

ACADEMIC CHALLENGE FOR
ACES
ENGINEERING AND SCIENCE



EASTERN ILLINOIS UNIVERSITY

2023 Academic Challenge

STATE MATHEMATICS EXAM

Mathematics Test Production Team

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

Time: 40 Minutes

Number of Questions: 30

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

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1. If $v(t) = 2t$ represents the velocity of a bicycle moving from rest on a straight road, where t is in seconds and v is in meters per second, how far does the bicycle travel in the first ten seconds?
a. 40 meters b. 80 meters c. 100 meters d. 200 meters e. none of these
2. Take the square root of Ben's age and add 15. Now square that result and round down to the nearest integer. If you end up with 467, how old is Ben?
a. 42 b. 45 c. 44 d. 41 e. 43
3. If $y = 3x$, then the graph of $\ln(y)$ vs. $\ln(x)$ is
a. a line with slope 3
b. a line with slope $\ln(3)$
c. a line with slope 1
d. a line through the origin
e. none of these
4. The sum of two real numbers equals their product. The sum, S , of their reciprocals satisfies
a. $S \leq -5$
b. $-5 < S \leq 0$
c. $0 < S \leq 5$
d. $5 < S \leq 10$
e. none of these
5. In $\triangle ABC$, $m\angle C = 106^\circ$, $m\angle A = 22^\circ$ and $BC = 14$. Then AB is most nearly
a. 36 b. 37 c. 38 d. 39 e. 40
6. The plane $2x - 3y + 4z = 77$ meets the xy -plane in a line L . If $(6, k, 0)$ is on L , then k is most nearly
a. 23 b. -12 c. -22 d. -8 e. 0
7. A' , B' and C' are the midpoints of the respective sides BC , CA and AB of a triangle. The sum of the lengths of the medians of $\triangle ABC$ is 864. The sum of the lengths of the medians of $\triangle A'B'C'$ is an integer N . The largest digit in N is
a. 4 b. 5 c. 6 d. 7 e. 8

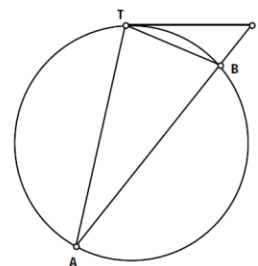
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8. The graphs of $y = 4 - |4 - x|$ and $y = 0$ determine a triangle. Ignoring units, the absolute value of the difference between this triangle's area and perimeter is most nearly

a. 1 b. 2 c. 3 d. 4 e. 5

9. In the figure to the right, tangent $PT = 10$. PA is a secant, with $AB = 21$ and $BP = 4$. If $TA = 17$, then TB is most nearly

a. 6.8 b. 6.5 c. 7.1 d. 6.3 e. 7.3



10. The foci of an ellipse are $F_1(-4, 0)$ and $F_2(4, 0)$. The point $Z(-5, 0)$ is on the ellipse. If $P(2, k)$ is on the ellipse, then $F_1P + F_2P$ is most nearly

a. 9.5 b. 9.8 c. 10.7 d. 10.4 e. 10.1

11. If $(0, 2, -5)$, $(1, -1, -3)$ and $(-2, 8, k)$ are collinear points, then k is

a. -10 b. -12 c. -8 d. -9 e. none of these

12. Each of five people is holding a standard deck of 52 cards. Each person draws one card from his/her deck. What is the probability that exactly two hearts are drawn?

a. $\frac{27}{1024}$ b. $\frac{90}{1024}$ c. $\frac{270}{1024}$ d. $\frac{2}{5}$ e. $\frac{81}{1024}$

13. Consider the set of 2×2 matrices

$$M = \left\{ \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} : k \text{ is real} \right\}$$

How many of the following statements are necessarily true?

- S1: M is closed under addition.
S2: M is closed under multiplication.
S3: Multiplication is commutative in M .
S4: If A is in M , then A^{-1} is also in M .

a. none b. one c. two d. three e. four

14. If $\cos q = -\frac{12}{13}$, $\tan q = -\frac{5}{12}$, and $\sin 2q = \frac{K}{169}$, then K is

a. -60 b. 60 c. -120 d. 120 e. none of these

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15. If $(4 + 5i)(-3 + 2i) = re^{i\theta}$, where $r > 0$ and $0 < \theta < 2\pi$, then θ is most nearly
- a. 2.8 b. 3.4 c. 0.31 d. 5.9 e. 3.1
16. A reflection across the y -axis is followed by a reflection across the line $x = 4$. This composite transformation is
- a. a translation 8 units to the right
b. a translation 8 units to the left
c. a translation 4 units to the right
d. a translation 4 units to the left
e. a translation not included in (a) - (d)
17. A cube is inscribed in a sphere of radius 1. The distance from the center of the cube to one of its faces is most nearly
- a. 0.506 b. 0.577 c. 0.552 d. 0.639 e. 0.707
18. A clockwise rotation of 90° about the origin sends the line $L: 3x + 2y = 6$ to the line L' . An equation of L' is
- a. $2x - 3y = 6$ b. $2x - 3y = -6$ c. $3x - 2y = 6$ d. $3x - 2y = -6$ e. none of these
19. The geometric transformation given by $x' = x + y$, $y' = y$
- a. preserves perpendicularity of lines
b. preserves distance
c. preserves betweenness of points
d. preserves shapes of figures
e. none of these
20. The plane α passes through the origin and is perpendicular to the vector $\mathbf{v} = \langle 1, -1, 2 \rangle$. Which of these points lies in α ?
- a. $(5, 1, 2)$ b. $(1, 2, 5)$ c. $(2, 5, 1)$ d. $(5, 2, 1)$ e. $(1, 5, 2)$
21. Dick and Jane have calculators which produce random numbers between 0 and 1, displayed as 8-place decimals. If Dick produces the random number x and Jane produces the random number y , what is the probability that the absolute value of the difference of x and y will be less than 0.4?
- a. 0.36 b. 0.40 c. 0.50 d. 0.60 e. 0.64

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22. In the nation of Hexaland there are five types of paper money: the u , which is equal to one hexollar; the t , which is worth 2 u 's; the f , which is worth 4 u 's; the e , which is worth 8 u 's; and the s , which is worth 16 u 's. If Nimadar hands a clerk ten s bills to pay for a uniform that costs exactly 145 hexollars, including tax, what is the minimum number of bills he could get back in change from the clerk?

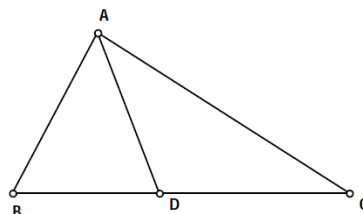
a. 5 b. 4 c. 3 d. 2 e. 1

23. What points are left unchanged by the transformation given by $x' = 0.6x + 0.8y$,
 $y' = 0.8x - 0.6y$?

a. only $(0, 0)$
b. all points on the line $y = x$
c. all points on the line $y = (1/2)x$
d. all points on the line $y = 2x$
e. none of these

24. In the figure, AD bisects $\angle BAC$. If $AB = 5$, $AC = 8$ and $BD = 3$, then CD is most nearly

a. 4.8 b. 4.9 c. 5.0 d. 5.1 e. 5.2



25. If $z = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$, and $S = \{z^n : n \text{ is a positive integer}\}$, then

a. S is empty.
b. S has exactly four elements.
c. S has exactly eight elements.
d. S has a finite number of elements, but not four or eight.
e. S has infinitely many elements.

26. The base three numeral for N is 2110. The base three numeral for $3N + N/3$ is

a. 22011
b. 21020
c. 22110
d. 21221
e. none of these

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27. A , B and P are non-collinear points. If X is a variable point on line AB , then $\overrightarrow{PA}$, $\overrightarrow{PB}$ and $\overrightarrow{PX}$ are vectors satisfying $\overrightarrow{PX} = t\overrightarrow{PA} + (1 - t)\overrightarrow{PB}$, where t is a real scalar. The inequality $t \leq 0$ describes
- the segment AB
 - the ray AB
 - the ray BA
 - the ray opposite to ray AB
 - the ray opposite to ray BA
28. Let $M = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \text{ are integers satisfying } ad - bc = 1 \right\}$. How many of the following statements are necessarily true?
- S1: M is closed under multiplication.
S2: If A is in M , then A^{-1} is also in M .
S3: There is a member U in M satisfying $UB = B$ for all B in M .
S4: If B is a 2×1 matrix with integer entries, and A is an arbitrary member of M , the equation $AX = B$ has a solution for X which has integer entries.
- none
 - one
 - two
 - three
 - four
29. The sum of the real parts of the five solutions to $x^5 - 33 = 0$ is most nearly
- 2.01
 - 0.62
 - 0.13
 - 1.63
 - 1.01
30. The vertex of an elliptical cone is at $(0, 0, 18)$. The base in the xy -plane is given by $4x^2 + 9y^2 = 144$. The volume of the cone is 144π . The plane $z = 9$ cuts off an elliptical cone whose base area is
- 6π
 - 12π
 - 16π
 - 18π
 - 48π