**2024 Academic Challenge**

**Sectional Mathematics Exam Answers**

1. A new product is revealed gradually to a company employing 1,200 people.
Every three days, one-fourth of the workforce learns about the product. After 10 days, how many people are still unaware of the product?
2. 250 b) 360 **c) 460** d) 500 e) 540

The decay equation is 1,200(0.75)^(10/3) = 459.96. This rounds to 460. The answer is c.

2. If log(3)25 = x and log(3)16 = y, what is log(3)20?

1. (x-y)/2 **b) (x+y)/2** c) x-y d) x+2y e) x+y

Answer: log(3)25 = log(3)5^2 = 2 log(3)5. Log(3)16 = log(3)4^2 = 2 log(3)4. Log(3)20 = log(3)(5\*4) = log(3)5 + log(3)4 = x/2 + y/2. The answer is b: (x+y)/2.

3. If 35 percent of students carry index cards, 47 percent carry sticky notes, and 19 percent carry index cards and sticky notes, what percent of students carry index cards or sticky notes?

1. 12 b) 54 **c) 63** d) 66 e) 82

Answer: 35 + 47 - 19 = 63. The answer is c.

4. Each side measure of a regular octagon is 12 centimeters. What is the area of the octagon?

**a) 695.28** $cm^{2}$ b) 1420.96 $cm^{2}$ c) 521.46 $cm^{2}$ d) 1390.56 $cm^{2}$ e) 764.36 $cm^{2}$

A central angle measure is 360/8 = 45. A way to solve is to break up the octagon into 8 regular triangles. Then divide these into right triangles. A vertex angle measure will be 45/2 = 22.5. The base measure will be 12/2 = 6. Then the tangent of 22.5 = 6/x. X = 6/tan(22.5) = 14.485. This is the apothem of the octagon. The area is 1/2(perimeter)(apothem) = 1/2(96)(apothem) = 695.28. The answer is a.

5. What is the slope of the tangent line to the equation 2x^2 + 3xy = 18 at the point (3,0)?

1. -3/4 **b) -4/3** c) 2/3 d) 3/4 e) 9/2

Take the derivative of the equation. It is 4x + 3y + 3x(dy/dx) = 0. Then plug in the x and y values to get 12 + 0 + 9(dy/dx) = 0. Solving for dy/dx leads to -12/9, which is -4/3. The answer is b.

6. If a sequence starts with the terms 14, 19, 24, 29 …, what will the 15th term of the sequence equal?

1. 59 b) 64 c) 74 **d) 84** e) 89

Each value goes up by 5. We take a1 and add 14\*5. 14 + 70 = 84. The answer is d.

7. If an object travels 4,800 degrees around the unit circle, how many revolutions has it completed?

a) 10 b) 12  **c) 13** d) 14 e) 15

Each revolution requires 360 degrees. 4800/360 = 13.3. The object has completed 13 revolutions. Answer C.

8. If the volume of a pyramid with a square base must hold at least 800 cubic meters, what do the dimensions need to be?

1. base with each side 12 meters; height of 15 meters
2. base with each side 10 meters; height of 22 meters
3. base with each side 15 meters; height of 10 meters
4. **base with each side 13 meters; height of 15 meters**
5. base with each side 12 meters; height of 16 meters

Square the base measure and multiply by the height. The answer that exceeds 800 is (13^2 \* 15)/3 = 845. Answer is d.

9. Solve for x. 3^(2x+4) = 81^(6x+12).

a) -3 **b) -2** c) 2 d) 3 e) 4

Convert 81 to 3^4 to get the bases the same. Then distribute the exponent of 4 on the right. The equation becomes 2x + 4 = 24x + 48. Solving for x, we get 22x = -44. X = -2. Answer is b.

10. During February, the probability of snow during a bus route is 32 percent. A bus is late to its stop 11 percent of the time if there is no snow. When there is snow, the bus is late 27 percent of the time. Given that the bus arrives on time, what is the probability there is snow?

a) 0.2155 b) 0.2500 c) 0.2635 **d) 0.2785** e) 0.3215

A tree probability yields the fraction of (0.32\*0.73)/((0.32\*0.73)+(0.68\*0.89)). This equals 0.2785. The answer is d.

11. What are the roots of x^2 - 4x + 27?

a) 4 +- sqrt(23) b) 1 +- isqrt(23) c) 4 +- isqrt(23) **d) 2 +- sqrt(23)** e) 2 +- isqrt(23)

Use the quadratic formula. [4 +- sqrt(16 - 4(1)(27)]/2 = [4 +- sqrt(-92)]/2 = [4 +- 2i(sqrt(23))]/2 = 2 +- isqrt(23). Answer is d.

12. For the function abs (x+2), what is the value of the derivative at x=-2?

1. 0 b) 1 c) -1 d) The function is not continuous at x = -2. **e) The function is not differentiable at x = -2.**

The vertex of the absolute value function is at (-2, 0). The derivative is undefined at a corner point. The answer is e.

13. Consider the following piecewise function:

x^2 - 4x; x < 6

Sqrt (bx + 3); x >= 6

What value of b would make the function continuous?

a) 3 b) 4.5 c) 12.5 **d) 23.5** e) 25

At 6, x^2 - 4x = 36 - 24 = 12. Set sqrt(bx+3) = 12 when x = 6. 6b + 3 = 144. 6b = 141. The b value is 23.5. Answer is d.

14. A parabola has a focus at (2, 6). The directrix for the parabola is at x = -8. What is the equation of the parabola?

1. y = 1/20(x+3)^2 + 6
2. **x = 1/20(y-6)^2 - 3**
3. x = 1/20(y-2)^2 + 3
4. y = 20(x+3)^2 - 6
5. x = 20(y-6)^2 + 3

The vertex is halfway between the directrix and the focus, so the vertex is at (-3, 6). The focal length is 5, so the focal width is 20. The parabola opens to the right. Using this information, the answer is b.

15. A card player is holding a five-card set. Three of the cards are clubs, with the highest rank of 10. (The ace is the high card in this game.) Assume that three other players each hold 5 other cards, but none of them is a club with a rank above 10. If the player draws a card from the deck, what is the probability of getting a club above 10? Assume the full, 52-card deck is used for this game.

a) 2/5 b) 4/13 c) 1/13 **d) 1/8** e) 3/13

The players hold 20 cards, so 32 are left in the deck. Four of the remaining cards are clubs with a rank above 10. 4/32 = 1/8. The answer is d.

16. A student draws a rectangle with a width of 7 inches and a length of 6 inches on a standard letter-size page of 8.5 inches by 11 inches. If another student, without looking, randomly puts a dot on the paper, what is the probability the dot will be within the rectangle?

1. 0.4017 **b) 0.4492** c) 0.5218 d) 0.5414 e) 0.5819

The rectangle has an area of 42 square inches. The paper has an area of 93.5 square inches. 42/93.5 = 0.4492. Answer is b.

17. A vehicle travels east at an average speed of 50 miles per hour. Two hours later, a second vehicle, starting from the same point as the first vehicle, travels north at an average speed of 45 miles per hour. After the second vehicle has been traveling for 3 hours, how far apart will the two vehicles be?

a) 156.6 miles b) 212.7 miles c) 235.4 miles **d) 284.1 miles** e) 312.3 miles

The vehicle traveling east goes 250 miles in 5 hours. The second vehicle travels 135 miles in 3 hours. 250^2 + 135^2 = 80725. The square root of 80725 is 284.1 miles. The answer is d.

18. Two angles form a linear pair. One angle has a measure of x^2 + 28x, while the other angle has a measure of 29x. What is the value of x?

1. 60 b) 45 **c) 3** d) -4 e) -60

Set up an equation of x^2 + 28x + 29x = 180. x^2 +57x - 180 - 0. This factors to (x+60)(x-3) = 0. The roots would be -60 and 3. However, -60 gives a negative angle measure. The answer is c.

19. A sound barrier is supported by a series of pillars. The first pillar is 5 feet tall, and the height of the pillars increases by 3 feet at each stage. If there are 40 pillars, what is the sum of the height of the pillars?

1. 600 feet b) 625 feet c) 2537 feet **d) 2540 feet** e) 2543 feet

A way to solve is to find the median height of the pillars, then multiply by the number of pillars. This would be the average of the 20th and 21st pillars. Pillar 20 is 5 + (19\*3) = 62. Pillar 21 is 62 + 3 = 65. (62+65) / 2 = 63.5. 63.5 \* 40 = 2540. The answer is d.

20. A regular polygon has an exterior angle measuring 18 degrees. What is the sum of the interior angle measures of the polygon?

1. 1440 b) 2160 c) 2880 d) 3040 **e) 3240**

Each interior angle is 180 - 18 = 162. Exterior angles must add up to 360 degrees, and 20 \* 18 = 360, so there are 20 angles. 162 \* 20 = 3240. Answer is e.

21. A series starts with the following numbers:

18, 27/2, 81/8, 243/32 …

What is the sum of the series?

1. 27/4 b) 36 c) 9/2 **d) 72** e) 27/2

Find the common ratio. In this problem, it is 3/4. If the absolute value of this ratio is less than 1, it converges, and a sum can be calculated. Take the first value, 18, and divide by (1 - 3/4). 18 / (1/4) = 18\*4 = 72. The answer is d.

22. Two classrooms each have 34 students. The mean height of Classroom A is 64 inches, and the standard deviation is 1.3 inches. The mean height of Classroom B is 67 inches, and the standard deviation is 2 inches. If the two classes are combined, what are the mean and the standard deviation of the height of the students?

1. mean of 65.5 inches and standard deviation of 3.3 inches
2. **mean of 65.5 inches and standard deviation of 2.4 inches**
3. mean of 65.5 inches and standard deviation of 1.7 inches
4. mean of 65 inches and standard deviation of 2.2 inches
5. mean of 65.5 inches and standard deviation of 5.7 inches

The mean is simply the average of 64 and 67, which is 65.5. To get the standard deviation, the values have to be converted to variances by squaring them. (1.3)^2 + 2^2 = 5.69. The square root of 5.69 is rounded to 2.4 inches. The answer is b.

23. A fruit stand expects to sell twice as many oranges as apples. If the stand can process no more than 160 orders during the lunch hour, and it earns a profit of 15 cents per apple and 20 cents per orange, what is the maximum profit it can earn during the lunch hour?

1. $19.00 b) $21.45 c) $25.75 d) $29.05 **e) $29.15**

The number of orders cannot be divided evenly into thirds. However, for 159 orders, the stand would sell 53 apples and 106 oranges. (53\*.15) + (106\*.20) = $29.15. The answer is e.

24. If a polyhedron has 12 faces and 16 vertices, how many edges will it have?

1. 18 b) 22 c) 24 **d) 26** e) 28

F + V = E + 2. 28 = E + 2. E = 26. The answer is d.

25. Which function yields a maximum output of y = 2 and a minimum output of y = -5?

1. y = 2 sin x - 5 b) y = 2 cos x - 5 c) y = 5 cos x - 2 d) y = 5 sin x - 2 **e) y = 3.5 sin x - 1.5**

Recognize the maximum value for either function is 1 and the minimum value is -1. With this information, answer E gives the correct minimum and maximum.

26. A right triangle has integer side measures. Its perimeter and its area are the same value. What is the smallest possible sum of the legs?

1. 22 **b) 14** c) 7 d) 4 e) 2

The smallest possible Pythagorean triple is 3-4-5. This gives a perimeter of 12 and an area of 6. If the sides are doubled to 6-8-10, both perimeter and area are 24. The sum of the legs is 14. The answer is b.

27. A square floor tile is expected to have a side measure of 5 inches. If each side has a measuring error of 1/50 inch, what is the expected error in the area of the tile?

a) 1/50 inch b) 1/25 inch c) 1/10 inch **d) 1/5 inch** e) 1/2 inch

The area formula would be s^2. The derivative is 2s ds. 2s = 10. Ds = 1/50. 10 \* 1/50 = 1/5. The answer is d.

28. If the polynomial 2x^3 - 6x^2 - 12x + 16 is factored, what are its roots?

1. -1, 3, -5. **b) -2, 1, 4**. c) 2, 1/2, 3/2. d) -3, -1, 2/3. e). 1/4, 3/4, -3/4

The answer is b. Factor out a GCF of 2. This leaves x^3 - 3x^2 - 6x + 8. From here, the best approach is to recognize that 1 would be a root, as it makes the output of the function go to 0. Then use synthetic division to get the other roots.

29. A student calculates the value of a log expression (written as y = log(b)x) as a negative number. Assuming the expression cannot return a value of zero, what must be true about the expression?

a) b < 0.

b) x < 0.

c) x = 0.

**d) 0 < x < 1**

e) x =1

The answer is d. Only x values between 0 and 1 will return a negative output for the log function.

30. For the graph of the polar equation $r$ = 4 sin 3Θ, what is the best description of the maximum and minimum values of the distance from the $x$-axis?

a) 2 positive, equal maximum values; 2 negative, equal minimum values

b) **2 positive, equal maximum values; 1 negative minimum value**

c) 2 positive, distinct maximum values; 2 negative, distinct minimum values

d) 2 positive, distinct maximum values; 1 negative minimum value

e) 2 maximum values of zero; 1 negative minimum value

The answer is b. The graph would have 3 petals, and the two maximum values would be above the x-axis, with the minimum value below the x-axis.