Sec 3.2 Truth Tables and Equivalent Statements

Conjunction: Given two statements p and q, their conjunction is $p \wedge q$.

Conjunction Truth Table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction Examples
Determine the truth values (T/F):

1. ___ Today is Tuesday **and** it is sunny.
2. ___ Today is Wednesday **and** it is sunny.
3. ___ The moon is made of green cheese **and** some violets are blue.
4. ___ It is daytime here **and** there are **not** 1000 desks in this classroom.
5. ___ This course is **MAT 1160** **and** we are learning calculus.
6. ___ This course is **MAT 4870** **and** we are learning physics.
7. ___ $3 < 5 \wedge 5 < 3$
8. ___ $3 < 5 \wedge 5 < 8$

Disjunction

Disjunction. Given two statements p and q, their (inclusive) **disjunction** is $p \vee q$.

Inclusive disjunctions are TRUE if either or both components are TRUE.

Disjunction Truth Table

p	q	$p \vee q$
T	**T**	**T**
T	**F**	**T**
F	**T**	**T**
F	**F**	**F**

Disjunction Examples
Determine the truth values:

1. ___ Today is Tuesday **or** it is sunny.

2. ___ Today is Wednesday **or** it is sunny.

3. ___ The moon is made of green cheese **or** some violets are blue.

4. ___ It is daytime here **or** there are not 100 desks in this classroom.

5. ___ This course is **MAT 1160** **or** we are learning calculus.

6. ___ This course is **MAT 4870** **or** we are learning physics.

7. ___ $3 < 5 \vee 5 < 3$

8. ___ $3 < 5 \vee 5 < 8$

Mathematical Examples

Using `or`

Statement	Reason It's True
$7 \geq 7$	$7 = 7$
$8 \geq 5$	$8 > 5$
$-7 \leq -3$	$-7 < -3$
$-3 \leq -3$	$-3 = -3$

The Porsche & The Tiger

A prisoner must make a choice between two doors: behind one is a beautiful red Porsche, and behind the other is a hungry tiger. Each door has a sign posted on it, but only one sign is true.

Door #1. IN THIS ROOM THERE IS A PORSCHE AND IN THE OTHER ROOM THERE IS A TIGER.

Door #2. IN ONE OF THESE ROOMS THERE IS A PORSCHE AND IN ONE OF THESE ROOMS THERE IS A TIGER.

With this information, the prisoner is able to choose the correct door... Which one is it?

Negation

Negation. Given a statement p, its negation is $\sim p$.

Negation Truth Table

p	$\sim p$
T	**F**
F	**T**

Negation Examples
Determine the truth values

Assume p is TRUE, q is FALSE, and r is FALSE

1. ___ p
2. ___ $\sim p$
3. ___ q
4. ___ $\sim q$
5. ___ r
6. ___ $\sim r$
7. ___ $\sim p \wedge p$
8. ___ $p \vee \sim p$
9. ___ $p \wedge \sim q$
10. ___ $p \vee \sim q$
11. ___ $\sim p \wedge (q \vee \sim r)$
12. ___ $p \wedge (\sim q \vee r)$

More Examples
Determine the truth values

Let p represent the statement $3 > 2$
q represent the statement $5 < 4$
r represent the statement $3 \leq 8$

1. ___ p
2. ___ $\sim p$
3. ___ q
4. ___ $\sim q$
5. ___ r
5. ___ $\sim r$
7. ___ $\sim p \wedge q$
8. ___ $\sim (p \wedge q)$
9. ___ $\sim p \vee (\sim q \vee r)$
10. ___ $(\sim p \wedge r) \vee (\sim q \wedge \sim p)$

Yet More Examples

11. ___ For some real number x, $x > 2$ and $x < 8$
12. ___ There exists a real number b, $b < 8$ or $b > 2$
13. ___ For at least one real number y, $y < 8$ and $y > 12$
14. ___ There is a real number m, $m < 8$ or $m > 12$
15. ___ For all real numbers x, $x < 8$ and $x > 2$
16. ___ For every real number b, $b < 8$ or $b > 2$
17. ___ For all real numbers y, $y < 8$ and $y > 12$
18. ___ For every real number m, $m < 8$ or $m > 12$
19. ___ For every real number n, $n^2 > 0$
20. ___ For every real number n, $n^2 \geq 0$

Constructing Truth Tables

Construct a Truth Table for: $(\sim p \wedge q) \vee \sim q$

p	q	$(\sim p \wedge q) \vee \sim q$
T	T	
T	F	
F	T	
F	F	

Construct a Truth Table for: $p \wedge (\sim p \vee \sim q)$

p	q	$p \wedge (\sim p \vee \sim q)$
T	T	
T	F	
F	T	
F	F	

Construct the Truth Table

p	q	r	$\sim p \wedge (q \vee \sim r)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Construct the Truth Table

p	q	r	$(\sim p \wedge r) \vee (\sim q \wedge \sim p)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Some Notes of Interest

A logical statement having n component statements will have 2^n rows in its truth table.

Two statements are **equivalent** if they have the same truth value in **every** possible situation.

In other words, two statements are **equivalent** if their columns in the same truth table have the same truth values.

De Morgan's Laws

p	q	$\sim p \wedge \sim q$	$\sim (p \vee q)$
T	T		
T	F		
F	T		
F	F		

p	q	$\sim p \vee \sim q$	$\sim (p \wedge q)$
T	T		
T	F		
F	T		
F	F		