



2024 Academic Challenge

MATH TEST – STATE

Math Production Team

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

Time: 40 Minutes Number of Questions: 30

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

**Academic Challenge
Math Test (State) - 2024**

1. Three of the factors of a polynomial are -2 , 3 , and $2 + i$. What is the polynomial?

- a) $x^4 + 5x^3 + 3x^2 + 19x - 30$ b) $x^4 - 5x^3 + 3x^2 + 19x - 30$ c) $x^4 + 5x^3 + 3x^2 + 19x + 30$
d) $x^4 - 5x^3 + 3x^2 + 19x + 30$ e) $x^4 - 5x^3 - 3x^2 - 19x - 30$

2. A series begins with the following values:

1, 16, 9, 256, 25, 1296 ...

Which terms in the sequence will be the first pair to have a difference with an absolute value of greater than 65,000?

- a) 9th and 10th b) 13th and 14th c) 14th and 15th d) 15th and 16th e) 17th and 18th

3. A piecewise function has the following conditions for the range $x \geq 0$:

$$f(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1}, & x \neq 1 \\ q, & x = 1 \end{cases}$$

What value of q will make the function continuous?

- a) 1 b) 0 c) $1/2$ d) $1/4$ e) 2

4. A spinner has four colors: red, green, yellow, blue. Each color has the same probability of being selected on each spin. If there are five spins, what is the probability that blue comes up exactly two times?

- a) 0.4 b) 0.323 c) 0.264 d) 0.25 e) 0.233

5. A supply company offers a 20 percent discount for the first \$5,000 order from a new customer. Each time the customer places another \$5,000 order within two months of the previous order, the company adds 1.5 percent to the previous discount. If the customer places 6 orders and receives all available discounts, how much money will the customer spend?

- a) \$22,875 b) \$23,125 c) \$23,875 d) \$24,125 e) \$24,875

6. Which concept is not part of non-Euclidean geometry?

- a) Two points can be included in more than one line.
- b) If two lines intersect, they do so at only one point.
- c) The value of a defined slope of a line does not change.
- d) a, b, and c
- e) a and b

7. An isosceles trapezoid has a base of 4 cm and a base of 64 cm. A midsegment divides the original trapezoid into two subtrapezoids. If this midsegment division process is repeated twice so that the original trapezoid is divided into eight subtrapezoids, what is the sum of all of the base measures?

- a) 224 b) 276 c) 294 d) 306 e) 336

8. A spherical object is deflated, with the radius changing at a constant rate. In 15 minutes, the radius decreases from 4.5 meters to 2.5 meters. How fast was the volume changing when the radius was 3.5 meters?

- a) -3.76π m/min b) -4.24π m/min c) -6.53π m/min d) -7.13π m/min e) -7.36π m/min

9. What are the irrational roots of $6x^2 + 17x + 29$?

- a) $\frac{-17 \pm \sqrt{174}}{12}$ b) $\frac{-17 \pm \sqrt{207}}{12}$ c) $\frac{17 \pm \sqrt{207}}{12}$ d) $\frac{17 \pm \sqrt{407}}{12}$ e) $\frac{-17 \pm \sqrt{407}}{12}$

10. An altitude of a right triangle divides the hypotenuse into segments of 18 inches and 25 inches. What is the measure of the altitude?

- a) $3\sqrt{30}$ b) $3\sqrt{15}$ c) $15\sqrt{2}$ d) $15\sqrt{5}$ e) $25\sqrt{2}$

11. A population of beetles grows at a rate of 2.3 percent per day during the summer. How long will it take for the population to triple?

- a) 33.81 days b) 37.33 days c) 41.23 days d) 45.63 days e) 48.31 days

12. Which trigonometric functions are undefined at $\pi/2$ and $3\pi/2$?

- a) tangent and cotangent b) cosecant and cotangent c) cosecant and tangent
d) secant and cotangent e) secant and tangent

13. Each of the letters A through E is arranged at random. How many distinct arrangements can have A as the first letter or E as the fifth letter?

- a) 30 b) 42 c) 48 d) 54 e) 63

14. What are the mean and the median of the set of prime numbers between 3 and 93?

- a) mean = 33.75; median = 37. b) mean = 35.52; median = 37
c) mean = 39.21; median = 41 d) mean = 41.78; median = 41
e) mean = 46.13; median = 43

15. What expression does not simplify to $\cos x$?

- a) $\sqrt{1 - \sin^2 x}$ b) $\cot x \sin x$ c) $\sec x (1 - \sin^2 x)$ d) first derivative of $\sin x$
e) second derivative of $\cos x$

16. What function will create a single intersection point with $3x^2 + 18x + 27$ and with the x-axis?

- a) $|(x - 3)|$ b) $|(x + 3)|$ c) $-|(x + 3)|$ d) $-|(x - 3)|$ e) $|(x + 4.5)|$

17. A capsule has a density of 0.33 g/cm^3 . If each capsule consists of a cylindrical center with a length of 0.5 cm and two hemispheres with a diameter of 0.1 cm, what is the mass of 170 capsules?

- a) 0.2497 g b) 0.2814 g c) 0.3099 g d) 0.3287 g e) 0.3414 g

18. In the expression $x(x - 8)$, what is the lowest possible output value?

- a) -128 b) -32 c) -24 d) -16 e) -8

19. At a park, a security camera is at the top of a 15-foot pole. If the camera auto-adjusts to an angle of depression of 54 degrees in order to focus on an object, how far is the object from the base of the pole?

- a) 9.4 ft b) 10.9 ft c) 14.6 ft d) 20.6 ft e) 21.5 ft

20. Which function will create a graph where (1) a tangent line with a slope of zero can be drawn at (0, 9), and (2) there are two points with an undefined slope on the x-axis?

- a) $x^2 + 9$ b) $-x^2 + 9$ c) $|9 - x^2|$ d) $9 - x^2$ e) $x^2 + 6x + 9$

21. In a popular word game, placing R as the second letter is correct 3 percent of the time. A particular player solves the puzzle 98 percent of the time when R is the second letter. When R is not the second letter, the player solves the puzzle 87.5 percent of the time. Given that the player solves the puzzle, what is the probability that R is the second letter of the word?

- a) 3 percent b) 3.12 percent c) 3.19 percent d) 3.35 percent e) 3.43 percent

22. What is the complement of an angle measuring 56 degrees, 19 minutes, and 27 seconds?

- a) 34 degrees, 41 minutes, 33 seconds b) 34 degrees, 40 minutes, 33 seconds
c) 33 degrees, 41 minutes, 33 seconds d) 33 degrees, 40 minutes, 33 seconds
e) 33 degrees, 31 minutes, 33 seconds

23. If a cyclist travels at a rate of 500 meters per minute, how fast will the cyclist travel in kilometers per hour?

- a) 12 kph b) 23.3 kph c) 30 kph d) 33.3 kph e) 38 kph

24. A circle has a radius of 3 cm. Point A is on the circle, and it is the vertex of an inscribed angle with a measure of 58 degrees. What is the length of the arc intercepted by angle A?

- a) 2.14 cm b) 2.28 cm c) 2.42 cm d) 2.76 cm e) 2.94 cm

25. A person jogs due east for 15 minutes at 6 miles per hour. Then the person turns north and jogs for 20 minutes at 9 miles per hour. At this point, the person reaches a diagonal path that goes back to the starting point. If the jogging speed on the diagonal path is 8 miles per hour, what is the jogger's average speed?

- a) 7.8 mph b) 8.1 mph c) 8.3 mph d) 8.5 mph e) 8.7 mph

26. An ellipse has an eccentricity of 0.8. If the length of the semimajor axis is 10, what is the length of the semiminor axis?

- a) 2 b) 4 c) 5 d) 6 e) 9

27. During a 10-day period, the chance of rain is 10 percent on five of the days, 25 percent on two of the days, 50 percent on two of the days, and 85 percent on a single day. What is the expected value for the number of days of rain during the 10-day period?

- a) 0.29 days b) 0.35 days c) 1.35 days d) 1.75 days e) 2.85 days

28. In the spring, one-fourth of a large snow pile melts each day. If the pile starts at a mass of 232 pounds, how long will it take for the pile to reach a mass of 100 pounds?

- a) 1.12 days b) 2.1 days c) 2.92 days d) 3.17 days e) 3.51 days

29. An infinite geometric series begins:

$\frac{3}{16}, \frac{9}{64}, \frac{27}{256}, \frac{81}{1024} \dots$

What is the sum of this series?

- a) $\frac{7}{16}$ b) $\frac{9}{16}$ c) $\frac{11}{32}$ d) $\frac{11}{16}$ e) $\frac{3}{4}$

30. Two sides of a triangle are 18.5 and 27.5. What are the correct inequalities for (1) the measure the third side has to be to have a triangle and (2) the measure the third side has to be to have an obtuse triangle?

- a) $9 < x < 17.429 \cup 32 < x < 46$ b) $9 < x < 20.347 \cup 33.1436 < x < 46$
 c) $9 < x < 22.147 \cup 33.1436 < x < 46$ d) $9 < x < 22.147 \cup 34.2374 < x < 46$
 e) $9 < x < 22.147 \cup 34.7461 < x < 46$

SCRATCH PAPER

