

WYSE – Academic Challenge
Physics Test Solutions (Regional) – 2017

1. **Correct Response: E**

$$F = -kx \Rightarrow k = \frac{-F}{x} \Rightarrow [k] = \frac{[F]}{[x]} = \frac{M \cdot L/T^2}{L} = \frac{M}{T^2},$$

where M is mass, L is length, and T is time.

2. **Correct Response: D**

$$\Delta K + \Delta U = 0 \Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + mgh - mgh_0 = 0$$

$$\Rightarrow h_0 = \frac{\frac{1}{2}v^2 - \frac{1}{2}v_0^2 + gh}{g} = \frac{\frac{1}{2}(20.0 \text{ m/s})^2 - 0 + 0}{(9.80 \text{ m/s}^2)} = 20.4 \text{ m}$$

3. **Correct Response: D**

$$6.03 \frac{\text{furlongs}}{\text{fortnight}} \times \frac{201 \text{ m}}{1 \text{ furlong}} \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{1 \text{ fortnight}}{14 \text{ days}} \times \frac{1 \text{ day}}{24 \text{ hrs}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}} = 0.100 \frac{\text{cm}}{\text{s}}$$

4. **Correct Response: D**

$$v^2 = v_0^2 + 2a(y - y_0) \Rightarrow$$

$$v = \sqrt{v_0^2 + 2a(y - y_0)} = \sqrt{(4.00 \text{ m})^2 + 2 \times 9.8 \frac{\text{m}}{\text{s}^2} \times 0.82 \text{ m}} = 5.66 \text{ m/s}$$

5. **Correct Response: E**

$$\mathbf{A} + \mathbf{B} = (A_x + B_x)\hat{\mathbf{x}} + (A_y + B_y)\hat{\mathbf{y}} = (2.00 + 5.00 \cos 110^\circ)\hat{\mathbf{x}} + (4.00 + 5.00 \sin 110^\circ)\hat{\mathbf{y}}$$

$$= 0.290\hat{\mathbf{x}} + 8.70\hat{\mathbf{y}}$$

6. **Correct Response: A**

$$m = \rho_1 V_1 = \rho_2 V_2 \Rightarrow V_2 = \frac{\rho_1 V_1}{\rho_2} = \frac{(804 \text{ kg/m}^3)(33.3 \text{ ml}) \left(1 \frac{\text{cm}^3}{\text{ml}}\right) \left(\frac{100 \text{ cm}}{\text{m}}\right)^3}{(1.21 \text{ kg/m}^3)} = 0.0221 \text{ m}^3$$

7. **Correct Response: B**

As the wall is frictionless, it can only exert a force normal to the wall. To be in equilibrium, the sum of the torques about any point must be zero. Considering the torques about the point of the board in contact with the floor,

$$\sum \tau = 0 \Rightarrow \tau_{\text{Wall}} + \tau_{\text{Weight of Board}} + \tau_{\text{Weight of Crate}} = 0$$

$$\Rightarrow -r_W F_W \sin \theta_W + r_{WB} F_{WB} \sin \theta_{WB} + r_{WC} F_{WC} \sin \theta_{WC} = 0$$

$$\Rightarrow F_W = \frac{r_{WB} m_B g \sin \theta_{WB} + r_{WC} m_C g \sin \theta_{WC}}{r_W \sin \theta_W}$$

$$= \frac{(1.50 \text{ m})(5.00 \text{ kg})(9.80 \text{ m/s}^2)(\sin 65.0^\circ) + (1.00 \text{ m})(10.0 \text{ kg})(9.80 \text{ m/s}^2)(\sin 65.0^\circ)}{(3.00 \text{ m})(\sin 155^\circ)}$$

$$= 123 \text{ N}$$

8. **Correct Response: C**

To be in equilibrium, the sum of the external forces on the system must be zero. Considering the vertical component of the forces (upward positive y direction),

$$\sum F_{iy} = F_{Wall\ y} + F_{Weight\ of\ Board\ y} + F_{Weight\ of\ Crate\ y} + F_{Floor\ y} = 0$$

$$\Rightarrow 0 - m_B g - m_C g + F_{N\ Floor} = 0$$

$$\Rightarrow F_{N\ Floor} = m_B g + m_C g = (5.00 \text{ kg})(9.80 \text{ m/s}^2) + (10.0 \text{ kg})(9.80 \text{ m/s}^2) = 147 \text{ N}$$

Considering the horizontal component of the forces (toward the right positive y direction),

$$\sum F_{ix} = F_{Wall\ x} + F_{Weight\ of\ Board\ x} + F_{Weight\ of\ Crate\ x} + F_{Floor\ x} = 0$$

$$F_{Floor\ x} = -F_{Wall\ x} - F_{Weight\ of\ Board\ x} - F_{Weight\ of\ Crate\ x} \Rightarrow -F_{fr\ Floor} = -F_{Wall} + 0 + 0$$

$$\Rightarrow F_{friction\ Floor} = 123 \text{ N}$$

$$F_{fr\ Floor} \leq \mu_s F_{N\ Floor} \Rightarrow \mu_s \geq \frac{F_{fr\ Floor}}{F_{N\ Floor}} = \frac{123 \text{ N}}{147 \text{ N}} = 0.834$$

9. **Correct Response: E**

$$\Delta K + \Delta U = 0 \Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + mgh - mgh_0 = 0$$

$$\Rightarrow v = \sqrt{-\frac{mgh - mgh_0 + \frac{1}{2}mv_0^2}{\frac{1}{2}m}} = \sqrt{-\frac{0 - g(l - l \cos \theta) + 0}{\frac{1}{2}}} = \sqrt{2g(l - l \cos \theta)}$$

10. **Correct Response: C**

The system will be in equilibrium where the change of the potential energy of the system with a minimal change in position is zero. These locations are the locations at which the tangent line to the curve on the graph is zero, points A, C, and E. An equilibrium position will be a stable equilibrium if the potential increases in both directions on either side of the equilibrium position, points A and E. An equilibrium position will be an unstable equilibrium if the potential decreases on either side of the equilibrium position, point C.

11. **Correct Response: B**

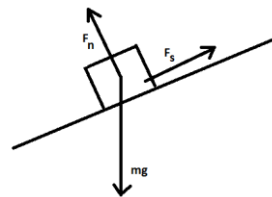
$$v_f = v_o + at = 22.0 - 0.85 \times 10.0 = 13.5 \text{ m/s}$$

12. **Correct Response: D**

$$v^2 = v_o^2 + 2a(y - y_o) \Rightarrow y - y_o = \frac{v^2 - v_o^2}{2a} = \frac{(0.00 \text{ m/s})^2 - (22.0 \text{ m/s})^2}{2(0.85 \text{ m/s}^2)} = 285 \text{ m}$$

13. **Correct Response: B**

Solve for the force that creates impending motion (the force with maximum static friction force).



$$\sum \vec{F}_i = 0 \Rightarrow \vec{F}_{incline} + \vec{F}_{gravity} + \vec{F}_{applied} = 0$$

In the direction perpendicular to the inclined plane (positive direction upward from incline),

$$F_{N\ incline} - mg \cos 10^\circ = 0$$

$$\Rightarrow F_{N\ incline} = mg \cos 10^\circ = (8.00\ \text{kg})(9.8\ \text{m/s}^2) \cos 10^\circ = 77.2\ \text{N}$$

In the direction parallel to the plane (positive direction down the incline),

$$-F_{fr\ incline\ max} + mg \sin 10^\circ + F_{applied} = 0$$

$$\Rightarrow F_{applied} = F_{fr\ incline\ max} - mg \sin 10^\circ = \mu_s F_{N\ incline} - mg \sin 10^\circ$$

$$= (0.400)(77.2\ \text{N}) - (8.00\ \text{kg})(9.8\ \text{m/s}^2) \sin 10^\circ = 17.3\ \text{N}$$

14. **Correct Response: D**

As the 50.0 N applied force is parallel to the plane, the forces in the direction perpendicular to the plane are unchanged from problem 13. In the direction parallel to the plane,

$$\sum F_i = ma \Rightarrow a = \frac{\sum F_i}{m} = \frac{F_{fr\ incline} - mg \sin 10^\circ + F_{applied}}{m}$$

$$\Rightarrow a = \frac{\mu_k F_{N\ incline} - mg \sin 10^\circ + F_{applied}}{m}$$

$$= \frac{-(0.300)(77.2\ \text{N}) + (8.00\ \text{kg})(9.8\ \text{m/s}^2) \sin 10^\circ + 50.0\ \text{N}}{(8.00\ \text{kg})} = 5.06\ \text{m/s}^2$$

15. **Correct Response: B**

$$P = Fv = mg\mu_k v = (650\ \text{kg})(9.8\ \text{m/s}^2)(0.63)(4.25\ \text{m/s}) = 17055\ \text{W} \times \frac{1\ \text{hp}}{746\ \text{W}} = 22.9\ \text{hp}$$

16. **Correct Response: C**

$$T_{WT} = aT_C + b \Rightarrow 15^\circ\text{WT} = a(0^\circ\text{C}) + b \quad \text{and} \quad 245^\circ\text{WT} = a(100^\circ\text{C}) + b$$

$$\Rightarrow 245^\circ\text{WT} - 15^\circ\text{WT} = [a(100^\circ\text{C}) + b] - a(0^\circ\text{C}) + b$$

$$\Rightarrow a = \frac{245^\circ\text{WT} - 15^\circ\text{WT}}{100^\circ\text{C} - 0^\circ\text{C}} = 2.30 \frac{^\circ\text{WT}}{^\circ\text{C}}$$

$$\Rightarrow b = 15^\circ\text{WT}$$

At liquid nitrogen temperature,

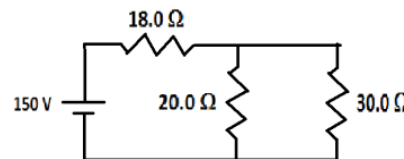
$$T_{WT} = 2.45 \frac{^{\circ}\text{WT}}{^{\circ}\text{C}} (-195.8^{\circ}\text{C}) + 15^{\circ}\text{WT} = -435^{\circ}\text{WT}.$$

17. **Correct Response: A**

$$\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I} = \frac{rF \sin \phi}{\frac{1}{3}mL^2} = \frac{\frac{1}{2}Lmg \cos \theta}{\frac{1}{3}mL^2} = \frac{3g}{2L} \cos \theta$$

18. **Correct Response: D**

The equivalent resistance of the 20.0 Ω and 30.0 Ω resistors in parallel is



$$R_{eq1} = \left(\frac{1}{R_{eq}} \right)^{-1} = \left(\frac{1}{R_{20.0\Omega}} + \frac{1}{R_{30.0\Omega}} \right)^{-1} = \left(\frac{1}{20.0\Omega} + \frac{1}{30.0\Omega} \right)^{-1} = 12.0\Omega.$$

This equivalent resistance is in series with the 18.0 Ω . The equivalent resistance of these two resistors in series is

$$R_{eq2} = R_{18.0\Omega} + R_{12.0\Omega} = 18.0\Omega + 12.0\Omega = 30.0\Omega.$$

$$P = \frac{V^2}{R} = \frac{(150.\text{V})^2}{30.0\Omega} = 750.\text{W}$$

19. **Correct Response: C**

The stretch of the spring, x , is the difference between the radius of the circular path and the unstretched spring length, l : $x = r - l \Rightarrow r = l + x$.

$$F = ma \Rightarrow kx = m \frac{v^2}{r} \Rightarrow k(r - l) = \frac{mv^2}{r} \Rightarrow kr^2 - klr - mv^2 = 0$$

$$r = \frac{kl \pm \sqrt{k^2l^2 + 4kmv^2}}{2k} = \frac{1}{2} \left\{ l \pm \sqrt{l^2 + \frac{4mv^2}{k}} \right\}$$

$$= \frac{1}{2} \left\{ (0.211\text{ m}) \pm \sqrt{(0.211\text{ m})^2 + \frac{4(0.211\text{ kg})(0.785\text{ m/s})^2}{1.65\text{ N/m}}} \right\} = 0.405\text{ m or } -0.195\text{ m}$$

The radius must be positive, so $r = 0.405\text{ m} = 40.5\text{ cm}$

20. **Correct Response: A**

$$V = \sum_i \frac{kq_i}{r_i} = \left(8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \left\{ \frac{-2 \times 10^{-6}\text{ C}}{0.10\text{ m}} + \frac{-2 \times 10^{-6}\text{ C}}{0.10\text{ m}} + \frac{2 \times 10^{-6}\text{ C}}{0.141\text{ m}} \right\} = -232\text{ kV}$$

21. **Correct Response: D**

$$W = \vec{F} \cdot \vec{s} = (150\text{ N})(7\text{ m})(\cos 25^{\circ}) = 952\text{ J}$$

22. **Correct Response: E**

$$\begin{aligned}\tau &= \mathbf{r} \times \mathbf{F} = (3\hat{\mathbf{x}} + 2\hat{\mathbf{y}}) \text{ m} \times (-5\hat{\mathbf{x}} + 7\hat{\mathbf{y}}) \text{ N} \\ &= (2 \times 0 - 0 \times 7) \text{ N} \cdot \text{m} \hat{\mathbf{x}} - (3 \times 0 - 0 \times (-5)) \text{ N} \cdot \text{m} \hat{\mathbf{y}} + (3 \times 7 - 2 \times (-5)) \text{ N} \cdot \text{m} \hat{\mathbf{z}} \\ &= 31 \text{ N} \cdot \text{m} \hat{\mathbf{z}}\end{aligned}$$

23. **Correct Response: A**

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(12.0 \text{ kg})(2.00 \text{ m}) + (7.00 \text{ kg})(4.00 \text{ m})}{12.0 \text{ kg} + 7.00 \text{ kg}} = 2.74 \text{ m}$$

$$y_{cm} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(12.0 \text{ kg})(5.00 \text{ m}) + (7.00 \text{ kg})(1.00 \text{ m})}{12.0 \text{ kg} + 7.00 \text{ kg}} = 3.53 \text{ m}$$

$$\Rightarrow \vec{r}_{cm} = (2.74\hat{\mathbf{x}} + 3.53\hat{\mathbf{y}}) \text{ m}$$

24. **Correct Response: A**

The collision being completely inelastic, implies $\vec{v}_{1f} = \vec{v}_{2f}$.

$$\sum \vec{p}_i = \sum \vec{p}_f$$

$$\Rightarrow m_1 v_{1xi} + m_2 v_{2xi} = m_1 v_{fx} + m_2 v_{fx} \Rightarrow v_{fx} = \frac{m_1 v_{1xi} + m_2 v_{2xi}}{m_1 + m_2}$$

$$v_{fx} = \frac{(45.0 \text{ g})(0.0 \text{ m/s}) + (62.0 \text{ g})(1.60 \text{ m/s})}{(45.0 \text{ g}) + (62.0 \text{ g})} = 0.927 \text{ m/s}$$

$$\Rightarrow m_1 v_{1yi} + m_2 v_{2yi} = m_1 v_{fy} + m_2 v_{fy} \Rightarrow v_{fy} = \frac{m_1 v_{1yi} + m_2 v_{2yi}}{m_1 + m_2}$$

$$v_{fy} = \frac{(45.0 \text{ g})(2.10 \text{ m/s}) + (62.0 \text{ g})(0.0 \text{ m/s})}{(45.0 \text{ g}) + (62.0 \text{ g})} = 0.883 \text{ m/s}$$

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(0.927 \text{ m/s})^2 + (0.883 \text{ m/s})^2} = 1.28 \text{ m/s}$$

25. **Correct Response: A**

$$\sum \vec{p}_i = \sum \vec{p}_f \Rightarrow m_w v_{wi} + m_c v_{ci} = m_w v_{wf} + m_c v_{cf}$$

$$\Rightarrow \Delta \vec{v}_c = \vec{v}_{cf} - \vec{v}_{ci} = \frac{m_w \vec{v}_{wi} - m_w \vec{v}_{wf}}{m_c}$$

$$= \frac{(65.0 \text{ kg})(0.0 \text{ m/s}) - (65.0 \text{ kg})(3.40 \text{ m/s})}{(29.0 \text{ kg})} = -7.62 \text{ m/s}$$

$$\Delta v_c = |\Delta \vec{v}_c| = |-7.62 \text{ m/s}| = 7.62 \text{ m/s}$$

26. **Correct Response: B**

$$v_f^2 = v_o^2 + 2ax \Rightarrow a = \frac{v_f^2 - v_o^2}{2x} = \frac{(0.0 \text{ m/s})^2 - (19.6 \text{ m/s})^2}{2(45.0 \text{ m})} = -4.27 \text{ m/s}^2$$

$$F = ma = (750. \text{ kg})(-4.27 \text{ m/s}^2) = 3.20 \times 10^3 \text{ N}$$

27. **Correct Response: B**

Newton's Laws say that the deceleration of the car must be caused by an external force. The force must be applied to the car. It is friction between the road and the car that causes the deceleration, therefore the force is applied by the road.

28. **Correct Response: B**

The force of gravity follows Newton's Universal law of gravitation, $F = G \frac{m_1 m_2}{r^2}$. According to Gauss' Law, inside the Earth it is only the mass of radius less than the radial position that contributes. Thus the mass is radius dependent according to the equation $m_2 = \rho V = \rho \frac{4}{3} \pi r^3$. Plugging this into the force gives $F = G \frac{m_1 \rho \frac{4}{3} \pi r^3}{r^2} = G m_1 \rho \frac{4}{3} \pi r$ so it increases linearly until the surface of the Earth is reached. After that point the law of gravitation indicates that the force drops off as $1/r^2$. This is sketch B.

29. **Correct Response: C**

First, calculate the time for the ball to reach the wall in the horizontal direction:

$$x = x_o + v_{ox}t \Rightarrow t = \frac{x - x_o}{v_{ox}} = \frac{x - x_o}{v_o \cos \theta} = \frac{142 \text{ m} - 0 \text{ m}}{(41.3 \text{ m/s}) \cos 28.0^\circ} = 3.894 \text{ s}$$

Then find the height of the ball when it reaches the wall:

$$y = y_o + v_{oy}t + \frac{1}{2}a_y t^2 = y_o + v_o \sin \theta t + \frac{1}{2}a_y t^2$$

$$= 0.92 \text{ m} + (41.3 \text{ m/s})(\sin 28.0^\circ)(3.894 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(3.894 \text{ s})^2 = 2.12 \text{ m}$$

This is above the ground and below the height of the wall, so the ball hits the wall first.

30. **Correct Response: E**

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}, \quad M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}, \quad \text{and} \quad f = \frac{R}{2} = \frac{96.8 \text{ cm}}{2} = 48.4 \text{ cm}$$

$$\Rightarrow d_i = \frac{f d_o}{d_o - f} = \frac{(48.4 \text{ cm})(41.3 \text{ cm})}{(41.3 \text{ cm}) - (48.4 \text{ cm})} = -281.5 \text{ cm}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \Rightarrow h_i = -\frac{d_i h_o}{d_o} = -\frac{(-281.5 \text{ cm})(2.58 \text{ cm})}{((41.3 \text{ cm}))} = 17.6 \text{ cm}$$

31. **Correct Response: D**

$$f = \frac{nc}{2L} \Rightarrow c = \frac{2Lf}{n} = \frac{2(0.650 \text{ m})(4.45 \times 10^3 \text{ Hz})}{1} = 5.79 \times 10^3 \text{ m/s}$$

32. **Correct Response: C**

$$M_A v_A + M_T v_T = 0 \Rightarrow v_A = -\frac{M_T v_T}{M_A} = -\frac{(6.00 \text{ kg})\left(-5.50 \frac{\text{m}}{\text{s}}\right)}{(200. \text{ kg})} = 0.165 \text{ m/s}$$

$$v = \frac{\Delta x}{\Delta t} \Rightarrow \Delta t = \frac{\Delta x}{v} = \frac{20.0 \text{ m}}{0.165 \text{ m/s}} = 121 \text{ s}$$

33. **Correct Response: D**

$$F = ma \Rightarrow G \frac{Mm}{r^2} = m \frac{v^2}{r} = m \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = m \frac{4\pi^2 r}{T^2}$$

$$\Rightarrow M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (4.22 \times 10^8 \text{ m})^3}{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.77 \text{ days}[8.64 \times 10^4 \text{ days/s}])^2}$$

$$\Rightarrow M = 1.90 \times 10^{27} \text{ kg}$$

34. **Correct Response: B**

The definition of work function is the minimum amount of work it takes to remove an electron from a metal.

35. **Correct Response: D**

$$P = \sigma e A_{\text{surface}} \{T_{\text{surroundings}}^4 - T_{\text{object}}^4\} = \sigma e 4\pi R^2 \{T_{\text{surroundings}}^4 - T_{\text{object}}^4\} =$$

$$= \left(5.6704 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4}\right) (0.780) 4\pi (0.250 \text{ m})^2 \{([129 + 273.15] \text{ K})^4 - ([29 + 273.15] \text{ K})^4\}$$

$$= 619 \text{ W}$$