# Graph Problems — Arm Wrestling

1. Faculty from two Math departments compete in arm-wrestling. Each department has a team of four professors. Each prof must compete against each prof on the opposing team. Draw a graph with vertices representing the teachers, and edges representing the arm-wrestling matches. How many matches must be completed in the competition?

# Chess Games

2. A chess master plays six simultaneous games with six other players. Draw a graph with vertices representing all the players, and edges representing the chess games. How many games are being played?

# Handshaking Hockey Teams

3. There are six members on a hockey team (including the goalie). At the end of a hockey game, each member of the team shakes hands with each member of the opposing team. Draw a graph to illustrate this hand-shaking. How many handshakes occur?

# Rumors

4. A lawyer is preparing her argument in a libel case. She has evidence that a libelous rumor about her client was discussed in various telephone conversations among eight people. Two of the people involved had four telephone conversations in which the rumor was discussed, one person had three, four had two, and one had one such telephone conversation. How many telephone conversations were there among the eight people in which the rumor was discussed?

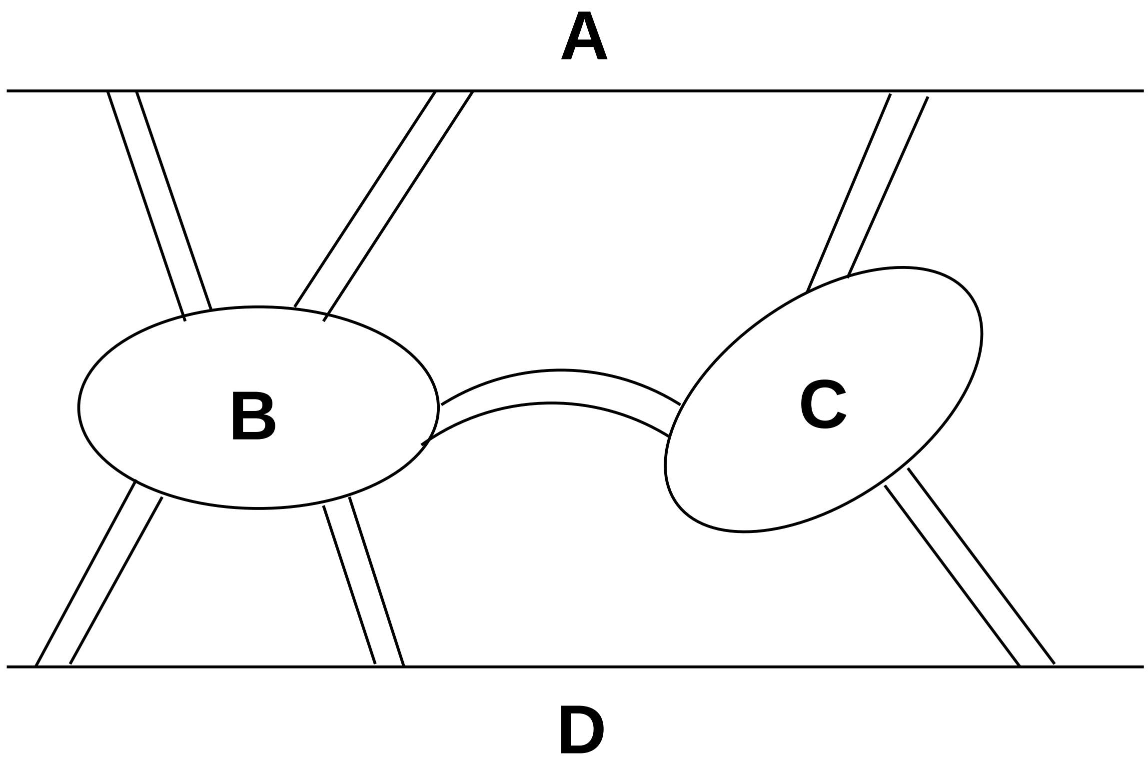
# Local Roads

5. Construct a graph to represent the following towns in the local area: Arcola, Tuscola, Mattoon, Hindsboro, Oakland, Ashmore, and Charleston, and the roads which connect them: Rte 45, Rte 133, Rte 16, Rte 130, and Rte 7. Rte 45 connects Tuscola, Arcola, and Mattoon. Rte 133 connects Arcola, Hindsboro, and Oakland. Rte 16 connects Mattoon, Charleston, and Ashmore. Rte 133 plus Rte 130 connects Arcola and Charleston. Rte 36 and Rte 133 connects Tuscola and Charleston. Rte 7 connects Oakland and Ashmore.

# Euler Circuits

## Recall: The Konigsberg Bridge Problem

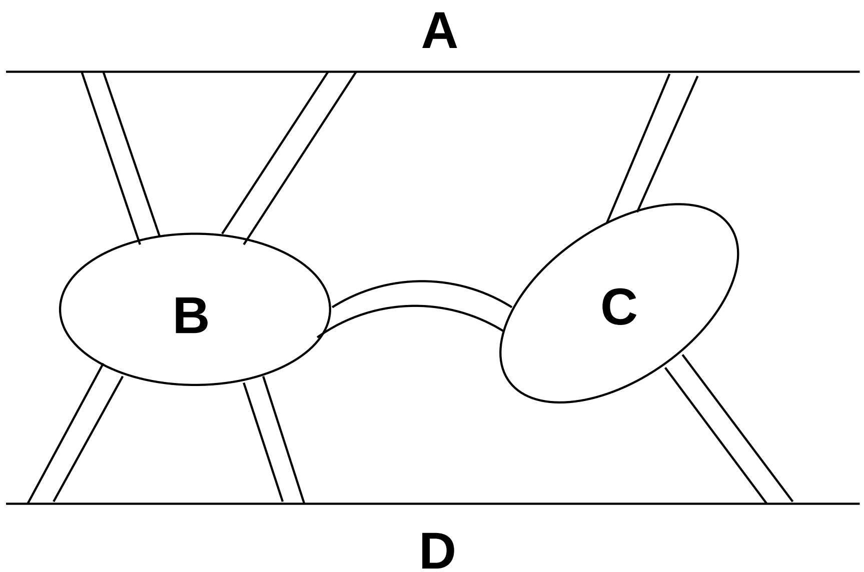
The city of Konigsberg, located on the banks of the Pregel River, included two islands which were joined to the banks and to each other by seven bridges:

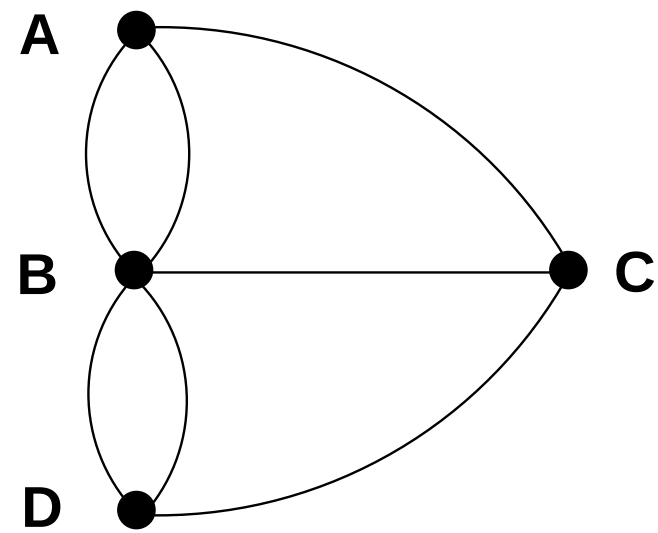



Determine a walk which begins on one side of the river, crosses each bridge exactly once, and return to the same side of the river...

# Turning The Problem Into A Graph

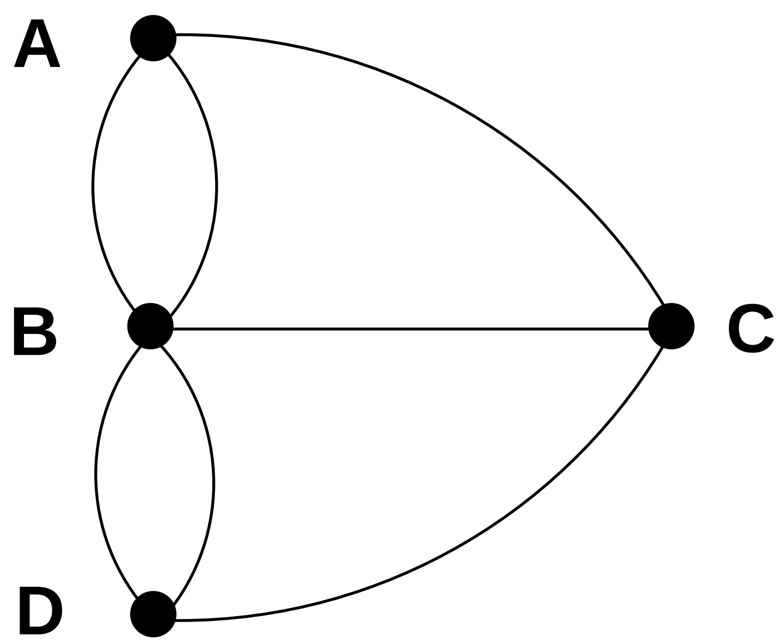
Recall: a **Multigraph** is a graph which allows multiple edges between the same vertices, as well as permitting self-loops.




Can we find a tour of the bridges which allows us to return home without crossing any bridge more than one time? Why or why not?

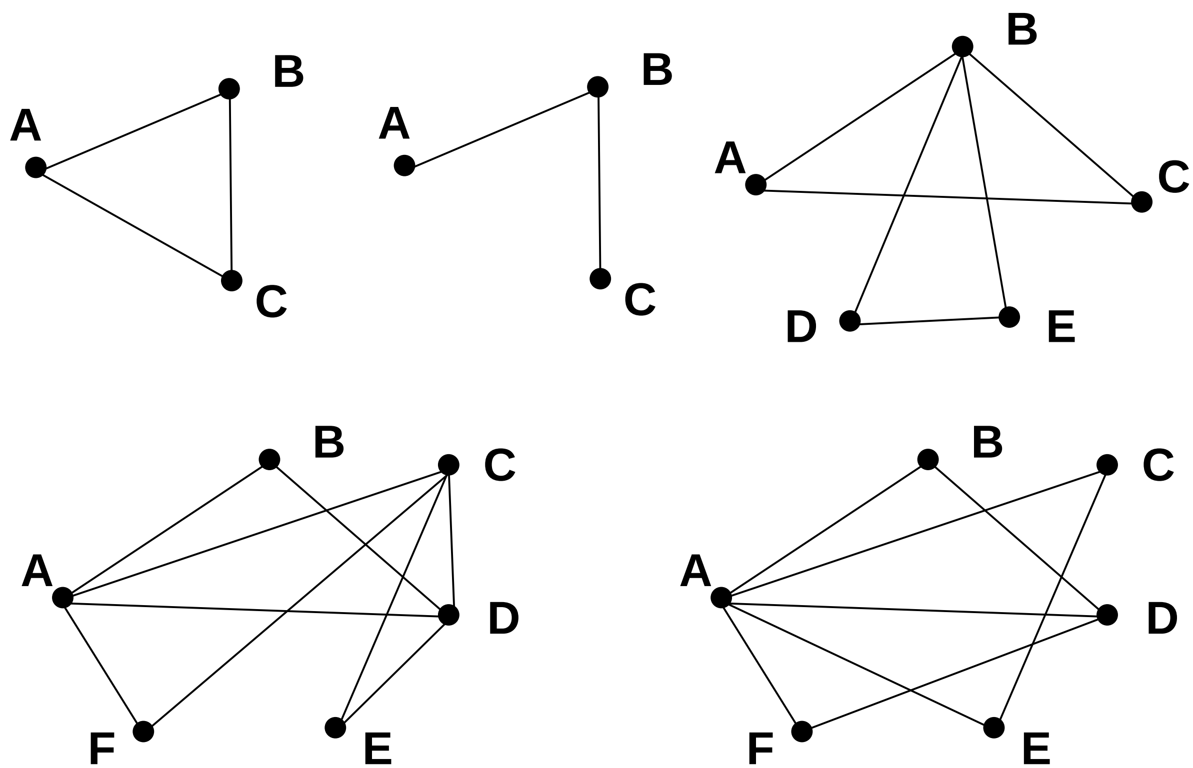
# Euler Paths and Circuits




**Euler Path**: a path that uses **every edge** of the graph **exactly once**.

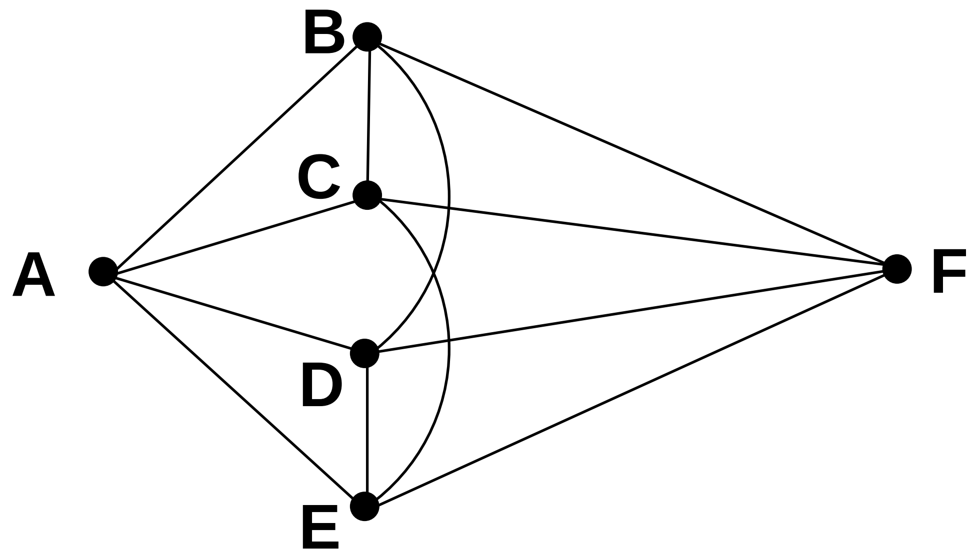
**Euler Circuit**: a **circuit** that uses every edge of the graph exactly once (and we arrive back where we started).

# Do these graphs contain Euler Circuits?



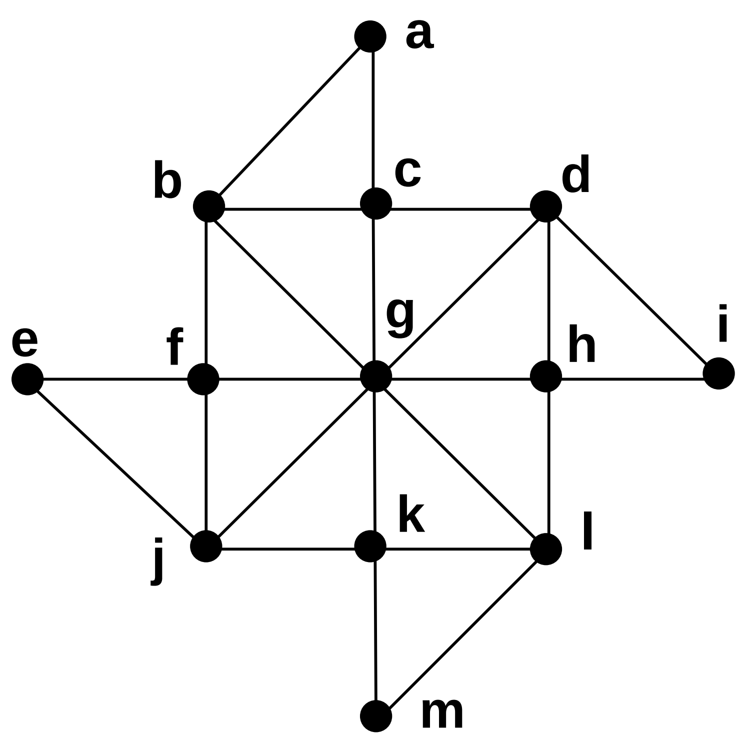

**Question**: If an Euler Circuit exists, does it matter *where* we start?

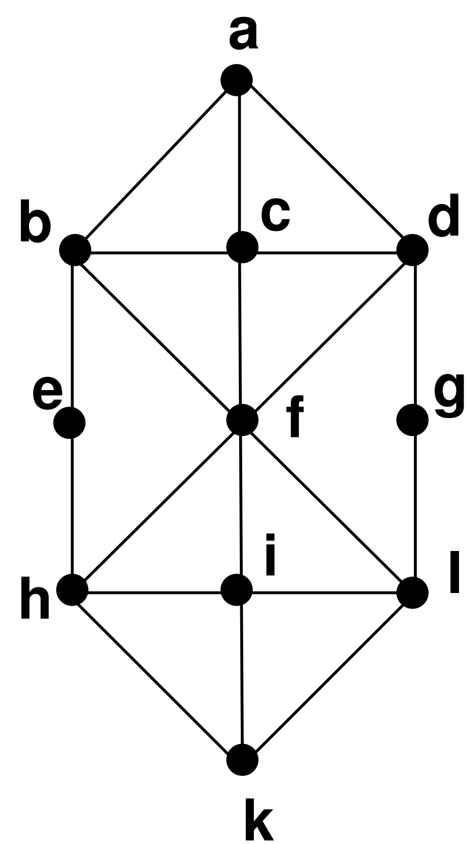
**Question**: Do we need to *produce* an Euler circuit to know if one exists?


B
C
A
F
D
E

**Euler's Theorem, Part I.** If G is connected and each vertex has even degree, then G has an Euler Circuit.

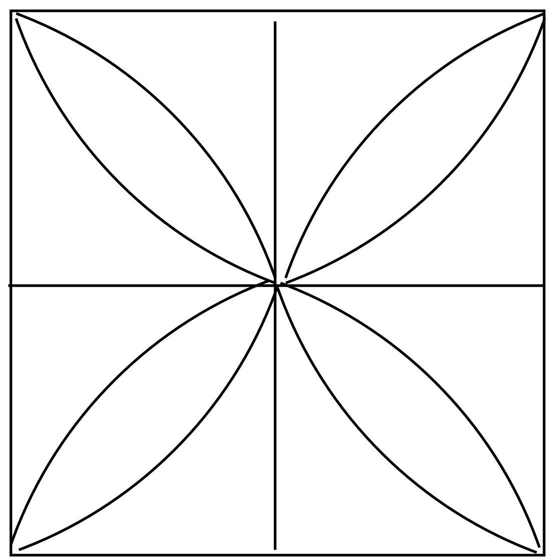
**Euler's Theorem, Part II.** If G is connected and has an Euler Circuit, then each vertex of the graph as even degree.

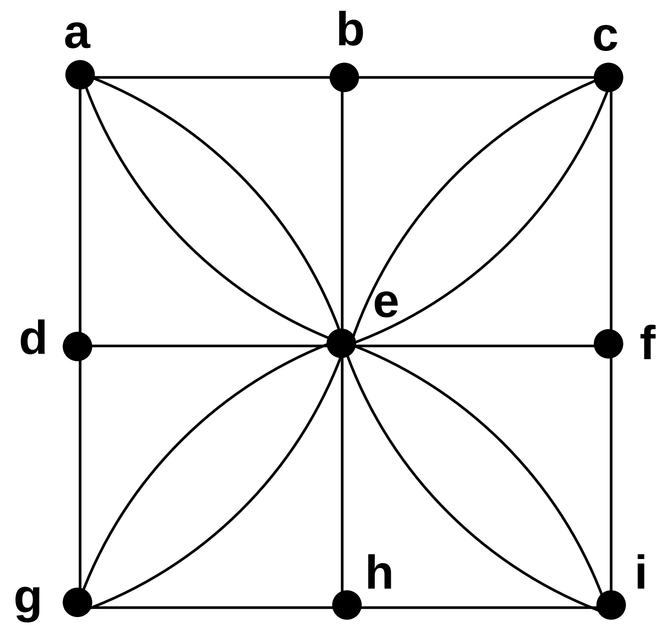



Do these graphs have Euler Circuits?

Beginning and ending at the same place, is it possible to trace the pattern shown in the figure below (left) without lifting the pencil off the page and without tracing over any line or curve in the pattern twice?






We can turn the pattern on the left into a graph by adding vertices at points where lines or curves of the pattern meet, obtaining the graph on the right.

Now we can use Euler's theorem to decide whether or not we can accomplish the task.

# Fleury's Algorithm

**Fleury's Algorithm** can be used to find an Euler circuit in any connected graph in which each vertex has even degree.

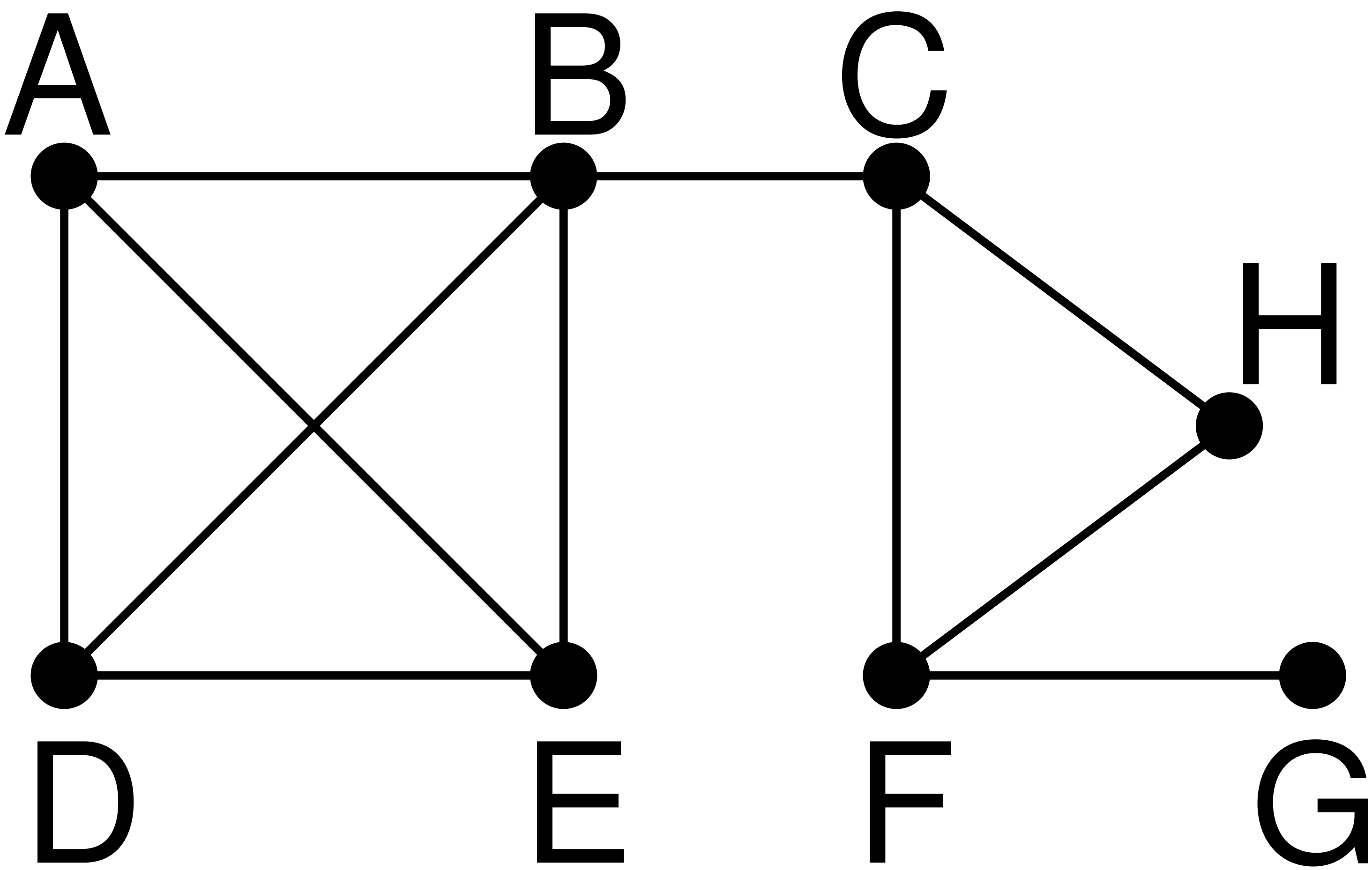
An algorithm is like a recipe: follow the specified steps and you achieve what you need.

First, we need a definition:

A **Cut Edge** in a graph is an edge whose removal disconnects a component of the graph. (aka **bridge**)

We call such an edge a cut edge, since removing the edge **cuts** a connected piece of graph into two pieces.

# Which are Cut Edges?



# Fleury's Algorithm

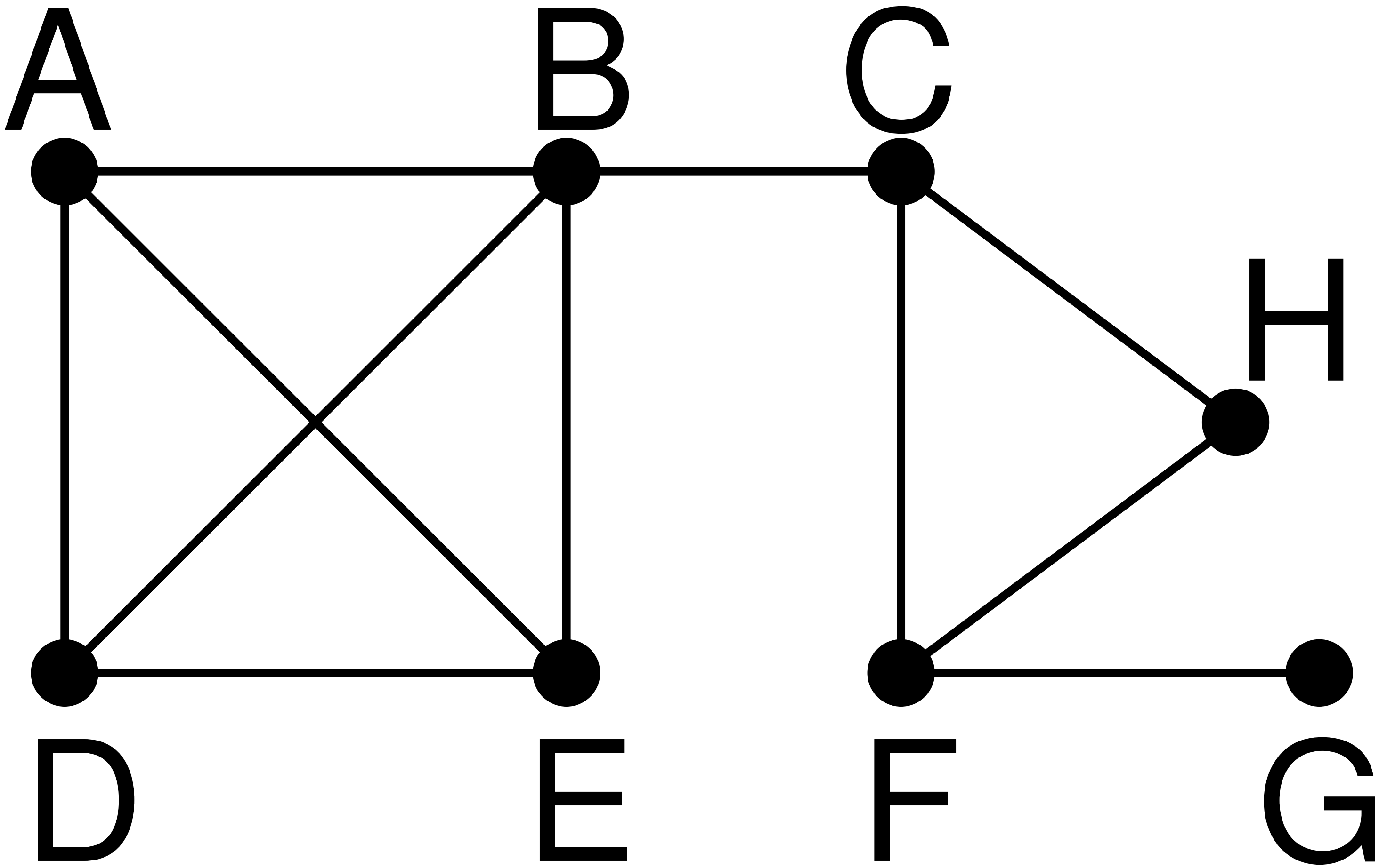
An algorithm for finding an Euler Circuit in a connected graph in which every vertex has even degree.

1. Start at any vertex. Go along any edge from this vertex to another vertex. *Remove this edge from the graph.*

2. You are now on a vertex of the revised graph. Choose any edge from this vertex, subject to only one condition: Do not use a cut edge (*of the revised graph*) unless you have no other options. Go along your chosen edge. *Remove this edge from the graph.*)

3. Repeat Step 2 until you have used all the edges and gotten back to the vertex at which you started.

# Apply Fleury's Algorithm



# Apply Fleury's Algorithm