ACES – Academic Challenge
Physics Solutions (Sectional) – 2024

1. Correct response: e

Time = Distance/Speed. Thus, $t = \frac{d}{c} = \frac{2\times 0.3048}{2.9979\times 10^{8}} = 2.0334 \times 10^{-9}$ s.

2. Correct response: e

The Cartesian plane is divided into four quadrants, numbered from I through IV, starting with the upper right and proceeding counterclockwise. As the angle θ is measured from the positive x-axis, it becomes larger as the vector is positioned in a higher-numbered quadrant. For instance, vector a. 48 $\hat{x}$ + 36 $\hat{y}$ is located in the first quadrant, vector c. -36 $\hat{x}$ + 48 $\hat{y}$ is in the second quadrant, and vectors b. -48 $\hat{x}$ - 36 $\hat{y}$ and d. -36 $\hat{x}$ - 48 $\hat{y}$ are situated in the third quadrant. Meanwhile, vector e. 36 $\hat{x}$- 48 $\hat{y}$ is found in the fourth quadrant. Consequently, vector e. 36 $\hat{x}$ - 48 $\hat{y}$ has the largest angle θ.

3. Correct response: c

The slope of the x-t graph represents the velocity v_x of the object along the x-axis, and it is most positive and greatest at point c.

4. Correct response: b

Let +x and +y denote the positive directions. The accelerations in the x and y directions are $a_x = 0$ and $a_y = -9.8$ m/s^2, respectively. With the initial positions $x_0 = y_0 = 0$, algebraic manipulation of the equations for horizontal and vertical motion shows that x and y are related by: $y = (tan\,\theta_0)x - \frac{g}{2{v_o}^2\,cos^2\,\theta_0}x^2$

Given $\theta_0 = 50°$, $x = 20m$, and $y = 10m$, solving for v_o yields

$v_0 = \sqrt{\frac{gx^2}{2(cos^2\,\theta_0)(x\,tan\,\theta_0 - y)}}$ =18.52 m/s.

5. Correct response: c

The object maintains a consistent horizontal speed, resulting in acceleration directed toward the center of the circle. With known values for v and T, the objective is to express the centripetal acceleration, a_c, in terms of these variables.

Using the relationships $a_c = \frac{v^2}{r}$ and $v = \frac{2\pi r}{T}$, where $v = \frac{2\pi r}{T} \Longrightarrow r = \frac{vT}{2\pi}$, we substitute and simplify to obtain $a_c = \frac{v^2}{(vT/2\pi)} = \frac{2\pi v}{T}$. To sustain the constant centripetal acceleration, the period should be doubled when the speed doubles.

6. Correct response: a

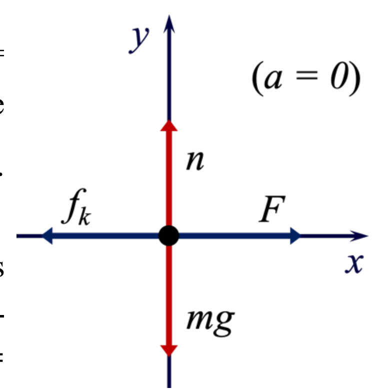
Plug in $v = \frac{2\pi r}{T}$ into the centripetal acceleration $a_c = \frac{2\pi v}{T}$. Then, $a_c = \frac{2\pi(2\pi r/T)}{T} = \frac{4\pi^2 r}{T^2}$. To maintain the constant centripetal acceleration, the period should be increased by $\sqrt{2}$ when the radius doubles.

7. Correct response: a
From $F = ma$ applied to various sets of masses we have $T_1 = (3m + 2m + m)a = 6ma$, $T_2 = 5ma$, and $T_3 = 3ma$. Therefore $T_1 > T_2 > T_3$.

8. Correct response: b
The elevator and its contents are undergoing upward acceleration, prompting the application of Newton's second law in the vertical direction. Designate $+y$ as upward, in the direction of the elevator's acceleration, and employ $\sum F_y = ma_y$. Additionally, $\sum F_y = N - W$, where N represents the scale reading, and $W = mg$ is your weight. Solving for N yields $N = W + ma_y = W\,(1 + a\,/g) = 600\ N(1 + 3/9.8) = 784$ N.

9. Correct response: d
Following Newton's third law, it's established that the car applies a force to the truck, and conversely, the truck exerts an equal force on the car. The magnitudes of these forces are consistently identical. Therefore, the force exerted by the truck on the car is 2000 N, matching the force applied by the car to the truck. Despite the truck's greater weight and bulk, it cannot exert a force on the car greater than the force the car applies to it.

10. Correct response: c
When maintaining a constant speed, the absence of acceleration (a = 0) allows us to apply Newton's first law to the box. The friction force f_k opposes the box's motion, denoted as positive in the x-direction. Let F represent the horizontal force applied.



Considering the y-axis, $\sum F_y = ma_y \Rightarrow n = mg$, where n is the normal force. This leads to $f_k = \mu_k n = \mu_k mg$. Examining the x-axis, $\sum F_x = ma_x \Rightarrow F = f_k$. Combining these results, we find $F = f_k = \mu_k n = \mu_k mg$. Thus, $F = \mu_k mg = 0.3 \times 20$ kg $\times$ 9.8 m/s^2 = 58.8 N.

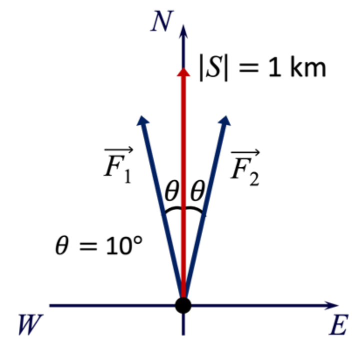
11. Correct response: b
With only the kinetic friction force acting on the box horizontally, apply Newton's second law to determine its acceleration. Once the acceleration is known, we can calculate the distance using a constant acceleration equation. The kinetic friction force is denoted as $f_k = \mu_k n = \mu_k mg$.

Analyzing the x-axis, $\sum F_x = ma_x \Longrightarrow -f_k = ma_x \Longrightarrow -\mu_k mg = ma_x$. Solving for a_x, we find $a_x = -\mu_k g = -0.3 \times 9.8$ m/s$^2 = -2.94$ m/s^2.

Then, use the constant acceleration equations to find the distance the box travels. ${v_x}^2 = {v_{0x}}^2 + 2a_x(x - x_0) \Longrightarrow \frac{{v_x}^2 - {v_{0x}}^2}{2a_x} = \frac{0-(4m/s)^2}{2(-2.94m/s^2)}$ =2.72 m

12. Correct response: d
Work is force, [M][L]/[T]^2, multiplied by displacement, [L]. Thus, work is [M][L]^2/[T]^2.

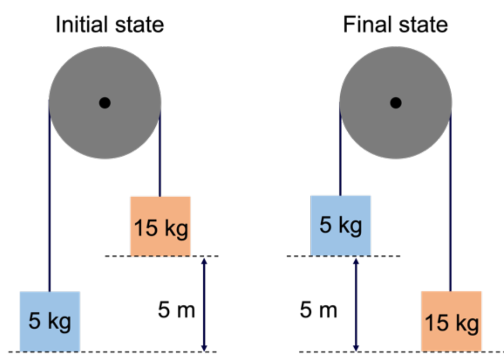
13. Correct response: d
In essence, work is determined by the equation: Work = Force $(\cos\theta) \times$ Distance. To calculate the total work, we aggregate the work carried out by two forces, $\vec{F_1}$ and $\vec{F_2}$. The work done by $\vec{F_1}$ is $W_1 = F_1 \cos 10° \times 1$ km $= (5 \times 10^5 \times 0.985)$ N $\times 10^3$ m = 4.925 $\times 10^8$ J.



Similarly, the work done by $\vec{F_2}$ is $W_2 =$ 4.925 $\times 10^8$ J. Consequently, the total work $W_{total} = W_1 + W_2 = 9.85 \times 10^8$ J.

14. Correct response: b
The magnitude of the work is determined by calculating the area beneath the graph. The area under a triangle is 0.5 × base × height. Since $F_x > 0$, the work done is positive as x increases during the displacement. The base is 15 m and the height is 20 N, so work = 0.5(15)20 = 150 J.

15. Correct response: a
Gravity is the sole contributor to work in the system, resulting in the equation $K_i + U_i = K_f + U_f$, with 'i' representing the initial state and 'f' representing the final state. Both figures illustrate the motion of both the initial and final states. The tension force contributes positive work on the 5 kg object and an equivalent amount of negative work on the 15 kg object, resulting in a net work of zero for the tension.



Given that the initial velocity is 0 ($K_i = 0$), K_f is expressed as $\frac{1}{2}m_{5kg}{v_{5kg}}^2 + \frac{1}{2}m_{15kg}{v_{15kg}}^2$, where $v_{5kg} = v_{15kg}$ since they move together with identical speed. Introducing $v = v_{5kg} = v_{15kg}$, K_f simplifies to $\frac{1}{2}(m_{5kg} + m_{15kg})v^2 = 10$ kg $\times v^2$.

The gravitational potential energy (U_i) for the 15 kg object is calculated as $m_{15kg}gh = 15$ kg × 9.8 m/s^2 × 5 m = 735 J. Similarly, U_f for the 5 kg object is $m_{5kg}gh = 5$ kg × 9.8 m/s^2 × 5 m = 245 J. Consequently, $K_i + U_i = K_f + U_f$ leads to $U_i = K_f + U_f$, which further results in 735 J = 10 kg × v^2 +245 J. Solving for v, we find $v = \sqrt{\frac{735J-245J}{10kg}}$ = 7 m/s.

16. Correct response: e
Let +x denote the right direction. Utilizing the conservation of momentum, $P_{i,x} = P_{f,x}$ (where 'i' represents the initial state and 'f' represents the final state) for the system of the two pucks, we derive the equation $P_{i,x} = P_{f,x} \Longrightarrow$ 0.4 kg × $v_{A,x}$ +0.55 kg × 0 =0.4 kg × (−0.15) m/s + 0.55 kg × 0.75 m/s. Solving for $v_{A,x}$, the result is $v_{A,x}$ = 0.88125 m/s.

17. Correct response: a
Let +x represent the right direction. Due to the absence of a net external horizontal force, the conservation of momentum in the x-component holds true: $m_A v_{A,x,i} + m_B v_{B,x,i} = m_A v_{A,x,f} + m_B v_{B,x,f}$, where 'i' denotes the initial state and 'f' denotes the final state. Additionally, as the collision is elastic and kinetic energy is conserved ($K_i = K_f$), we can utilize the relative velocity equation for an elastic collision: $v_{B,x,f} - v_{A,x,f} = -(v_{B,x,i} - v_{A,x,i})$, with 'i' and 'f' representing the initial and final states, respectively.

Applying the conservation of the x-component of momentum, we derive $m_A v_{A,x,i} + m_B v_{B,x,i} = m_A v_{A,x,f} + m_B v_{B,x,f} \Longrightarrow m_A v_{A,i} - m_B v_{B,i} = m_A v_{A,x,f} + m_B v_{B,x,f} \Longrightarrow$ 0.2 kg × 0.9 m/s – 0.3 kg × 2 m/s = 0.2 kg × $v_{A,x,f}$ + 0.3 kg × $v_{B,x,f}$ $\Longrightarrow$ −2.1 m/s = $v_{A,x,f}$ + 1.5$v_{B,x,f}$.

Using the relative velocity equation for an elastic collision, $v_{B,x,f} - v_{A,x,f} = -\left(v_{B,x,i} - v_{A,x,i}\right)$ = −(−2 m/s – 0.9 m/s) = 2.9 m/s $\Longrightarrow$ 2.9 m/s = $-v_{A,x,f} + v_{B,x,f}$.

We now have two equation and two unknowns. Combining them, we obtain 0.8 m/s = 2.5$v_{B,x,f}$. Solving for $v_{B,x,f}$, we find $v_{B,x,f}$ = 0.32 m/s. Consequently, $v_{A,x,f}$ = −2.58 m/s. In conclusion, after the collision, the 0.2 kg glider (Glider A) will move to the left at 2.58 m/s, and the 0.3 kg glider (Glider B) will move to the right at 0.32 m/s.

18. Correct response: e
With a volume of 0.125 m^3, the cube has a side length of 0.5 m. Consequently, the center of mass of this cube is positioned 0.25 m above the floor. The center of mass for the sphere is 0.3 m above the top of the cube due to its 0.6 m diameter. Therefore, the respective center of mass locations are y_{cube} = 0.25 m and y_{sphere} = 0.5 m + 0.3 m = 0.8 m.

Now, applying the center of mass formula with the origin at the floor:

$$y_{cm} = \frac{m_{cube}y_{cube}+m_{sphere}y_{sphere}}{m_{cube}+m_{sphere}} = \frac{(1kg)\times(0.25m)+(0.9kg)\times(0.8m)}{1kg+0.9kg} = 0.511 \text{ m above the floor.}$$

19. Correct response: c
By establishing connections between angular quantities and linear quantities and applying the equations for constant-angular acceleration, we can ascertain the resultant acceleration after 2

seconds. Initially, determine the instantaneous angular speed ω of the flywheel using $\omega = \omega_0 + \alpha t$, where ω_0 represents the angular speed at time 0, α is the constant angular acceleration, and t is the time. Consequently, $\omega = \omega_0 + \alpha t = 0 + (1.2\ \text{rad/s}^2) \times (2\ \text{s}) = 2.4\ \text{rad/s}$.

To find the tangential acceleration a_{tan} of a point on the rotating flywheel, apply the formula $a_{tan} = r\alpha$, where r is the distance of that point from rotation axis. This yields $a_{tan} = r\alpha = (0.5\ \text{m}) \times (1.2\ \text{rad/s}^2) = 0.6\ \text{m/s}^2$. Additionally, determine the centripetal acceleration a_c of a point on the rotating flywheel through the formula $a_c = \frac{v^2}{r} = \omega^2 r$, where v is the linear speed of that point. Hence, $a_c = (2.4\ \text{rad/s})^2 \times (0.5\ \text{m}) = 2.88\ \text{m/s}^2$. The resultant acceleration a can be computed as $a = \sqrt{{a_{tan}}^2 + {a_c}^2} = \sqrt{(0.6m/s^2)^2 + (2.88m/s^2)^2} = 2.94\ \text{m/s}^2$.

20. Correct response: d

This is a statement of the law of conservation of angular momentum.

21. Correct response: c

The torque magnitude for each force is determined by $\tau = Fl$, where F is the force magnitude and l is the lever arm magnitude. We define the clockwise direction as having a positive torque. The torque exerted by $\vec{F_1}$ is clockwise, with a magnitude calculated as $\tau_1 = F_1 \times (5m) = (10N) \times (5m) = 50\text{N·m}$ (clockwise). On the other hand, the torque exerted by $\vec{F_2}$ is counterclockwise and is calculated as $\tau_2 = -F_2 \times (l + 5m) = -(10N) \times (l + 5m) = -50\ \text{N·m} - (10N)\, l$.

The system's net torque is $\tau_{net} = \tau_1 + \tau_2 = 50\text{N·m} - 50\ \text{N·m} - (10N)\, l = -(10N)\, l$. To achieve a net torque of -7 N·m, indicating counterclockwise torque, we set $\tau_{net} = -(10N)\, l = -7$ N·m and solve for l, resulting in $l = 0.7\text{m}$.

22. Correct response: c

To determine the gauge pressure at Point 2, apply Bernoulli's equation, which is expressed as $p_1 + \rho g y_1 + \frac{1}{2}\rho {v_1}^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho {v_2}^2$. Here, p denotes pressure, ρ is fluid density, g is the acceleration due to gravity, y represents elevation, and v is the flow speed. Subscripts 1 and 2 refer to Points 1 and 2 along the flow tube.

By utilizing the continuity equation $v_1 A_1 = v_2 A_2$, we establish that $v_2 = \frac{1}{9} v_1$ due to the pipe diameter at Point 2 being three times larger than that at Point 1, resulting in the area at Point 2 being nine times larger.

Manipulating Bernoulli's equation yields $p_2 = p_1 + \frac{1}{2}\rho({v_1}^2 - {v_2}^2) + \rho g(y_1 - y_2)$. Substituting $v_2 = \frac{1}{9} v_1$ into the expression, we arrive at $p_2 = p_1 + \rho\left[\frac{1}{2} \times \frac{80}{81} {v_1}^2 + g(y_1 - y_2)\right]$. Utilizing the provided values, we obtain $p_2 = 7 \times 10^4\ \text{Pa} + (1 \times 10^3\ \text{kg/m}^3)\left(\frac{80}{162} \times (4m/s)^2 + (9.8m/s^2) \times 15m\right) = 2.249 \times 10^5\ \text{Pa}$.

Given that Point 2 is situated lower than Point 1 and the speed at Point 2 is reduced, the pressure at Point 2 is expected to be greater than that at Point 1, and the mathematical calculation supports this conclusion.

23. Correct response: c

The magnitude of total acceleration is given by $|a_{total}| = \sqrt{a_{centripetal}^2 + a_{vertical}^2}$. To determine the value of v such that the magnitude of the sphere's total acceleration is twice the acceleration due to gravity g, we set $|a_{total}| = 2\,g$. Thus, $|a_{total}| = 2\,g = \sqrt{a_{centripetal}^2 + a_{vertical}^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + g^2}$, where r is the length of the pendulum string. Square both sides, we obtain $4g^2 = \left(\frac{v^2}{r}\right)^2 + g^2$, which simplifies to $3g^2 = \frac{v^4}{r^2}$. Solving for v, we get $v = (3g^2r^2)^{1/4} = (3 \times (9.8m/s^2)^2 \times (3m)^2)^{1/4} =$ 7.136 m/s.

24. Correct response: a

The formula for the period of a simple harmonic motion is given by $T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$, where m represents the mass of the object and k is the force constant of the spring. By manipulating this equation, we can solver for k as $k = \omega^2 m = \left(\frac{2\pi}{T}\right)^2 m$. Utilizing the provided values, we find $k = \left(\frac{2\pi}{T}\right)^2 m = \left(\frac{2\pi}{2\times 3s}\right)^2 \times$ (0.4 kg) = 0.439 N/m. It's crucial to note, during calculations, that the time interval between the instances when the glider is at the equilibrium position is doubled to determine the period T.

25. Correct response: e

While examining the Doppler effect, we can determine the frequency of the sound perceived by the listener using the expression, $f_L = \left(\frac{v+v_L}{v+v_S}\right) f_S$. Here, v_L and v_S represent the velocity components for the listener and the source, respectively. v denotes the speed of sound relative to the medium, and f_S is the frequency emitted by the sound source. Let's consider the direction from your car toward the truck as the positive direction.

Since the truck, the source of the sound, is stationary ($v_S = 0$), the expression simplifies to $f_L = \left(\frac{v+v_L}{v+v_S}\right) f_S = \left(1 + \frac{v_L}{v}\right) f_S$. Solving for v_L, we get $v_L = v\left(\frac{f_L}{f_S} - 1\right) =$ 344 m/s $\times \left(\frac{470Hz}{500Hz} - 1\right) = -20.64$ m/s $= -74.304$ km/h.

The negative sign indicates that to detect a frequency of 470 Hz, your car should move away from the stationary truck at a speed of 74.304 km/h.

26. Correct response: b

For a process at constant pressure, the system's work, denoted by $W = p(V_f - V_i)$,involves the pressure p, final volume V_f, and initial volume V_i. Using the provided values, we find the work done by the system, W = (1.8 × 10^5 Pa) × (0.25 m^3 −0.1 m^3) = 2.7 × 10^4 J.

Now, applying the first law of thermodynamics to analyze the scenario, we use the equation $\Delta U = Q - W$. Here, ΔU represents the internal energy change of the thermodynamic system, Q is the added heat to the system, and W is the work done. Therefore, $\Delta U = Q - W$ =1.3 × 10^5− 2.7 × 10^4= 1.03 × 10^5 J.

27. Correct response: a

For a heat engine, the net work done by the working substance is expressed as $W = |Q_H| - |Q_C|$, where Q_H and Q_C denote the amounts of heat absorbed and rejected by the engine during one cycle. Consequently, $W = |Q_H| - |Q_C|$ = 10000 J – 7000 J = 3000 J.

The thermal efficiency of the engine, denoted by e is determined by $e = \frac{W}{Q_H}$. In this case, $e = \frac{3000J}{10000J}$ = 30 %.

28. Correct response: d

The electric flux through the cylinder's surface is given by $\Phi_E = \int \vec{E} \cdot d\vec{A} = EA\,cos\,\emptyset$, where $\vec{E}$ is the electric field, $d\vec{A}$ is the vector element of the surface area, and $\emptyset$ is the angle between $\vec{E}$ and the surface normal.

According to Gauss's law, $\Phi_E = \frac{Q_{enc}}{\epsilon_0}$, where Q_{enc} is the total enclosed charge, and ϵ_0 is the permittivity of vacuum. We determine the electric flux using the known quantities: $\Phi_E = EA\,cos\,\emptyset = \frac{Q_{enc}}{\epsilon_0} \Longrightarrow E2\pi rl = \frac{\lambda l}{\epsilon_0} = \frac{(5\times10^{-6}C/m)\times(2m)}{8.854\times10^{-12}C^2/N\cdot m}$ =1.129 × 10^6 N·m^2/C. Importantly, note that the cylinder's radius does not influence the determination of the total electric flux through the surface.

29. Correct response: e

If the voltage remains constant while the separation is halved, the capacitance of the parallel-plate vacuum capacitor doubles. This observation is confirmed by the capacitance formula $C = \frac{\epsilon_0 A}{d}$, where ϵ_0 represents the permittivity of vacuum, A is the area of each plate, d is the separation or distance between plates.

The stored energy in a parallel-plate capacitor can be expressed in two ways: $U = \frac{Q^2}{2C}$ or $U = \frac{CV^2}{2}$, where Q denotes the magnitude of charge on each plate, and V is the potential difference between the two plates. Consequently, a doubling of the capacitance results in a doubling of the stored energy to 20 J, as per the formula $U = \frac{CV^2}{2}$, while maintaining a constant V.

30. Correct response: b

The voltage from the power supply is divided according to V (or use E for emf) = IR + Q/C, so I = (V - Q/C)/R. When the capacitor has reached half of its maximum charge, Q/C = V/2, so I = (V – V/2)/R = V/2R = 15 V/2 (15 ohm) = 0.5 A.

31. Correct response: b

The focal length f of a spherical mirror is related to its radius of curvature R by the equation $f = \frac{R}{2}$. The mirror's focal length is determined as $f = \frac{30cm}{2}$ =15 cm. The process of image formation in the mirror follows the law of reflection, and it is not influenced by the medium through which light travels. As a result, the focal length remains unchanged at 15 cm even when the mirror is placed in a liquid with a refractive index of 1.5.

32. Correct response: a

In the context of the photoelectric effect, the maximum kinetic energy of ejected electrons can be expressed as $\frac{1}{2}mv_{max}{}^2 = hf - \emptyset = \frac{hc}{\lambda} - \emptyset$. Here, m represents the mass of an electron, v_{max} is the maximum speed of an electron, h is Planck's constant, f is the frequency of an incident photon, $\emptyset$ is the work function of a material, c is speed of light in vacuum, and λ is the wavelength of an incident photon.

Utilizing known values, including $\emptyset$ for silver, the equation becomes $\frac{1}{2}mv_{max}{}^2 = \frac{(6.626\times10^{-34}J\cdot s)(2.998\times10^{8}m/s)}{250\times10^{-9}m} -$ (4.3 eV)$(1.602 \times 10^{-19}J/1eV) = 7.946 \times 10^{-19}$ J $- 6.889 \times 10^{-19}$ J $= 1.057 \times 10^{-19}$ J. Now, when we solve for v_{max}, we obtain $v_{max} = \sqrt{\frac{2\times(1.057\times10^{-19}J)}{9.1094\times10^{-31}kg}} = 4.817 \times 10^{5}$ m/s.

33. Correct response: a

The expression for the energy levels of a particle confined in a box is given by $E_n = \frac{n^2h^2}{8mL^2}$, where n represents the quantum number, h is Planck's constant, m is the mass of the electron, and L is the width of the box.

For the ground level ($n = 1$), the energy $E_1 = \frac{h^2}{8mL^2}$. It is required that E_1 equals 13.6 eV. Solving for L in terms of the given quantities results in

$L = \frac{h}{\sqrt{8mE_1}} = \frac{6.626\times10^{-34}J\cdot s}{\sqrt{8\times(9.1094\times10^{-31}kg)\times(13.6eV)\times(1.602\times10^{-19}J/eV)}}$ $=1.663 \times 10^{-10}$ m.

34. Correct response: e

The phenomenon of length contraction is mathematically expressed as $l = l_0\sqrt{1 - \frac{u^2}{c^2}} = \frac{l_0}{\gamma}$, where l represents the length in second frame of reference moving parallel to the object's length. Here, l_0 denotes the proper length of the object, measured in the rest frame, u is the speed of the second frame relative to the rest frame, c is the speed of light in vacuum, and γ is the Lorentz factor that connects the two frames.

The measured length of the moving spaceship is $l = 50$ m. Solving for l_0, the proper length of the object, we find $l_0 = \frac{l}{\sqrt{1-u^2/c^2}} = \frac{50m}{\sqrt{1-(0.700c/c)^2}} = 70.01$ m. As anticipated, the moving spaceship appears shortened in the direction of motion as observed by the physicist on the planet.

35. Correct response: d

The definition of the half-life of an isotope is the amount of time that must pass for one-half of the nuclei to decay. If one-half of the nuclei decay, the activity of the sample also reduces to one-half the original activity.