Sec 3.3 The Conditional & Circuits

Conditional statement: a compound statement that uses the connective **if** . . . **then**.

Conditional statements are also known as **implications**, and can be written as:

$p \rightarrow q$ (pronounced "p implies q")

The statement p is called the **antecedent**.

The statement q is called the **consequent**.

Conditional Examples

- **If** you are not home by midnight,
  (**then**) you'll be grounded.

- **If** he hits a home run,
  (**then**) he'll beat the old record.

- **If** you scratch my back,
  (**then**) I'll scratch yours.

- **If** you exceed the speed limit,
  (**then**) you'll get a ticket.

- The English are bad cooks.
  translation: **If** you are English, **then** you are a bad cook.

- College students are immature.
  translation: **If** you are a student, **then** you are immature.

Truth Table for Conditional Statements

There are four possible combinations of truth values for the two component statements

p	q	$p \rightarrow q$
T	T	?
T	F	?
F	T	?
F	F	?

Let's consider: ***If* you are not home by midnight, *then* you'll be grounded**.

Is the implication true when:

1. _____ You are not home by midnight and you are grounded
2. _____ You are not home by midnight but you are not grounded
3. _____ You are home by midnight but you are grounded
4. _____ You are home by midnight and you are not grounded.

Another Example

Let's consider: ***If*** **he hits a home run,**

then **he'll beat the old record.**

p	q	$p \rightarrow q$ T or F?
he hits a home run	he beats the old record	
he hits a home run	he doesn't beat the old record	
he doesn't hit a home run	he beats the old record	
he doesn't hit a home run	he doesn't beat the old record	

Another Example

How about: *If* you are English,

then you are a bad cook.

p	q	$p \rightarrow q$ T or F?
you are English	you are a bad cook	
you are English	you are not a bad cook	
you aren't English	you are a bad cook	
you aren't English	you are not a bad cook	

Another Example

And finally: *If* you are a college student,

then you are immature.

p	q	$p \rightarrow q$ T or F?
you are a college student	you are immature	
you are a college student	you aren't immature	
you aren't a college student	you are immature	
you aren't a college student	you aren't immature	

Truth Table for the Conditional

If p, then q

p	q	$p \rightarrow q$
T	T	T
T	F	**F**
F	T	T
F	F	T

If the moon is made of green cheese, . . .

If my name isn't < My name here > . . .

If I finish my homework, . . .

If I had a million dollars, . . .

If wishes were fishes, . . .

Notes

- $p \rightarrow q$ is **false only** when the antecedent is **true** and the consequent is **false**
- If the antecedent is **false**, then $p \rightarrow q$ is automatically **true**
- If the consequent is **true**, then $p \rightarrow q$ is automatically **true**

true or false?

____ **true** → (6 = 6)

____ **true** → (6 = 3)

____ **false** → (6 = 6)

____ **false** → (6 = 3)

____ (6 = 6) → true

____ (6 = 3) → true

____ (6 = 6) → false

____ (6 = 3) → false

Let p, q, and r be **false**

____ (p → q)

____ (∼ r → q)

____ (p → ∼ q)

____ (p → ∼ q) → (∼ r → q)

Truth Table: $(\sim p \rightarrow \sim q) \rightarrow (\sim p \wedge q)$

p	q	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$	$\sim p \wedge q$	$(\sim p \rightarrow \sim q) \rightarrow (\sim p \wedge q)$
T	T					
T	F					
F	T					
F	F					

Truth Table: $(p \rightarrow q) \rightarrow (\sim p \vee q)$

p	q	$p \rightarrow q$	$\sim p$	$\sim p \vee q$	$(p \rightarrow q) \rightarrow (\sim p \vee q)$
T	T				
T	F				
F	T				
F	F				

Tautology: a statement that is **always** true, no matter what the truth values of the components.

Truth Table: $p \vee \sim p$

p	$\sim p$	$p \vee \sim p$
T		
F		

Truth Table: $p \rightarrow p$

p	$\sim p$	$p \rightarrow p$
T		
F		

Truth Table: $(\sim p \vee \sim q) \rightarrow \sim (q \wedge p)$

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$\sim (q \wedge p)$	$(\sim p \vee \sim q) \rightarrow \sim (q \wedge p)$
T	T					
T	F					
F	T					
F	F					

Truth Table: Negation of $p \rightarrow q$

p	q	$p \rightarrow q$	$\sim (p \rightarrow q)$	$\sim q$	$p \wedge \sim q$
T	T				
T	F				
F	T				
F	F				

Recall: You are not home by midnight,
you are not grounded...
the only false result, and thus the negation

The negation of $p \rightarrow q$ is $p \wedge \sim q$

Write the negation of each statement

- *If* you are not home by midnight, *then* you'll be grounded.
- *If* he hits a home run, (*then*) he'll beat the old record.
- *If* you scratch my back, (*then*) I'll scratch yours.
- *If* you exceed the speed limit, (*then*) you'll get a ticket.

- *If* it's Smucker's, it's got to be good!

- *If* that is an authentic Persian rug, I'll be surprised.

- The English are bad cooks.
  translation: **If** you are English, **then** you are a bad cook.

- College students are immature.
  translation: **If** you are a student, **then** you are immature.

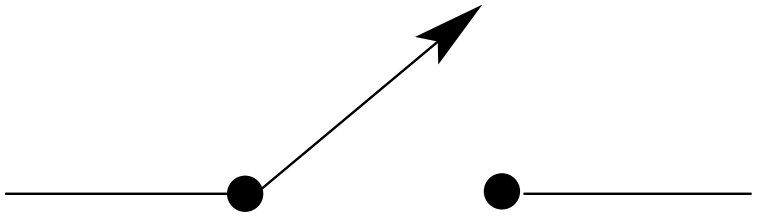
$p \rightarrow q$ is equivalent to $\sim p \vee q$

Rewrite as a statement that doesn't use the *if. . . then* connective

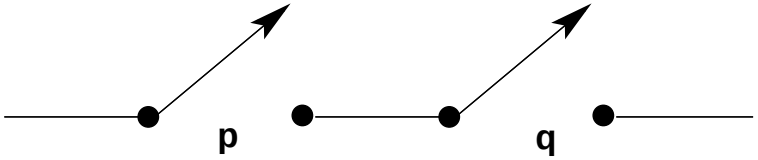
- *If* you are not home by midnight, *then* you'll be grounded.
- *If* he hits a home run, (*then*) he'll beat the old record.
- *If* you scratch my back, (*then*) I'll scratch yours.
- *If* you exceed the speed limit, (*then*) you'll get a ticket.

- *If* it's Smucker's, it's got to be good!
- *If* that is an authentic Persian rug, I'll be surprised.
- *If* you give your plants tender, loving care, they flourish.
- *If* she doesn't, he will.
- *If* you are a student, then you are immature.

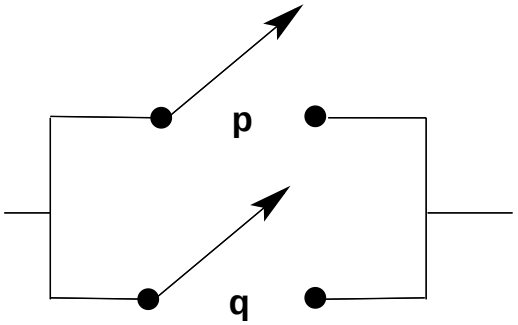
CIRCUITS
When will current flow through the switch and wire?



A Switch — On or Off

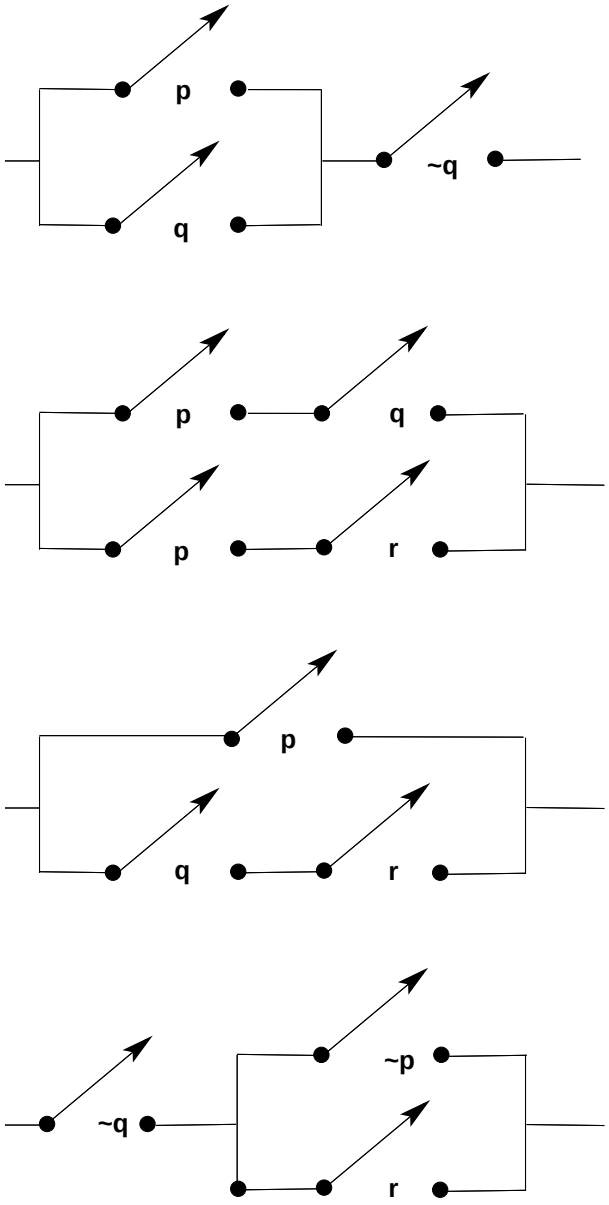


A Series Circuit



A Parallel Circuit

What is the corresponding logic statement?



Equivalent Statements Used to Simplify Circuits

$$
\begin{aligned}
p \vee T &\equiv T \\
p \wedge F &\equiv F \\
p \vee \sim p &\equiv T \\
p \wedge \sim p &\equiv F \\
p \vee p &\equiv p \\
p \wedge p &\equiv p \\
\sim (p \wedge q) &\equiv \sim p \vee \sim q \\
\sim (p \vee q) &\equiv \sim p \wedge \sim q \\
p \vee (q \wedge r) &\equiv (p \vee q) \wedge (p \vee r) \\
p \wedge (q \vee r) &\equiv (p \wedge q) \vee (p \wedge r) \\
p \rightarrow q &\equiv \sim q \rightarrow \sim p \\
p \rightarrow q &\equiv \sim p \vee q
\end{aligned}
$$

Draw a circuit for: $p \vee (\sim q \wedge \sim r)$

Draw a circuit for $p \rightarrow (q \wedge \sim r)$.
(Rewrite it first)