Sec 2.5 Infinite Sets & Their Cardinalities—Review

- **Cardinality**:
- **One-to-one (1-1) Correspondence**:
- $\aleph_0$, Aleph-naught or Aleph-null:
- If we can show a 1-1 correspondence between some set, *A*, and the natural numbers, we say that *A* also has cardinality $\aleph_0$.
- Thus to show a set has cardinality $\aleph_0$, we need to find a 1-1 correspondence between the set and $\mathbb{N}$, the set of natural numbers

Show the Cardinality of each set is $\aleph_0$

- the positive even integers, {2, 4, 6, . . .}
- The negative integers, {−1, −2, −3, . . .}
- The positive odd integers, {1, 3, 5, . . .}

Chapter 3. Introduction to Logic

- How can we draw logical conclusions from the facts we have at hand?
- How can we know when someone is making a valid argument?
- How can we determine the truth or falsity of statements with many parts?
- Why should we care about these things?

Sec 3.1 Statements and Quantifiers

- The Greek philosopher Aristotle was one of the first to attempt to codify "right thinking," or **irrefutable reasoning processes**.
- His famous syllogisms provided **patterns for argument structures that always gave correct conclusions given correct premises**.

For example: SOCRATES IS A MAN
ALL MEN ARE MORTAL
THEREFORE SOCRATES IS MORTAL.

Laws of Logic

- These laws of thought were supposed to govern the operation of the mind, and initiated the field of **logic**.
- **Logic** is based on **knowledge** and **reasoning**.
- We have some facts and from them draw **conclusions**, perhaps about our next course of **action** or to extend our **knowledge**.
- **Logic** consists of:
  1. a formal language (such as mathematics) in which knowledge can be expressed
  2. a means of carrying out **reasoning** in such a language

Logic Values

- **Logic values**: `True` and `False`
- **Statement**: a declarative (factual) sentence that is either TRUE or FALSE, but **not** both. Examples:
  - Salt lowers the melting point of ice.
  - 3 + 5 = 9
  - The outdoor temperature in Charleston today is 26° F
- Some sentences are not statements. For example:
  - The best way to melt ice is to move to Florida.
  - Get outta here!
  - Are you feeling okay today?
  - This sentence is false.

  Opinions, commands, questions, and ***paradoxes*** are not statements.

Compound Statements

- **Compound Statement**: a statement formed by combining two or more statements.

  Ex: You are my student and we are studying mathematics.

- **Component Statements**: the statements used to form a compound statement.

  In the above example, **You are my student** and **we are studying mathematics** are the two component statements.

Logical Connectives

- **Logical Connectives** (or **connectives**) are used to form compound statements: **and, or, not**, and **if ... then**
  - Today it is sunny **and** there is a slight breeze.
  - Yesterday it was raining **or** snowing.
  - The back tire on my bicycle is**n't** flat.
  - **If** the moon is made of green cheese, **then** so am I.

Negation

- **Negation**: an opposite statement.
  The **negation** of a TRUE statement is FALSE
  The **negation** of a FALSE statement is TRUE

Statement	Negation
My car is red.	My car is not red.
My car is not red.	My car is red.
The pen is broken.	The pen isn't broken.
Four is less than nine.	Four is not less than nine (i.e., $4 \geq 9$).
$a \geq b$	$a < b$

Remember: a negation must have the **opposite** truth value from the original statement.

Symbolic Logic

- **Symbolic logic** uses **letters** to represent statements, and **symbols** for words such as **and, or**, and **not**.

- The letters used are often p and q. They will represent **statements**.

Connective	Symbol	Statement Type
and	$\wedge$	Conjunction
or	$\vee$	Disjunction
not	$\sim$	Negation

Symbolic Logic to English Statements

- If p represents "**Today is Thursday**," and
  q represents "**It is sunny**,"
  translate each of the following into an English sentence:

1. $p \wedge q$
2. $p \vee q$
3. $\sim p \wedge q$
4. $p \vee \sim q$
5. $\sim (p \vee q)$
6. $\sim p \wedge \sim q$

Quantifiers

- **Quantifiers** in mathematics indicate **how many** cases of a particular situation exist.

- **Universal Quantifier**: indicates the property applies to **all** or **every** case. Universal quantifiers are:
  all, each, every, no, and **none**
  - **All** athletes must attend the meeting.
  - **Every** math student enjoys the subject.
  - There are **no** groundhogs which are purple.

- **Existential Quantifier**: indicates the property applies to **one or more** cases. Existential quantifiers include:
  some, **there exists**, and **(for) at least one**
  - **Some** athletes must attend the meeting.
  - **At least one** math student enjoys the subject.
  - **There exists** a groundhog which is brown.

Negating Quantifiers

- Care must be taken when negating statements with quantifiers.

Negations of Quantified Statements	
Statement	**Negation**
All do	**Some do not** (Equivalently: **Not all do**)
Some do	**None do** (Equivalently: **All do not**)

Practice with Negation

- What is the negation of each statement?

1. **Some** people wear glasses.
2. **Some** people **do not** wear glasses.
3. **Nobody** wears glasses.
4. **Everybody** wears glasses.
5. **Not everybody** wears glasses.

Practice with Quantifiers

- TRUE or FALSE?

1. **All** Whole numbers are Natural numbers.
2. **Some** Whole number **isn't** a Natural number.
3. **Every** Integer is a Natural number.
4. **No** Integer is a Natural number.
5. **Every** Natural number is a Rational number.
6. **There exists** an Irrational number that **is not** Real.