocity per unit time, and velocity is change in position per unit time. So, the dimensions of power are $([M][L]/[T]^2)[L]/[T] = [M][L]^2/[T]^3$.

The dimensions of area are $[L]^2$. So, the dimensions of power per area are $[M]/[T]^3$.

14. Correct Response: C

The kinetic energy is the initial kinetic energy plus the work that the force does on the object. So we must calculate

$$\int_0^7 F dx = \int_0^6 \left(-\frac{6}{4}x + 6\right) dx + \int_6^7 (-3) dx = \left(-\frac{6}{8}x^2 + 6x\right)\Big|_0^6 + (-3x)\Big|_6^7 = 9J - 3J = 6J$$

Of course there are other ways to calculate that integral including calculating the areas of triangles and squares in the figure. Finally adding the initial kinetic energy we get:

$$\frac{1}{2}mv^2 + 6J = 9J + 6J = 15J$$

15. Correct Response: A

Here, instantaneous power P = 10.0 W. $P = F_{\text{net, }x}v_x$. So, $F_{\text{net, }x} = P/v_x$ = 10.0 W/5.00 m/s = 2.00 N. Acceleration $a_x = F_{\text{net, }x}/m$ = 2.00 N/0.500 kg = 4.00 m/s^2.

16. Correct Response: C

The trajectory of the center-of-mass follows the same trajectory of the original firecracker. Apply kinematics equations in 2D.

$$x_{cm} = (v_0 \cos\theta)t = (3.00 \text{ m/s})(\cos 60.0^\circ)t = (1.50 \text{ m/s})t$$

$$y_{cm} = (v_0 \sin\theta)t - (1/2)gt^2 = (3.00 \text{ m/s})(\sin 60.0^\circ)t - (1/2)(9.80 \text{ m/s}^2)t^2$$
$$= (2.598 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2 \approx (2.60 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$$

17. Correct Response: B

$$x_{\text{cm}} = (m_1x_1 + m_2x_2)/(m_1 + m_2) = (2m_2x_1 + m_2x_2)/(2m_2 + m_2) = (2x_1 + x_2)/3$$

$$y_{\text{cm}} = (m_1y_1 + m_2y_2)/(m_1 + m_2) = (2m_2y_1 + m_2y_2)/(2m_2 + m_2) = (2y_1 + y_2)/3$$

Solving for x_2 and y_2, we have

$$x_2 = 3x_{\text{cm}} - 2x_1 = 3(1.50 \text{ m/s})t - 2x_1 = (4.50 \text{ m/s})t - 2x_1$$

$$y_2 = 3y_{\text{cm}} - 2y_1 = 3[(2.598 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2] - 2y_1 = (7.79 \text{ m/s})t - (14.7 \text{ m/s}^2)t^2 - 2y_1$$

18. Correct Response: B

Static equilibrium means that the sum of the torques is zero. Using the torque about the point of the fulcrum and noting that the center of mass of the meter stick is at the 50 cm point we get:

$$m_s g(50cm - 36.3cm) - (175g)g\{36.3 - 23.4\} = 0$$

Solving this equation for m_s gets us 164.8g which rounds to 165g.

19. Correct Response: D

Set the +*x*-axis to the east, the +*y*-axis to the north, and the origin at the starting point. Use **i** and **j** to represent the unit vectors in the *x*- and *y*-directions, respectively. The position vectors of students A and B are $\mathbf{r}_A$ = 500**i** (m) and $\mathbf{r}_B$ = 700**j** (m), respectively. So, the displacement of student B from student A is $\Delta\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A$ = – 500**i** + 700**j** (m).

$|\Delta\mathbf{r}| = [(-500\text{ m})^2 + (700\text{ m})^2]^{1/2} = 860$ m

$\tan^{-1}[(700\text{ m})/(-500\text{ m})] = -54.5°$. Since $\Delta\mathbf{r}$ is in the II quadrant, $\Delta\mathbf{r}$ is directed 54.5° north of west.

20. Correct Response: C

Gravitational acceleration at the surface of a planet is $g = GM/R^2$. For the planet in question, $g_p = GM_p/R_p^2$ and for the earth, $g_e = GM_e/R_e^2$. Since $M_p = 5M_e$ and $g_p = 2g_e$, $G(5M_e)/R_p^2 = 2GM_e/R_e^2$. So, $R_p = (5/2)^{1/2}R_e$.

21. Correct Response: B

The moment of inertia is given by:

$$I = \sum m_i r_i^2 = 3.6kg \times (3m)^2 + 7.2kg \times (1.5m)^2 + 14.4kg \times (0.75m)^2 = 56.7\ kg\ m^2$$

And the torque is related to the angular acceleration by $\tau = I\alpha$ so we can now find α to be 1.4109 rad/s^2.

The angular velocity can now be found through:

$$\omega = \omega_i + \alpha t = 0 + 1.4109\frac{rad}{s^2} \times 26.6s = 37.53\frac{rad}{s}$$

We can now calculate the rotational kinetic energy:

$$KE_{rot} = \frac{1}{2}I\omega^2 = 39933J$$

Second approach: We can consider the kinetic energy directly:

$$KE_{rot} = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$$

but the angular momentum and the torque are related as follows when the system starts from rest and has a constant torque:

$$L = \tau \times t$$

so after we find the moment of inertia of the system as above we can now get the rotational kinetic energy from the information given:

$$KE_{rot} = \frac{(\tau \times t)^2}{2 \times 56.7\ kg\ m^2} = 39933J$$

22. Correct Response: E

Conservation of momentum applied to rockets leads to the result that the speed of a rocket is given by:

$$v - v_o = v_{ex} \ln\left(\frac{m_o}{m}\right)$$

Where m_o is the initial mass of the rocket, m is the final mass of the rocket, and v_o is the initial velocity of the rocket. Since the fuel is expelled the ratio m_o/m can be arbitrarily large (up to relativistic speeds) so the final velocity can greatly exceed the exhaust velocity.

23. Correct Response: D

The power transmitted in a transverse wave on a string is given by:

$$P_{ave} \propto \sqrt{\mu T} f^2 A^2$$

Of the choices in the problem the only one with variables that are in the same ratios as this is A which states $P_{ave} \propto f^2\sqrt{T}$

24. Correct Response: D

For double-slit interference, the condition for constructive interference is that the path length difference is a whole number of wavelengths, that is, $d \sin \theta = m\lambda$. So, $\sin \theta = m\lambda/d$. For small angles, $\sin \theta \approx \tan \theta = y/L$. So, $m\lambda/d = y/L$ and $y = Lm\lambda/d$.

For $m = 1$, for red light, $y = (3.50\ \text{m})(6.50 \times 10^{-7}\ \text{m})/(1.75 \times 10^{-3}\ \text{m}) = 1.30 \times 10^{-3}\ \text{m}$; for green light, $y = (3.50\ \text{m})(5.50 \times 10^{-7}\ \text{m})/(1.75 \times 10^{-3}\ \text{m}) = 1.10 \times 10^{-3}\ \text{m}$

Separation between bright fringes of red and green light of the order $m = 1$ is 1.30 mm – 1.10 mm = 0.20 mm.

25. Correct Response: E

The minimum speed, v, at the top of the track is determined by Newton's 2nd law for circular motion: $mg = mv^2/R$. So, $v = (gR)^{1/2}$. Apply conservation of energy. $(1/2)mv_0^2 = (1/2)mv^2 + mg(2R) = (1/2)mgR + mg(2R) = (5/2)mgR$.

So, minimum initial speed required is $v_0 = (5gR)^{1/2}$.

26. Correct Response: B

Apply work-energy relation: $W_f = \Delta E$, where W_f is the work done by friction and ΔE is the change in mechanical energy.

$W_f = E_f - E_i = (1/2)mv^2 + mg(2R) - (1/2)mv_0^2 = (1/2)m(v^2 + 4gR - v_0^2) = (1/2)(0.200\ \text{kg})[(2.25\ \text{m/s})^2 + 4(9.80\ \text{m/s}^2)(0.500\ \text{m}) - (6.00\ \text{m/s})^2] = -1.13\ \text{J}$.

27. Correct Response: D

The power that is available in the circuit is $P = IV = 15A \times 120V = 1800W$. If each bulb uses 4 watts we can see that $\frac{1800W}{4W} = 450$.

28. Correct Response: C

In relativistic velocities the relative velocity is given by:

$$u = \frac{v + u'}{1 + \frac{u'v}{c^2}}$$

Where u' is the speed of the antiproton in the lab frame, v is the speed of the proton in the lab frame, and u is the speed of the proton in the reference frame of the antiproton. Plugging in we get:

$$u = \frac{0.805c + 0.190c}{1 + \frac{0.805c \times 0.190c}{c^2}} = \frac{0.995c}{1.15295} = 0.863c$$

29. Correct Response: E

One might remember that the wave function is given by the following.

$$\Psi_3(x) = \sqrt{\frac{2}{L}} \sin\frac{3\pi x}{L}$$

However, we might also remember that the general statement of a wave function for a particle in a box is:

$$\Psi_n(x) = A \sin\frac{n\pi x}{L}$$

So for n=3 we see the functional relationship but to normalize this wave function we see:

$$1 = \int_0^L |\Psi_n(x)|^2 dx = A^2 \int_0^L \sin^2\frac{n\pi x}{L} dx = A^2 \frac{L}{2}$$

And solving for that amplitude and plugging it in above we get the wave function given.

30. Correct Response: C

The lensmaker's equation is used here. After that it is simply plugging in the numbers given.

$$\frac{1}{f} = (n-1)\left\{\frac{1}{R_1} - \frac{1}{R_2}\right\} = (0.78)\left\{\frac{1}{-21.4cm} - \frac{1}{33.8cm}\right\} = -0.0595255\frac{1}{cm}$$

So we get:

$$f = \frac{1}{-0.0595255\frac{1}{cm}} = -16.8\,cm$$

31. Correct Response: A

Using a Kirchhoff loop to follow the potential around the circuit and noting that because there are no junctions so that the current is constant all the way around the loop we get:

$$10V - 18\Omega I - 17\Omega I - 6V - 25\Omega I = 0$$

Solving for current we get:

$$I = \frac{4V}{60\Omega} = 0.0666667\,A = 66.7\,mA$$

32. Correct Response: D

The activity of a radioactive material is time dependent according to:

$$A(t) = A_o e^{-\lambda t}$$

where A_o is the initial activity, t is the time, and λ is the decay constant. We are given the initial activity as well as the activity after a time so we can solve this equation for λ.

$$\lambda = \frac{-1}{0.952\,days} \times ln\left(\frac{11.1\,Ci}{14.7\,Ci}\right) = 0.295\frac{1}{days}$$

And now we look at the definition of the decay constant to find the halflife:

$$\lambda = \frac{\ln 2}{T_{1/2}}$$

Solving for the halflife we get:

$$T_{1/2} = \frac{\ln 2}{0.295\frac{1}{days}} = 2.35\,days$$

33. Correct Response: B

The first thing to note is that the temperatures are given in Fahrenheit but need to be in Celsius.

$$T_i = (T_F - 32) \times \frac{5}{9} = -6.1111℃$$

And

$$T_f = (T_F - 32) \times \frac{5}{9} = 16.6667℃$$

And the freezing point is

$$T_{fp} = (T_F - 32) \times \frac{5}{9} = 6.1111℃$$

So the number of Celsius degrees from the initial temperature to the freezing point is 12.2222°C and the number of degrees from the freezing point to the final temperature is 10.5555°C.
Now we have three cases of heat added. From the initial temperature to the freezing point, the latent heat of fusion, and from the freezing point up to the final temperature.

$$Q_1 = mc\Delta T = 1.36kg \times 118000\frac{J}{kg\,°\mathrm{C}} \times 12.2222°\mathrm{C} = 1961400J$$

$$Q_2 = mL = 1.36kg \times 45000\frac{J}{kg} = 61200J$$

$$Q_3 = mc\Delta T = 1.36kg \times 133000\frac{J}{kg\,°\mathrm{C}} \times 10.5556°\mathrm{C} = 1909300J$$

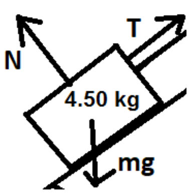
Adding these three heats together gives us 3931900J which is $3.93\text{x}10^6$J.

34. Correct Response: B

"The entropy of the Universe increases in all natural processes" is one statement of the 2nd Law of Thermodynamics and therefore is true.

35. Correct Response: A

We can draw force diagrams for each of the two masses:



Where N is the normal force, T is the force of tension in the string, and the mg on the hanging mass relates to gravity on the 3.00 kg mass and mg on the mass on the incline is also gravity only this time it is the 4.50 kg mass. This should be broken down into the force perpendicular to the incline and the force parallel to the incline. The force parallel (and down the incline) will be $F_{para} = mgsin\theta = 4.50kg \times 9.8\frac{m}{s^2} \times sin39.5° = 28.051N$ If we assume that the up direction on the hanging mass is positive for the acceleration we can write force equations for each of these

$$T - 3.00kg \times 9.8\frac{m}{s^2} = 3.00kg \times a$$

And

$$T - 4.50kg \times 9.8\frac{m}{s^2} \times sin39.5° = -4.50kg \times a$$

If we subtract the second from the first the result is

$$4.50kg \times 9.8\frac{m}{s^2} \times sin39.5° - 3.00kg \times 9.8\frac{m}{s^2} = (3.00kg + 4.50kg) \times a$$

$$a = \frac{28.051N - 29.4N}{7.5\ kg} = \frac{-1.34895N}{7.5\ kg} = -0.17986\frac{m}{s^2}$$

The negative sign means that the hanging weight is accelerating downward which means that the mass on the incline must be moving up the incline. The result is that the acceleration is 0.180 m/s^2 up the incline for the 4.50 kg mass.