Chap 2. Basic Concepts of Set Theory
2.1 Symbols & Terminology

- **Terminology**
  - **Set**: a collection of objects
  - **Element** or **Member** of a set: an object belonging to the set

- **Three ways to designate sets**:
  - **word description**
    ex: the set of odd counting numbers between 2 and 12
  - the **listing method**
    ex: {3, 5, 7, 9, 11}
  - **set-builder notation**
    ex: $\{x \mid x \in \mathbb{N},\ x \text{ is odd, and } x < 12\}$

- Notes:
  - use curly braces to designate sets,
  - use commas to separate set elements
  - the variable in the set-builder notation doesn't have to be x.
  - use ellipses (. . .) to indicate a continuation of a pattern established before the ellipses
    ex: {1, 2, 3, 4, . . . , 100}

Important Number Sets

- $\mathbb{N}$ — Natural or Counting numbers: {1, 2, 3, . . .}
- $\mathbb{W}$ — Whole Numbers: {0, 1, 2, 3, . . .}
- $\mathbb{I}$ — Integers: {. . . , -3, -2, -1, 0, 1, 2, 3, . . .}
- $\mathbb{Q}$ — Rational numbers: $\{\frac{p}{q} \mid p, q \in \mathbb{I}, q \neq 0\}$
- $\Re$ — Real Numbers: { x | x is a number that can be written as a decimal }
- Irrational numbers: { x | x is a real number and x cannot be written as a quotient of integers }.
  Examples are: π, $\sqrt{2}$, and $\sqrt[3]{4}$
- $\emptyset$ — Empty Set: { }

Notes

- The symbols { x | x . . .} is read "x such that x . . . has some property
- The symbol $\in$ means "is an element of"
- Any **rational** number can be written as either a TERMINATING decimal (like 0.5, 0.333, or 0.8578966) or a REPEATING decimal (like $0.\overline{333}$ or $123.392\overline{545}$)
- The decimal representation of an **irrational** number **never terminates** and **never repeats**
- The set { $\emptyset$ } is ***not*** empty, but is a set which ***contains*** the empty set

Set Cardinality

- **Cardinality** of a set: the number of distinct elements in the set
  - textbook: n(A) — or we can use |A|
  - If the cardinality of a set is a particular whole number, we call that set a **finite** set
  - If a set is too large to ever finish the counting process, it is called an **infinite** set

- **Well-Defined** set: one for which we can determine **membership**, i.e., given any arbitrary value we can determine conclusively whether or not that value is in the set

Set Membership

- **Well-Defined** means that given a set and an object, we can determine if the set contains that object
  - Is $2 \in \{0, 2, 4, 6\}$?
  - Is $2 \in \{1, 3, 5, 7, 9\}$?
  - Is $\emptyset \in \{a, b, c\}$?
  - Is $\emptyset \in \{\emptyset, \{\emptyset\}\}$?
  - Is $\emptyset \in \{\{\emptyset\}\}$?
  - Is $\frac{1}{3} \notin \{x \mid x = \frac{1}{p},\ p \in \mathbb{N}\}$

Set Equality

- **Set Equality**: the sets A and B are equal (written A = B) provided:
  - every element of A is an element of B, and
  - every element of B is an element of A

  i.e., if they contain exactly the same elements
  - Does { a, b, c } = { b, c, a } = { a, b, a, b, c } ?
  - Does { 3 } = { x | x $\in$ $\mathbb{N}$ and 1 < x < 5} ?
  - Does { x | x $\in$ $\mathbb{N}$ and x < 0} =
    { y | y $\in$ $\mathbb{Q}$ and y is irrational} ?

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