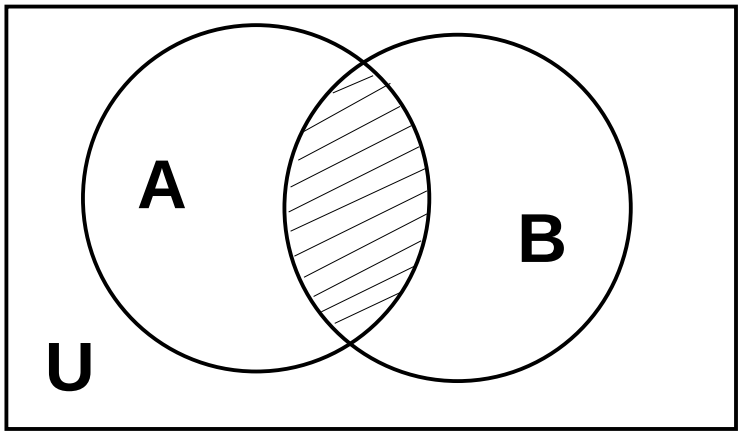
Sec 2.3 Set Operations & Cartesian Products

- **Intersection** of sets: $A \cap B$ is the set of elements common to both: $A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$



- Find the intersections of the following sets:

$\{ a, b, c \}$ and $\{ b, f, g \}$ _____

$\{ a, b, c \}$ and $\{ a, b, c \}$ _____

$\{ a, b, c \}$ and $\{ a, b, z \}$ _____

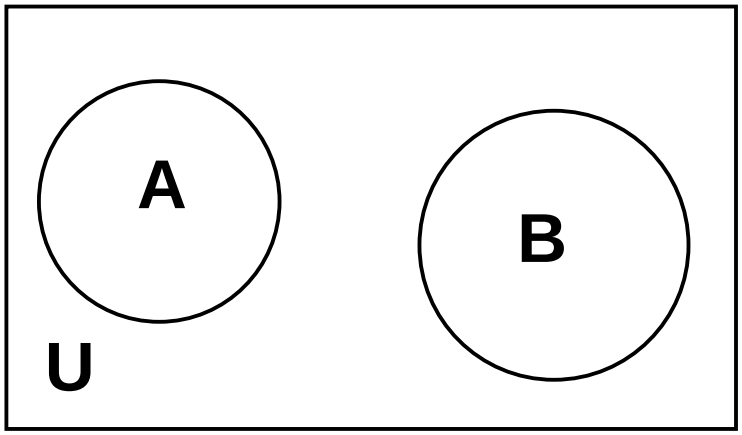
$\{ a, b, c \}$ and $\{ x, y, z \}$ _____

$\{ a, b, c \}$ and $\emptyset$ _____

Disjoint Sets

- **Disjoint** sets: two sets which have no elements in common.

  I.e., their intersection is empty: $A \cap B = \emptyset$



- Are the following sets disjoint?

  { a, b, c } and { d, e, f, g } _____

  { a, b, c } and { a, b, c } _____

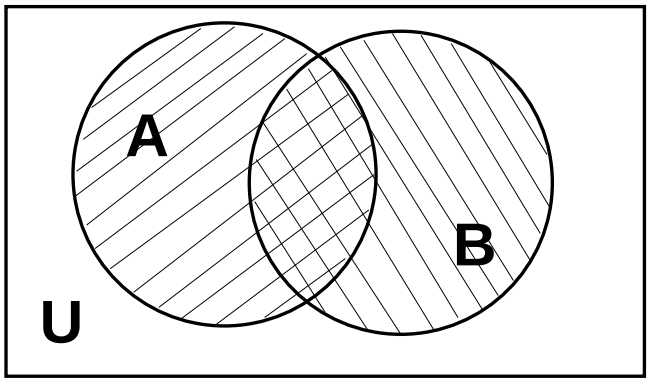
  { a, b, c } and { a, b, z } _____

  { a, b, c } and { x, y, z } _____

  { a, b, c } and ∅ _____

Set Union

- **Union** of sets: $A \cup B$ is the set of elements belonging to either of the sets: $A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$



Note: an element in the union of sets A and B may be a member of A, a member of B, *or* a member of ***both*** sets.

- Find the unions of the following sets:

$\{ a, b, c \}$ and $\{ b, f, g \}$ _____

$\{ a, b, c \}$ and $\{ a, b, c \}$ _____

$\{ a, b, c \}$ and $\{ a, b, z \}$ _____

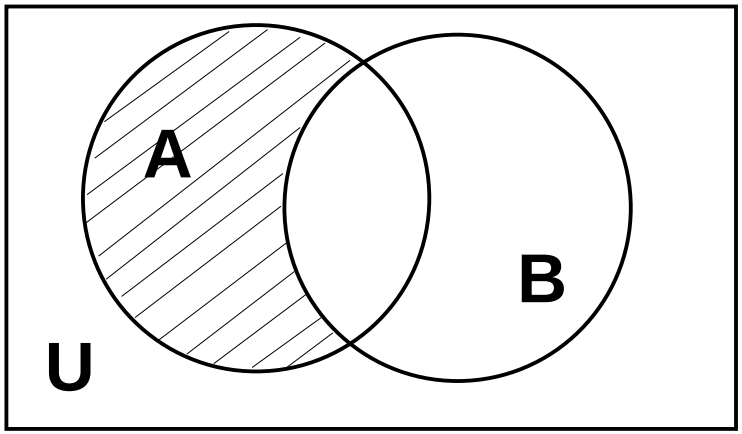
$\{ a, b, c \}$ and $\{ x, y, z \}$ _____

$\{ a, b, c \}$ and $\emptyset$ _____

Set Difference

- **Difference** of two sets: $A - B$ is the set of all elements belonging to set A and *not* to set B.

$$A - B = \{ x \mid x \in A \text{ and } x \notin B \}$$



$$\{ 1, 2, 3, 4, 5 \} - \{ 2, 4, 6 \} = \{ 1, 3, 5 \}$$

but $\{ 2, 4, 6 \} - \{ 1, 2, 3, 4, 5 \} = \{ 6 \}$

Note: $x \notin B \rightarrow x \in B'$ (the complement of B)

$$\begin{aligned} \text{Thus, } A - B &= \{ x \mid x \in A \text{ and } x \notin B \} \\ &= \{ x \mid x \in A \text{ and } x \in B' \} \\ &= A \cap B' \end{aligned}$$

- Given the sets:

  $U = \{1, 2, 3, 4, 5, 6, 9\}$
  $A = \{1, 2, 3, 4\}$
  $B = \{2, 4, 6\}$
  $C = \{1, 3, 6, 9\}$

  Find each of these sets:

  - $A \cup B =$

  - $A \cap B =$

  - $A \cap U =$

  - $A \cup U =$

$U = \{1, 2, 3, 4, 5, 6, 9\}$
$A = \{1, 2, 3, 4\}$
$B = \{2, 4, 6\}$
$C = \{1, 3, 6, 9\}$

- $A' =$
- $A' \cap B =$
- $A' \cup B =$
- $A \cup B \cup C =$
- $A \cap B \cap C =$

- Describe each of the following sets in words:
  - $A' \cup B' =$
  - $A' \cap B' =$
  - $A \cap (B \cup C)$
  - $(A' \cup C) \cap B$

- Given the sets:

  $U = \{1, 2, 3, 4, 5, 6, 7\}$

  $A = \{1, 2, 3, 4, 5, 6\}$

  $B = \{2, 3, 6\}$

  $C = \{3, 5, 7\}$

  Find each set:

  - $A - B =$

  - $B - A =$

  - $(A - B) \cup C' =$

  Note, in general, $A - B \neq B - A$

Ordered Pairs

- **Ordered Pair**: a group of two objects designated as **first** and **second** components.

  In the **ordered pair** (a, b):
  a is called the **first component**
  b is called the **second component**

- In general (a, b) ≠ (b, a), so **order** is **important**!

- Two ordered pairs (a, b) and (c, d) are **equal**
  provided a = c and b = d

  (1, 3) = (1, 3) (1, 3) ≠ (3, 1)
  (4, 9) = (4, 9) (9, 4) ≠ (4, 9)
  (2+2, 3×3) = (2×2, 6+3)

- Sets can contain ordered pairs:
  { (–3, 3), (–12, –6), (13, 29), (8, 7) }
  { (1, 3), (2, 6), (3, 9), . . . }

Cartesian Products

- The **Cartesian product** of sets A and B is:

$$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$$

The Cartesian product of $\{a, b, c\} \times \{1, 2\}$ =

$$\{ (a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2) \}$$

The Cartesian product of $\{1, 2\} \times \{a, b, c\}$ =

$$\{ (1, a), (1, b), (1, c), (2, a), (2, b), (2, c) \}$$

What's the difference between the two resulting sets above?

If set $A = \{x, y, z\}$, what is $A \times A$?

Cardinality of Cartesian Products

- If set A has cardinality 5 and set B has cardinality 4, what is the cardinality of $A \times B$?

  Of $B \times A$?

  If $|A| = n$ and $|B| = m$, what is $|A \times B|$?

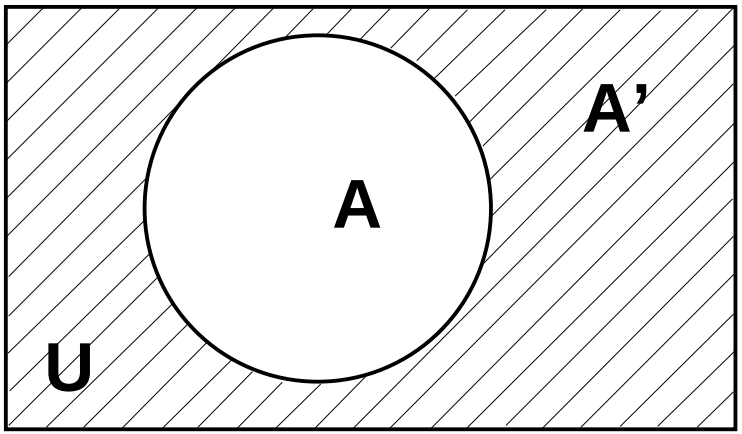
Set Operations

- Finding intersections, unions, differences, Cartesian products, and complements of sets are examples of **set operations**

- An **operation** is a rule or procedure by which one or more objects are used to obtain another object (usually a set or number).

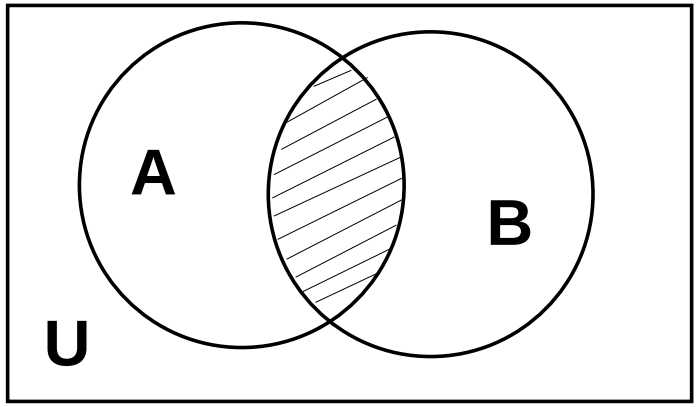
- **Common Set Operations**
  Let A and B be any sets, with U the universal set.

  - **Complement** of A is:
    $A' = \{x \mid x \in U \text{ and } x \notin A\}$

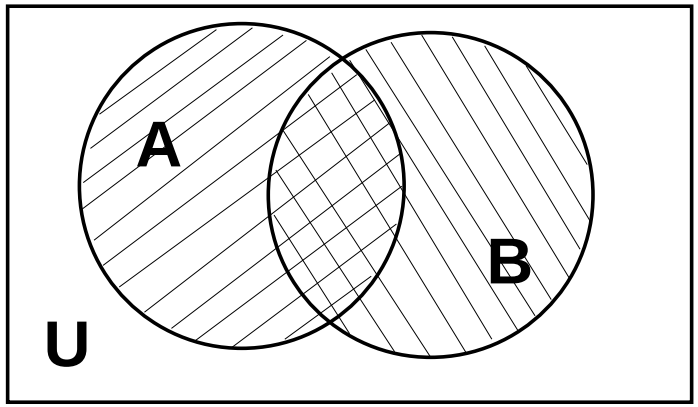


Set Intersection and Union

- **Intersection** of A and B is: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

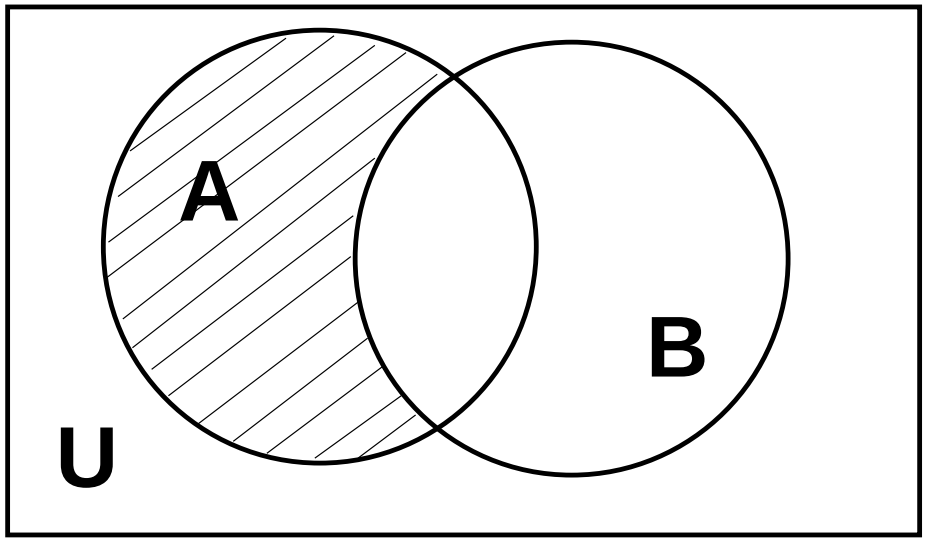


- **Union** of A and B is: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$



Set Difference

- **Difference** of A and B is: $A - B = \{x \mid x \in A \text{ and } x \notin B\}$



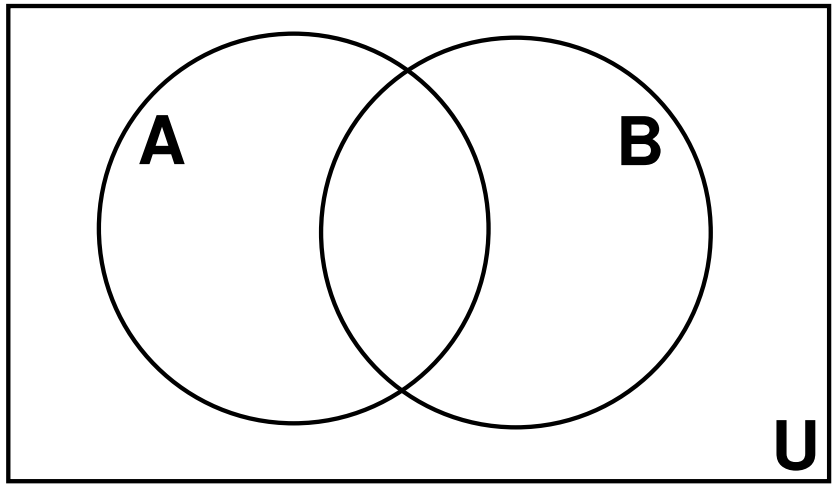
Cartesian Product

- The **Cartesian product** of A and B is:

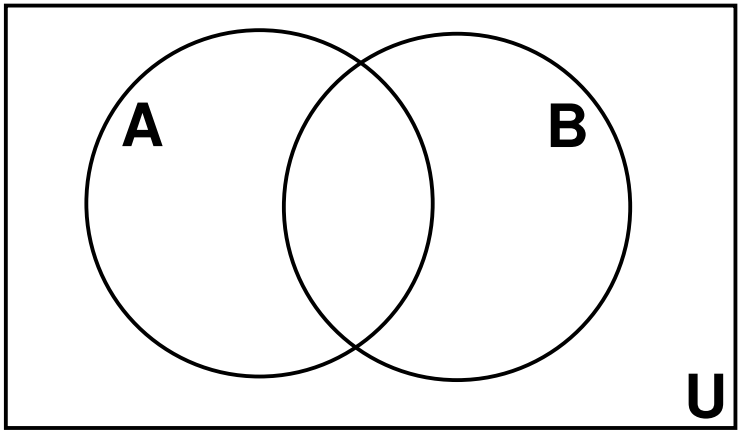
$$A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$$

- Let U = {q, r, s, t, u, v, w, x, y, z}
  A = {r, s, t, i, v} B = {t, v, x}

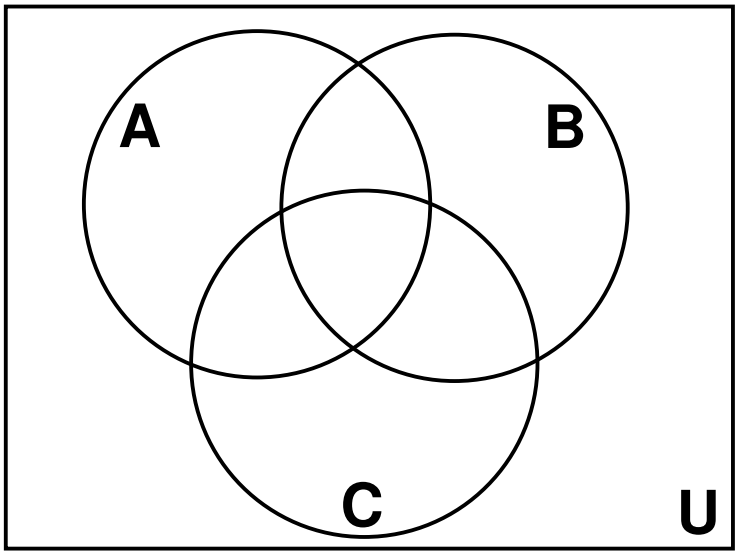
  - Complete the Venn Diagram to represent U, A, and B



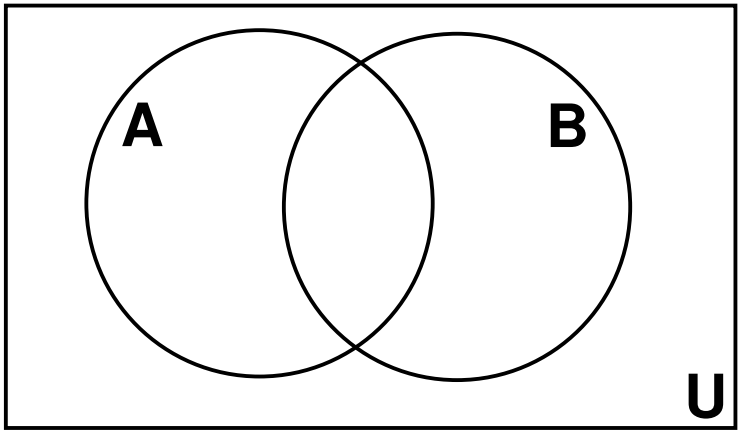
- Shade the Diagram for: $A \cap B$



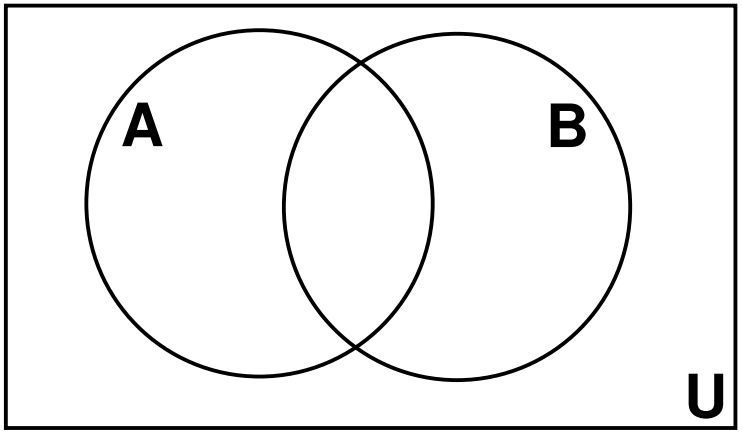
- Shade the Diagram for: $(A' \cap B') \cap C$



- Shade the Diagram for: $(A \cap B)'$



- Shade the Diagram for: $A' \cup B'$



Did we get these last two correct?

De Morgan's Laws

- **De Morgan's Laws**. For any sets A and B
  - $(A \cap B)' = A' \cup B'$
  - $(A \cup B)' = A' \cap B'$

- Using A, B, C, $\cap$, $\cup$, $-$, and $'$, give a symbolic description of the shaded area in each of the following diagrams. Is there more than one way to describe each?