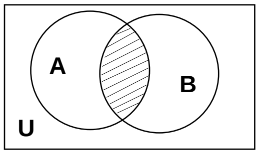
Sec 2.3 Set Operations & Cartesian Products

- **Intersection** of sets: $A \cap B$ is the set of elements common to both: $A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$



- Find the intersections of the following sets:

  { a, b, c } and { b, f, g } ________

  { a, b, c } and { a, b, c } ________

  { a, b, c } and { a, b, z } ________

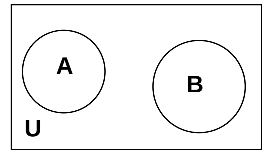
  { a, b, c } and { x, y, z } ________

  { a, b, c } and $\emptyset$ ________

Disjoint Sets

- **Disjoint** sets: two sets which have no elements in common.

  I.e., their intersection is empty: $A \cap B = \emptyset$



- Are the following sets disjoint?

  { a, b, c } and { d, e, f, g } ________

  { a, b, c } and { a, b, c } ________

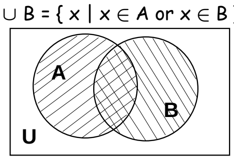
  { a, b, c } and { a, b, z } ________

  { a, b, c } and { x, y, z } ________

  { a, b, c } and $\emptyset$ ________

Set Union

- **Union** of sets: $A \cup B$ is the set of elements belonging to either of the sets: $A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$



Note: an element in the union of sets A and B may be a member of A, a member of B, ***or*** a member of ***both*** sets.

- Find the unions of the following sets:

  { a, b, c } and { b, f, g } ________

  { a, b, c } and { a, b, c } ________

  { a, b, c } and { a, b, z } ________

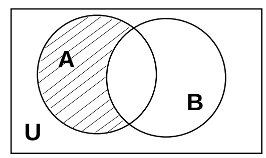
  { a, b, c } and { x, y, z } ________

  { a, b, c } and $\emptyset$ ________

Set Difference

- **Difference** of two sets: $A - B$ is the set of all elements belonging to set A and ***not*** to set B.

  $A - B = \{ x \mid x \in A \text{ and } x \notin B \}$



$\{ 1, 2, 3, 4, 5 \} - \{ 2, 4, 6 \} = \{ 1, 3, 5 \}$

but $\{ 2, 4, 6 \} - \{ 1, 2, 3, 4, 5 \} = \{ 6 \}$

Note: $x \notin B \rightarrow x \in B'$ (the complement of B)

Thus, $A - B = \{ x \mid x \in A \text{ and } x \notin B \}$

$= \{ x \mid x \in A \text{ and } x \in B' \}$

$= A \cap B'$

- Given the sets:

  U = {1, 2, 3, 4, 5, 6, 9}

  A = {1, 2, 3, 4}

  B = {2, 4, 6}

  C = {1, 3, 6, 9}

  Find each of these sets:

  - $A \cup B =$
  - $A \cap B =$
  - $A \cap U =$
  - $A \cup U =$

U = {1, 2, 3, 4, 5, 6, 9}

A = {1, 2, 3, 4}

B = {2, 4, 6}

C = {1, 3, 6, 9}

- $A' =$
- $A' \cap B =$
- $A' \cup B =$
- $A \cup B \cup C =$
- $A \cap B \cap C =$

- Describe each of the following sets in words:
  - $A' \cup B'$ =
  - $A' \cap B'$ =
  - $A \cap (B \cup C)$
  - $(A' \cup C) \cap B$

- Given the sets:
  - U = {1, 2, 3, 4, 5, 6, 7}
  - A = {1, 2, 3, 4, 5, 6}
  - B = {2, 3, 6}
  - C = {3, 5, 7}

  Find each set:
  - A – B =
  - B – A =
  - $(A - B) \cup C'$ =

  Note, in general, $A - B \neq B - A$

Ordered Pairs

- **Ordered Pair**: a group of two objects designated as **first** and **second** components.

  In the **ordered pair** (a, b):
  a is called the **first component**
  b is called the **second component**

- In general $(a, b) \neq (b, a)$, so **order** is **important**!

- Two ordered pairs (a, b) and (c, d) are **equal** provided a = c and b = d

  (1, 3) = (1, 3) (1, 3) ≠ (3, 1)
  (4, 9) = (4, 9) (9, 4) ≠ (4, 9)
  (2+2, 3×3) = (2×2, 6+3)

- Sets can contain ordered pairs:
  { (–3, 3), (–12, –6), (13, 29), (8, 7) }
  { (1, 3), (2, 6), (3, 9), . . . }

Cartesian Products

- The **Cartesian product** of sets A and B is:

  $A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$

  The Cartesian product of {a, b, c} × {1, 2} =

  { (a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2) }

  The Cartesian product of {1, 2} × {a, b, c} =

  { (1, a), (1, b), (1, c), (2, a), (2, b), (2, c) }

  What's the difference between the two resulting sets above?

  If set A = {x, y, z}, what is A × A ?

Cardinality of Cartesian Products

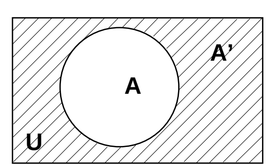
- If set A has cardinality 5 and set B has cardinality 4, what is the cardinality of A × B?

  Of B × A?

  If |A| = n and |B| = m, what is |A × B|?

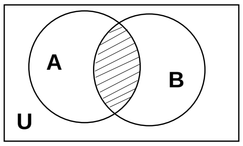
Set Operations

- Finding intersections, unions, differences, Cartesian products, and complements of sets are examples of **set operations**
- An **operation** is a rule or procedure by which one or more objects are used to obtain another object (usually a set or number).
- **Common Set Operations**
  Let A and B be any sets, with U the universal set.
  - **Complement** of A is:
    $A' = \{x \mid x \in U \text{ and } x \notin A\}$

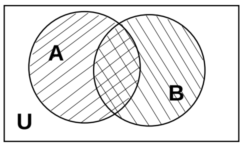


Set Intersection and Union

- **Intersection** of A and B is: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

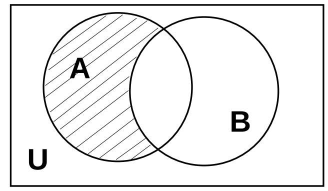


- **Union** of A and B is: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$



Set Difference

- **Difference** of A and B is: $A - B = \{x \mid x \in A \text{ and } x \notin B\}$



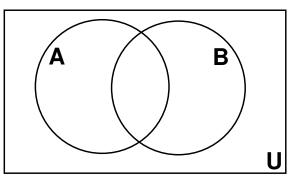
Cartesian Product

- The **Cartesian product** of A and B is:

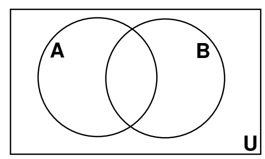
  $A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$

- Let U = {q, r, s, t, u, v, w, x, y, z}
  A = {r, s, t, i, v} B = {t, v, x}

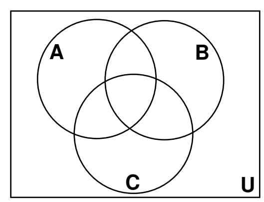
  - Complete the Venn Diagram to represent U, A, and B



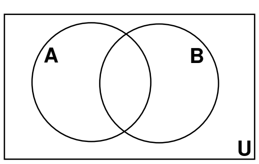
- Shade the Diagram for: $A \cap B$



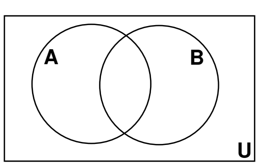
- Shade the Diagram for: $(A' \cap B') \cap C$



- Shade the Diagram for: $(A \cap B)'$



- Shade the Diagram for: $A' \cup B'$



Did we get these last two correct?

De Morgan's Laws

- **De Morgan's Laws**. For any sets A and B
  - $(A \cap B)' = A' \cup B'$
  - $(A \cup B)' = A' \cap B'$
- Using A, B, C, ∩, ∪, −, and ′, give a symbolic description of the shaded area in each of the following diagrams. Is there more than one way to describe each?