



2015 Academic Challenge

MATHEMATICS TEST - STATE

This Test Consists of 40 Questions

Mathematics Test Production Team

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. Only one oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

***** TIME: 40 MINUTES *****

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

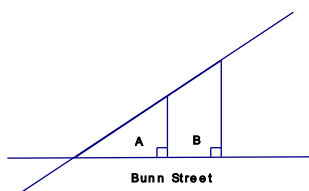
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WYSE Academic Challenge
Mathematics Test (State) – 2015

1. $\sinh x = \frac{e^x - e^{-x}}{2}$, $\cosh x = \frac{e^x + e^{-x}}{2}$ and $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$. So what is $\frac{d}{dx} \cosh x$?
 a) $\sinh x$ b) $-\sinh x$ c) $\cosh x$ d) $-\cosh x$ e) $\tanh x$
2. Find the a value for the parabola in standard form ($y = ax^2 + bx + c$) with vertex at $(35, 1225)$ and passes through $(70, 0)$.
 a) -1 b) 1 c) -35 d) 35 e) -70
3. A spherical watermelon grows three inches in length per month. If one starts to grow at the beginning of April, at what rate is its surface area increasing at the beginning of September? Round the answer to the nearest whole number of square inches per month.
 a) 226 b) 905 c) 942 d) 1131 e) 2827
4. In a business district downtown, there are 2 lots (A and B) for sale (see diagram below). A has the frontage on Bunn Street of 140 feet. B has the frontage of 84 feet. The boundary between lots is 120 feet. Find the square area of lot B.



- a) $8,400 \text{ ft}^2$ b) $13,104 \text{ ft}^2$ c) $13,440 \text{ ft}^2$ d) $16,128 \text{ ft}^2$ e) $21,504 \text{ ft}^2$
5. What is the area of the region between $x^2 - x - 12$, the x-axis, and the lines $x = -4$ and $x = 4$?
 a) $-\frac{160}{3}$ b) 61 c) $\frac{160}{3}$ d) $\frac{494}{3}$ e) $\frac{3869}{3}$
6. Which of the following is $\log_{xy} x^6 y^6$?
 a) 0 b) 6 c) 12 d) 36 e) Undefined
7. Bob has a coin collection worth \$8.00. In this collection, there are twice as many dimes as quarters, and five less nickels than dimes. How many nickels are in the collection?
 a) 10 nickels b) 15 nickels c) 25 nickels d) 30 nickels e) 35 nickels
8. Jack travels for 4 hours at 55 mph in a direction that is 63 degrees east of south and then turns down a road that is 32 degrees south of east on which he travels for 2 hours at the same speed. How far is he from his original starting point? Round to the nearest mile.

- a) 110mi b) 185 mi c) 319 mi d) 330 mi e) 405 mi
9. Rewrite the integration $\int_0^3 \sqrt{x^7 + 9x^4} dx$ by substitution in terms of u .
- a) $\int_0^3 \sqrt{u^3 + 9u^4} du$ b) $\frac{1}{3} \int_0^{81} \sqrt{u^3 + 9u^4} du$ c) $\int_9^{36} \sqrt{u} du$
d) $\frac{1}{3} \int_9^{36} \sqrt{u} du$ e) $\frac{2}{3} \int_0^9 \sqrt{u^3 + 9u} du$
10. Find the value of x that satisfies the given equation: $y + 2 + 7i = xi - 2yi$.
- a) -3 b) -2 c) 1 d) 2 e) 3
11. In which quadrant(s) is $\csc^2 x - \cot^2 x$ negative?
- a) I b) II c) III d) I and III e) None
12. Which graph below is not a conic section?
- a) cardioids b) circle c) ellipse
d) hyperbola e) parabola
13. Which of the following are equivalent?
- I. $\frac{\ln 8}{\ln 7}$ II. $\log_7 6 \bullet \log_6 8$ III. $\log_7 8$
- a) I, II b) I, III c) II, III d) All three e) None
14. Solve the given equation for x : $3\log_2 3 - \log_2 x = \log_2 45$
- a) $x = .20$ b) $x = .60$ c) $x = 20$ d) $x = 60$ e) $x = 80$
15. How much money must be invested at 6% compounded daily for 26 years in order to end up with \$50,000? Round your answer to the nearest dollar.
- a) \$10,508 b) \$10,805 c) \$11,215 d) \$11,352 e) \$12,008
16. A solid cylinder shaped pole with diameter of 4 inches and a height of 8 feet is to be painted with 1 coat of glossy red paint. If a can of paint can cover 350 square inch, how many cans of paint will be needed to completely cover the pole (this includes the ends)?
- a) 2 b) 3 c) 4 d) 5 e) 6
17. Which of the following is $2i$ raised to the 15th power?
- a) 32,768 b) -32,768 c) -16,384i d) -32,768i e) 16,384

18. When adding a matrix to a $m \times n$ matrix, the matrix to be added must be _____.
 a) a square matrix b) an invertible matrix c) an $n \times m$ matrix
 d) a $m \times n$ matrix e) a zero matrix
19. Given the equation $x^2 + 6x + y^2 - 12y = -45$, which one of the following is true about its graph?
 a) Its graph is nonexistent.
 b) Its graph is the single point (6, -12).
 c) Its graph is the single point (-3, 6).
 d) Its graph is a circle with radius $3\sqrt{5}$.
 e) Its graph is a circle with a radius that is a positive integer.
20. Choose the statement below that best describes the graph of the parametric equation $x = \cos t$; $y = \sin t$ where $0 \leq t \leq 2\pi$.
 a) self-intersecting
 b) simple but not closed
 c) neither closed nor simple
 d) closed but not simple
 e) both simple and closed
21. Which of the following is the directrix for $y = x^2 + 2x - 3$?
 a) The point (-3, 0)
 b) The line $x = -1$
 c) The point (-1, -4)
 d) The point $\left(-\frac{3}{4}, -\frac{63}{16}\right)$
 e) The line $y = -\frac{17}{4}$
22. Write the polar equation of the line that has a slope of -1 and passes through the point $(-1, 2)$.
 a) $r = \frac{1}{\sin \theta + \cos \theta}$ b) $r = \frac{1}{\sin \theta}$ c) $r = \frac{1}{\cos \theta}$
 d) $r = \frac{\sin \theta}{\cos \theta}$ e) $r = \frac{1}{\cos^2 \theta}$
23. Which of the following is the area of the largest square peg that can fit in a round hole with area π square units?
 a) 1 square unit b) 2 square units c) 3 square units

- d) 4 square units e) 5 square units

24. Using the phase shift value as the x coordinate of the point (x,y) on the graph of $y = -2\cos\left(\pi x + \frac{2\pi}{3}\right) + 3$, find the x coordinate value for the next corresponding point for which the value of y repeats.
- a) $x = 2$ b) $x = \frac{4}{3}$ c) $x = \frac{3}{4}$ d) $x = \frac{-2}{3}$ e) $x = \frac{-3}{2}$
25. The graph of function g is obtained by reflecting the graph of the function $f(x) = x^2 + 2$ across the line $y = 1$. What is the value of $g(-2)$?
- a) -6 b) -5 c) -4 d) -2 e) 0
26. If $x = 3 + 2t$ and $y = t^2 + 5t - 9$, what is y as a function of x?
- a) $y = \frac{1}{4}x^2 + 5x - \frac{57}{4}$ b) $y = \frac{1}{4}x^2 + 5x + \frac{15}{4}$ c) $y = \frac{1}{4}x^2 + x - \frac{57}{4}$
- d) $y = \frac{1}{4}x^2 + x + \frac{15}{4}$ e) $y = \frac{1}{9}x^2 + 3x - \frac{9}{4}$
27. On a 10 question multiple choice test where the answers are randomly selected from a list of five, what is the probability of getting 2 As, 2 Bs, 2 Cs, 2 Ds and 2 Es? Round to the nearest percent.
- a) 0% b) 1% c) 2% d) 3% e) 4%
28. If $A = \{2, 4, 6, 8, 10\}$ and $B = \{4, 8, 12\}$, What is the cardinality of $A \cup B$?
- a) 16 b) 15 c) 8 d) 6 e) 3
29. A student has an IQ of 190. If IQ scores are normally distributed with a mean of 100 and a standard deviation of 15, what is the probability that another randomly selected student has a higher IQ? Round to the nearest percent.
- a) 0% b) 1% c) 2% d) 3% e) 4%
30. Jack, Will, and Barbossa must draw one card from 6 cards in turn, with replacement until one of them draws the card with the black spot. If Jack is first and Barbossa is last, what is the probability that Barbossa draws the black spot?

- a) $\frac{35}{99}$ b) $\frac{1}{3}$ c) $\frac{25}{91}$ d) $\frac{5}{18}$ e) $\frac{1}{4}$

31. Which of the following computations cannot be guaranteed to be done with two square matrices of the same size?
- Sum of transposes
 - Product of transposes
 - Difference of inverses
 - Inverse of product of transposes
 - Both c and d
32. Jon is among the 8 finalist to be selected for a part in the play. Three people will be selected from the finalist. Each will play a unique part in the play. Out of the different possible selections, how many contain Jon?
- 21
 - 35
 - 56
 - 126
 - 210
33. What is the identity for $\sin x \tan x + \cos x$?
- $-\cot x$
 - $\cot x$
 - $\tan x$
 - $\csc x$
 - $\sec x$
34. Which of the following is the end behavior for $f(x) = -x^3 + x^4 - x^5 + 7$?
- As $x \rightarrow \infty$, $y \rightarrow \infty$. As $x \rightarrow -\infty$, $y \rightarrow \infty$.
 - As $x \rightarrow \infty$, $y \rightarrow -\infty$. As $x \rightarrow -\infty$, $y \rightarrow -\infty$.
 - As $x \rightarrow \infty$, $y \rightarrow -\infty$. As $x \rightarrow -\infty$, $y \rightarrow \infty$.
 - As $x \rightarrow \infty$, $y \rightarrow \infty$. As $x \rightarrow -\infty$, $y \rightarrow -\infty$.
 - None of these
35. Which of the following is the type of discontinuity shown on the graph of $f(x) = \frac{x^2 - 25}{x + 6}$?
- A vertical asymptote
 - A jump
 - A hole
 - A removed discontinuity
 - It is continuous on all of the real numbers
36. A triangular parcel of land has two sides, one that is 45 feet long, the other side is 67 feet long with the included angle of 35° . What is the area of this parcel of land? Round to one decimal place.
- 865.2 ft^2
 - 854.4 ft^2
 - 764.9 ft^2
 - 749.0 ft^2
 - 664.4 ft^2

37. Find a polynomial function with minimal degree, leading coefficient of 1, all real coefficients and a root of $-3 + 4i$.
- a) $x^2 + 6x + 9$ b) $x^2 - 6x + 25$ c) $x^2 + 25x + 625$
- d) $x^2 + 6x + 25$ e) We cannot have such a function.
38. Two pirates pull horizontally on ropes attached to a treasure chest sunk in the mud. One pirate pulls with a force of 500N directly to the right, and the other pirate pulls with a force of 350N at 40° from the first force. What is the resultant force on the treasure? Round to the nearest whole number.
- a) 850N b) 800N c) 768N d) 630N e) 610N
39. Two giraffes denude one acacia tree in four hours. How many giraffes would it take to denude a forest of 1000 acacia trees in five hours? For obvious reasons, round up to the nearest whole giraffe. (Is half of a giraffe a large horse?)
- a) 625 b) 800 c) 1,000 d) 1,600 e) 3,200
40. When George Clooney lived in Cincinnati, there were 4 main Cincinnati establishments (Frisch's, Grippos, LaRosa's, and Skyline) that he visited. Each establishment was known for a specialty food item (the Big Boy, chili, pizza or potato chips). At each establishment there is a special food item (cheese cake, hot fudge cake, pretzels or Ziti Alfredo) that George ordered each time he visited one of these establishments. Using the clues below, where did George order his cheese cake?
- i) The establishment known for pizza is not Frisch's nor is it where George buys cheese cake. George buys his pretzels from Grippos. Grippos never serves chili.
- ii) Grippos does not serve pizza or Big Boys. Hot fudge cakes are not sold at neither Grippos nor LaRosa's. George always orders Chili at the establishment that he buys cheese cake.
- iii) George orders Ziti Alfredo from LaRosa's which is not known for the Big Boy or chili. The establishment that sells the great hot fudge cake is not known for pizza nor is it called Skyline.
- a) Frisch's & LaRosa's b) Frisch's c) Skyline
- d) LaRosa's e) Grippos