



2023 Academic Challenge

SECTIONAL MATHEMATICS EXAM

Mathematics Test Production Team

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

Time: 40 Minutes

Number of Questions: 30

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

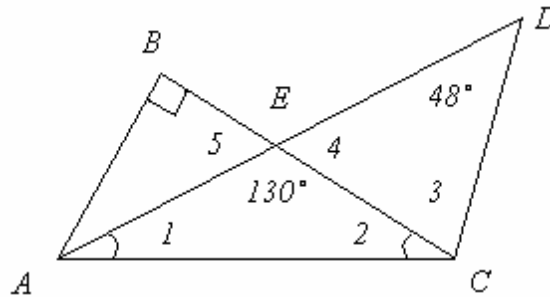
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Academic Challenge
2023 Sectional Mathematics Exam

1. George plans on walking from his house up to the top of a hill and then back down the hill to his house. It is 600 feet from his house to the top of the hill. He walks 2 mph on the trip up the hill and 4 mph on the trip back down the hill. How long does it take George to complete the entire trip? (Round your answer to the nearest second.)

- a) 53 seconds
- b) 2 minutes, 16 seconds
- c) 2 minutes, 33 seconds
- d) 4 minutes, 33 seconds
- e) 5 minutes, 7 seconds

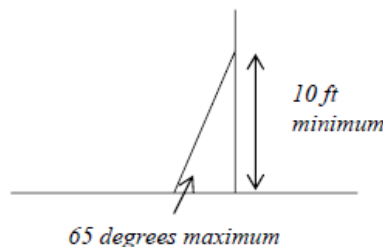
2. In the drawing below, $\angle 1 \cong \angle 2$ and $\angle ABC$ is a right angle. Find the measure of $\angle 3$.



- a) 40°
- b) 48°
- c) 50°
- d) 65°
- e) 82°

3. For safety reasons, a ladder must be set at angle no more than 65 degrees. The ladder also needs to reach at least 10 feet up a wall – see picture below. What is the minimum length the ladder needs to be in order to meet both requirements? (Round your answer to the nearest inch.)

- a) 10 feet, 11 inches
- b) 11 feet, 0 inches
- c) 11 feet, 10 inches
- d) 12 feet, 1 inch
- e) 14 feet, 3 inches



4. The set S consists of $S = \{1, 2, 3, 4, 6, 8, 10, 14, 18, 24\}$. What is the difference between the mean and median of the set S?

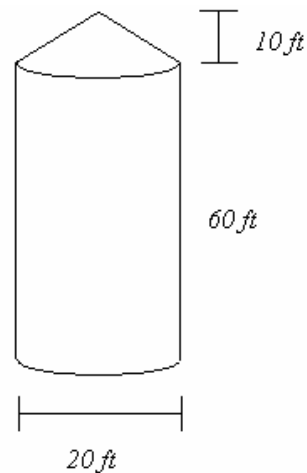
- a) 0
- b) 1
- c) 2
- d) 3
- e) Can't calculate the difference

5. The radioactive gas known as Radon decays exponentially, and has a half-life of 3.8 days. A basement has a Radon concentration that is 50 times the level that is considered safe. How long would it take the Radon in this basement to decay down to a safe level? (Round your answer to the nearest day.)

- a) 22 days
- b) 25 days
- c) 95 days
- d) 190 days
- e) Not enough information was given

6. The silo, pictured to the right, is formed of a circular cylinder 60 feet tall and 20 feet in diameter with a cone shaped top of height 10 feet. Grain is poured into the silo at a rate of 50 cubic feet per minute. How long will it take to fill the circular cylinder as well as the cone shaped top? (Round your answer to the nearest minute.)

- a) 2 hrs 27 min.
- b) 6 hrs 17 min.
- c) 6 hrs 38 min.
- d) 6 hrs 59 min.
- e) 7 hrs 20 min.



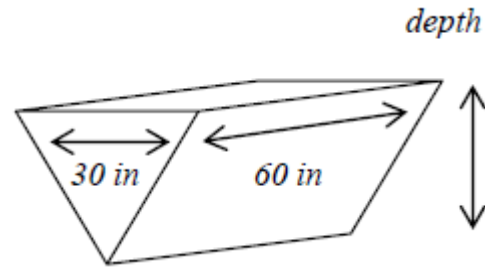
7. A code consists of 6 digits or 4 letters. If the code consists of digits, then the first two numbers must be even. If the code consists of letters, then the first letter must be a vowel (Y included). How many different codes are available with these restrictions?

- a) 250,000
- b) 355,456
- c) 854,740
- d) 906,240
- e) 1,456,976

8. A person can row a boat in still water at 4 miles per hour. It takes the person 3 hours to go a certain distance upstream, while it only takes 1 hour to go down stream the same distance. Assuming that the current rate is steady, what is the current rate? (Round your answer to the nearest mph.)

- a) 1 mph b) 2 mph c) 3 mph d) 4 mph e) 5 mph

9. A feeding trough is a triangular cylinder/prism as shown to the right. The triangular side is an equilateral triangle with 30 inches per side, and the length of the trough is 60 inches. If water is poured into the trough at a steady rate of 100 cubic inches per second, how fast is the depth of the water rising when the water reaches a depth of 12 inches (1 foot)? (Round your answer to the nearest tenth of an inch per second.)

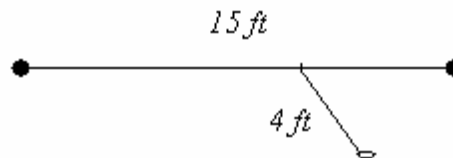


- a) 0.1 inches per second
 - b) 0.2 inches per second
 - c) 0.3 inches per second
 - d) 0.5 inches per second
 - e) 0.9 inches per second
10. Select 5 cards from a standard 52 card deck without replacement. What is the probability that you will get 3 of a kind? (Round to the nearest percent.)
- a) 1% b) 2% c) 3% d) 4% e) 5%
11. Find the midpoint of the line segment AB if A is (3, -2) and B is (9, 4).
- a) (6, 3) b) (6, 1) c) (6, -1) d) (3, 3) e) (3, 1)
12. How many times does a 20 inch diameter tire completely rotate if it is used to drive 7.5 miles? (Round your answer to the nearest ten rotations.)
- a) 1,010 rotations
 - b) 1,230 rotations
 - c) 5,830 rotations
 - d) 7,560 rotations
 - e) 9,020 rotations
13. Rewrite the base seven number 362 as a base ten number.
- a) 95 b) 148 c) 191 d) 488 e) 1337
14. The first five numbers in a geometric sequence are 64, 96, 144, 216, 324. What is the sum of the next five numbers in the sequence?
- a) 2460.375
 - b) 3690.5625
 - c) 6409.125
 - d) 7253.125
 - e) 18683.47266
15. Solve the following for x: $10^x = 6^{x+2}$. (Round to the nearest hundredth.)
- a) 3.92 b) 7.02 c) 15.23 d) 25.97 e) No solution

16. Solve the following for x : $\frac{x}{x-3} + \frac{2}{x+1} = \frac{8x}{x^2-2x-3}$

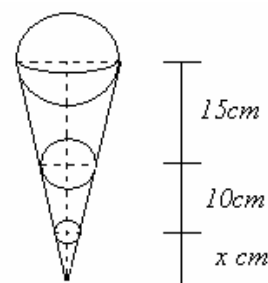
- a) $x = 3, -2$ b) $x = 3, 2$ c) $x = 6, -1$ d) $x = 6, 1$ e) $x = 6$

17. A rope is tightly hung between two poles 15 feet apart. A dog leash, 4 feet long, is attached to the rope so that it may glide smoothly between the two poles, as shown below. How much area would a dog have to move freely if he was placed on the leash? (Round your answer to the nearest tenth of a square foot.)



- a) 110.7 ft² b) 152.0 ft² c) 170.3 ft² d) 184.0 ft² e) 321.1 ft²

18. Three different size balls are placed in a right circular cone, as shown below. The smallest has a diameter of 3 cm. The medium sized ball has diameter 8 cm. The centers of these two balls are 10 cm apart. The centers of the medium and largest size balls are 15 cm apart. What is the diameter of the largest ball? (Round your answer to the nearest tenth of an inch.)



- a) 6.0 in. b) 12.0 in. c) 12.5 in. d) 15.5 in. e) 40.5 in.

19. Pump A can fill a pool in 22 hours. Pump B can empty the same pool in 16 hours. Pump A begins to fill the pool, and two hours later pump B begins to empty the pool. How many hours after pump A started will the pool be empty? (Round your answer to the nearest minutes.)

- a) 5 hours and 15 minutes
b) 6 hours and 45 minutes
c) 7 hours and 20 minutes
d) 8 hours and 40 minutes
e) No answer – pool will fill up

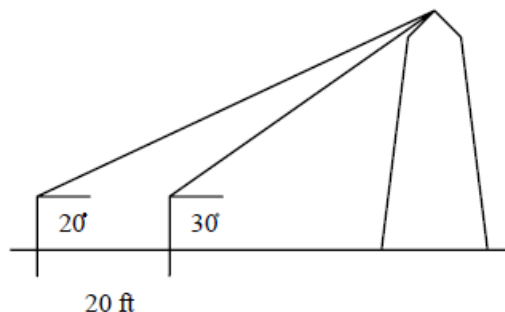
20. How many different five letter “words” can be formed using the letters of the word “analytical”? One example of a five letter word would be “laaty”.

- a) 252 b) 2520 c) 4180 d) 5040 e) 30240

21. Point A is on the origin, and point B is at (6,0). An ellipse is created by taking all points such that the distance from the point to A plus the distance from the point to B always adds up to 10. What is the equation of this ellipse?

- a) $\frac{x^2}{3^2} + \frac{y^2}{4^2} = 1$ b) $\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$ c) $\frac{(x-3)^2}{4^2} + \frac{y^2}{3^2} = 1$ d) $\frac{(x-3)^2}{5^2} + \frac{y^2}{3^2} = 1$ e) $\frac{(x-3)^2}{5^2} + \frac{y^2}{4^2} = 1$

22. A girl wants to estimate how tall a particular monument is. She stands at a particular point, and notices that she must look up at a 30 degree angle to see the top of the monument. She steps back twenty feet, and now she only has to look up at a 20 degree angle to see the top of the monument (see picture below for reference). She also factors in that her eyes are approximately five feet above the ground. Approximately how tall is the monument? (Round your answer to the nearest foot.)



- a) 18 feet b) 20 feet c) 25 feet d) 28 feet e) 30 feet

23. Determine the equation of the slant asymptote of the function:

$$f(x) = \frac{2x^2 - 4x + 3}{\frac{1}{2}x - 2}$$

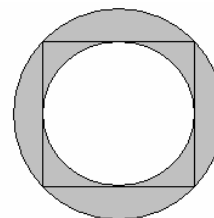
- a) $y = 2x - 4$
 b) $y = x - 2$
 c) $y = 4x - 8$
 d) $y = 4x + 8$
 e) $f(x)$ does not have a slant asymptote

24. Point A is 25 feet away from the base of a flag pole. At point A, the angle of elevation to the top of the flag pole is 42 degrees. How tall is the flag pole? (Round your answer to the nearest tenth of a foot.)

- a) 16.7 feet b) 18.6 feet c) 22.5 feet d) 24.9 feet e) 27.8 feet

25. A square is inscribed in a circle, and another circle is inscribed in the square. What fraction of the area of the larger circle is shaded in?

- a) $1/2$ b) $2/2$ c) $1/3$ d) $3/8$
 e) Not enough information.



26. What is the area of the shape created by the polar equation $r(\theta) = \sin \theta$? (Round your answer to the nearest square unit.)

- a) 1 square unit
 b) 3 square units
 c) 4 square units
 d) 9 square units
 e) 13 square units

27. Solve the following for x . $\log_6 x + \log_6(x - 1) = 1$

- a) $x = 1$ b) $x = 1,6$ c) $x = -2,3$ d) $x = 3$ e) $x = 6$

28. What is the area of a triangle with sides of 62 inches, 54 inches, and 60 inches? (Round your answer to the nearest square inch.)

- a) 954 square inches
- b) 1,476 square inches
- c) 1,527 square inches
- d) 1,804 square inches
- e) none of these

29. You are given the vectors $A = i + 3j - k$ and $B = 3i - j + 2k$. What is the length of the vector $B - A$? (Round your answer to the nearest whole unit.)

- a) 1 unit b) 2 units c) 3 units d) 5 units e) 7 units

30. Evaluate the derivative of $3x^2 - 4y^5 = 7$ with respect to x at the point $(4, -1)$.

- a) -1.2 b) -0.8 c) 0.0 d) 0.8 e) 1.2

