ACES – Academic Challenge

Physics Solutions (State) – 2024

**1. Correct response: C**

21.45 g/cm3 (1 kg/1000 g) (100 cm/ 1 m)3 = 2.145 104 kg/m3

**2. Correct response: D**

The velocity vector can be determined as , where is a vector given by .   
This vector forms a 45° angle with each axis when its *x*- and *y*-components are of equal magnitude. Expressing as , where and . To have an angle of 45, , thus .

**3. Correct response: C**

We can determine the acceleration vector as The velocity vector is found as and . At s,

The net force can be determined as .

**4. Correct response: D**

When the object returns to the ground, . Thus, from , we have . Solving for , which is the time in the air, we obtain . Additionally, we find .

The total work done is , with 'i' representing the initial state and 'f' representing the final state. Here, , thus .

**5. Correct response: A**

As demonstrated in the previous problem, . If is increased four times, also increases accordingly. Consequently, the total work will be sixteen times greater than in the previous problem. Thus, .

**6. Correct response: B**

Assuming the final height () is zero and designating the initial height as , where denotes the total spring stretch, we find . The work () done by the force over an elongation of is , with representing the spring’s force constant.

Applying the work-energy theorem yields . Solving for results in . Substituting the provided values gives 0.049 m = 4.9 cm.

**7. Correct response: A**

To tackle this problem, we start by determining the velocity of the combined object after the collision. Let’s designate the direction in which the shark is moving as the positive direction. Utilizing the conservation of momentum, we express leading to . Solving for yields, m/s

As the two objects combine, leading to an inelastic collision, the overall kinetic energy should decrease. Let denote the initial kinetic energy and the final kinetic energy. The initial kinetic energy is 625 J. The final kinetic energy is520.917 J. Therefore, the total energy dissipated during this event is  J.

**8. Correct response: C**

The angular velocity of a rigid body rotating around the *z*-axis at time is expressed as , where represents the angular velocity of the body at time 0, and denotes the constant angular acceleration of the body. Solving for gives us the equation Utilizing the provided values from the problem, we can calculate as follows: s.

**9. Correct response: C**

The rotational motion about the center of mass is described by the rotational analog of Newton’s second law. Thus, the net torque on a rigid body about *z*-axis through center of mass is expressed as where is the moment of inertia of rigid body about *z*-axis and is angular acceleration of rigid body about *z*-axis. In this problem, , where is the moment of the inertia of the person and is the moment of inertia of the empty turntable. Solving for gives .

**10. Correct response: A**

A more in-depth analysis using conservation of momentum of the combined system of rocket & fuel would show that this result is not just dimensionally consistent, but is in fact the correct solution.

**11. Correct response: B**

The speed of the mechanical wave is influenced by both the tension in the wire and its mass per unit length (), represented by the equation , where *F* is the tension in the wire. Neglecting the weight of the wire, *F* = *Mg* with *M* = 35 kg. Solving for, we find .

To find , we use the formula m/s. Substituting the provided values, we calculate as kg/m. Finally, the mass () of the wire is determined by ( kg/m)(2 m)kg.

A more complex calculation, taking into account the fact that the tension at any point in the wire is equal to the combined weight of the 35 kg mass and the portion of the wire below that point, would give the more accurate result m = 0.2755 kg.

**12. Correct response: E**

The sound intensity () is given by the formula The change in sound intensity at two distinct points, labeled 1 and 2, is expressed as . The expected value for is 15 dB. Consequently, 15 dB, leading to 31.62.

**13. Correct response: A**

For an infinitely long straight wire, Gauss’s law gives the electric field strength as , where represents the line charge density, is the permittivity of vacuum, and is the distance from the wire. Solving for gives . Substituting the given values, we find 1.1235 m

**14. Correct response: E**

When the wire undergoes a change in geometry, its resistance changes accordingly. The resistance () is articulated as with representing the resistivity of the wire material, denoting the length of the wire, and signifying the cross-sectional area.

In this particular scenario, the wire’s length is stretched fourfold, making the length of the stretched wire ( equal to . Despite the stretching, the volume of the wire remains constant, given that the amount of material does not alter. The volume () is defined as , and alternatively expressed as , where is the reduced cross-sectional area due to stretching.

This relationship leads to . With this information, the resistance of the stretched wire () can be determined: .

**15. Correct response: E**

Start by using the given critical angle information to find the index of refraction of the liquid. Apply Snell’s law, , where ‘1’ represents vacuum and ‘2’ represents the liquid. This simplifies to , given that the index of vacuum () is 1. To meet the condition for total internal reflection, . Solving for , yielding the index of refraction of the liquid as 1.346.

Next, apply Snell’s law again to find the angle of refraction in vacuum when the incident angle from the liquid is 30 . Solve for , resulting in 42.299.

**16. Correct response: B**

The phenomenon of thin-film interference arises from the interaction between light reflected from both surfaces of a thin coating. To optimize light transmission through the rod, those two reflected waves should interfere destructively.

An extra half-cycle phase shift occurs when light reflects from the vacuum/coating interface (*i.e*., from a lower index material to a higher index material), whereas no such shift occurs at the coating/rod interface (*i.e*., from a higher index material to a lower index material). Therefore, destructive interference occurs if the path length difference between the two waves (2*t*, where *t* is the thickness of the dielectric coating) is equal to an integer multiple of the wavelength within the coating, where is the wavelength in vacuum and is the index of refraction in the coating, or 2*t* = *m*(.

Solving 2 for with = 1 yields 172.58 nm, representing the minimum thickness required for the thin coating. Note that = 1 for the minimum thickness.

**17. Correct response: B**

Stefan’s law addresses the total power of blackbody radiation emitted across the spectrum of wavelengths at a given temperature. The total power corresponds to the area under the blackbody radiation curve for a specific temperature, and as the temperature of a blackbody rises, so does the total emitted power. In a quantitative form, Stefan’s law is expressed as , where represents the Stefan-Boltzmann constant, is the surface area, and is the temperature in Kelvin.

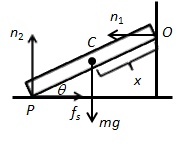
In the context of this problem, ​ denotes the emitted power at 300 °C, while ​ denotes the emitted power at 0 °C. To apply Stefan’s law, it is necessary to convert the temperature unit from Celsius degrees to Kelvin, leading to the following expression: 19.39 W. Note that the conversion from Celsius to Kelvin is given by Kelvin = Celsius + 273.15.

**18. Correct response: D**

A lepton, characterized by its half-integer spin, is an elementary particle that does not experience strong interactions. An example of a lepton is an electron.

**19. Correct Response: D**

Draw a free-body diagram of the object, as shown below.



Assume that the center of mass is a distance *x* from point *O*.

Apply condition for static equilibrium, net force = 0, in the vertical direction.

*n*2 = *mg*. Therefore, the normal force exerted by the floor on the object does not change when the center of mass changes.

Choose point *O* as the point of rotation. Apply condition for static equilibrium, net torque = 0, about point *O*.

*n*2*L*cos *θ* = *mgx*cos *θ* + *f*s *L*sin *θ* 🡪 *mgL*cos *θ* = *mgx*cos *θ* + *f*s*L*sin *θ*

* *mg*cos *θ*(*L* – *x*) = *f*s*L*sin *θ* 🡪 tan *θ* = *mg*(*L* – *x*)/*f*s*L*.

The maximum value for the static frictional force is *f*s, max = *μ*s*n*2 = *μ*s*mg*. So, for a given *x*,

tan *θ*min = *mg*(*L* – *x*)/*μ*s*mgL* = (*L* – *x*)/*μ*s*L*. When *C* shifts downward to *C*’, *x* increases, and thus *θ*min decreases.

**20. Correct Response: C**

**21. Correct Response: B**

When things are in orbit it can be said that they are falling into the Earth but their speed means that they also continuously miss the Earth. Thus, they are in a state of free fall toward the Earth.

**22. Correct Response: C**

We are told:

The conversion is

We can plug the first into the second to get:

This can be solved:

**23. Correct Response: D**

The relativistic energy is related to rest energy and momentum:

We are given that E is 3.33x and that

Solving for momentum

**24. Correct Response: D**

A black and white symbol

Description automatically generated

The first thing that we can notice is that the symmetry of the system means that the center of mass is somewhere on the y axis. So we start with the center of mass in the y direction. The center of mass in that direction is given by:

This gives us

where we have used the definition of the linear density of the hoop. This means that the position of the center of mass is (0, 2R/).

**25. Correct Response: E**

The work done is P dV and we can integrate that between the initial and final volumes.

This is an integral that is easily written:

**26. Correct Response: C**

The force of static friction is given by:

The force of static friction is in the vertical direction and must overcome the force of gravity which is in the negative vertical direction.

We solve this for the acceleration

**27. Correct Response: A**

The force diagram for this problem is

A black triangle with arrows and letters

Description automatically generated

The x direction is parallel to the incline and the y direction is perpendicular to the incline surface. The length of the incline surface is L.

The x component of gravity is

Where Fg = mg. The magnitude of the normal force on the block is:

The force of kinetic friction is:

From the force diagram we can see that Newton’s second law gives:

Substituting in the previous results we get

So that we have the acceleration:

Plugging in the acceleration and solving for t gives us:

**28. Correct Response: B**

This is a projectile motion problem. To answer the question we determine the y position when the x position is at the fence. The equations of motion are:

Where we realize that xo = 0 and the y component of the initial velocity is zero, the acceleration in the x direction is also zero. We also are given that the initial y position is 11.7 m and the final x position is 9.33 m.

The y position at that time is:

The fence is 2.0 m high so this ball will hit the front of the fence before it strikes the ground.

**29. Correct Response: B**

This time we do have an x and a y component to the velocity. So we must write them out:

The equations of motion for this system are

Plugging the x velocity component into the x equation of motion and solving for t gives us

Then plugging what we have into the y equation of motion gives us

We know the initial and final x and y position, the angle, and, of course, g. So we can solve this equation for vo to get:

So

**30. Correct Response: E**

In general a potential energy function must be related to the force in two dimensions

If we take the derivative with respect to x and then with respect to y we should get the same result no matter what order this is done in. So we can write

Putting these together would result in

We can now use this as the test for whether a given force has a potential energy function:

And

And we see that these two are not equal so that means that there is no potential energy function for this force and the force is not conservative.

**31. Correct Response: E**

This is an application of the parallel axis theorem for moment of inertia:

For a sphere the moment of inertia through the center of mass is stated in the problem.

We are given all of the data we need to complete this calculation. The radius of Mimas, the mass of Mimas, and the orbital radius of Mimas (D in the equation above):

**32. Correct Response: D**

The angular momentum is given by:

Where we have used the orbital radius because the velocity is the circumference of the orbit divided by the period of the orbit and that gets multiplied by the distance from the axis of rotation to the object.

**33. Correct Response: B**

Radioactive decay follows the exponential function

The decay constant is related to the half-life according to

Dividing the first equation by No and realizing that the ratio of the Ns is also the percentage of carbon 14 in the tissue.

We can solve this equation for t,

**34. Correct Response: B**

By Newton’s third law there is an action-reaction force that must be equal for both items. However, the mass of the tennis ball is much less than the racquet so that the acceleration of the ball is greater than the acceleration of the racquet.

**35. Correct Response: E**

The position is the total velocity vector times the time. With x to the east and y to the north, we add the boat velocity relative to the water to the water current velocity vector to get the total vector. The position would then be the time times each of those vectors.

Of course, these components can be tied back to the initial vectors of the velocity.

In this set of equations we have two equations and two unknowns so it should be possible to solve it. Let us rearrange them first

Dividing the 2nd equation by the 1st gives

˚ West of North

Inserting = 51.0785˚ into either equation then gives