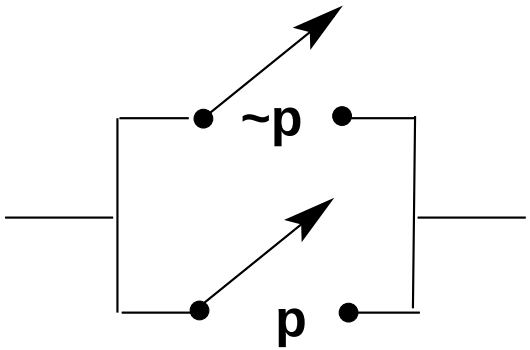
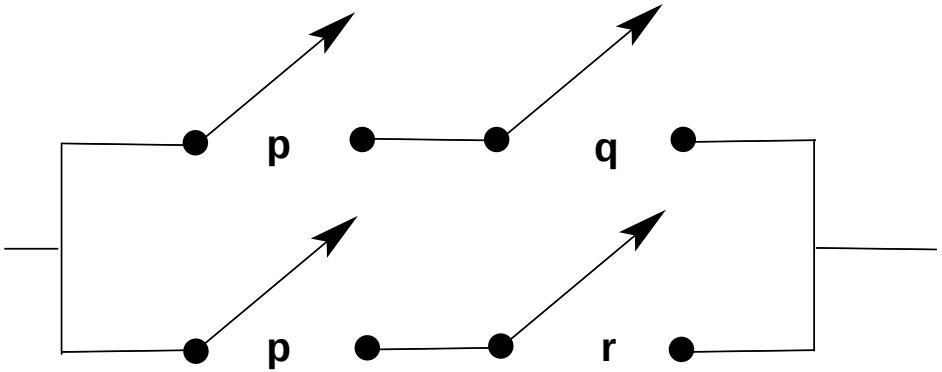
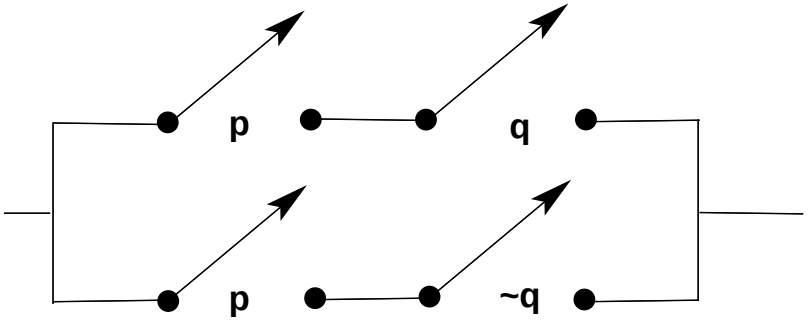
# Sec 3.3 Review

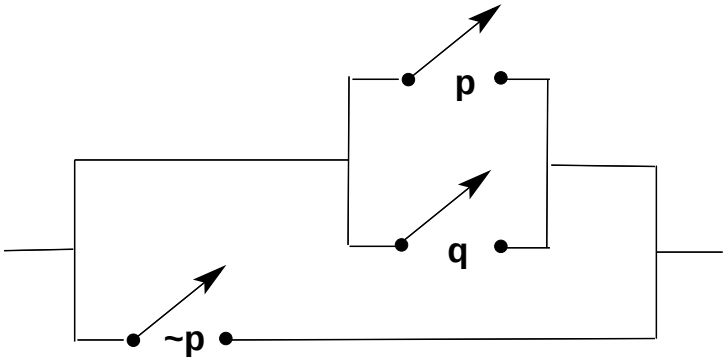
- A **conditional** statement uses implication ($\rightarrow$) or **`if...else`**
- $p \rightarrow q$ is **`false only`** when p is **`true`** and q is **`false`**.
- $p \rightarrow q$ is equivalent to $(\sim p \vee q)$
- The negation of $p \rightarrow q$ is $(p \wedge \sim q)$
- We can use Truth Tables to show two conditional expressions are equivalent (their truth values will be the same)
- A **tautology** is a statement which is always TRUE.
- Circuits in **series** correspond to conjunctions (**`and`**s)
- Circuits in **parallel** correspond to disjunctions (**`or`**s)
- Some circuits can be simplified.

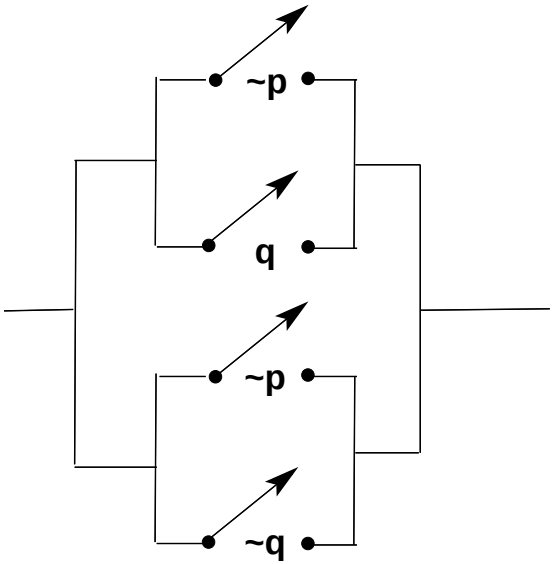
# Rewrite as Boolean Expressions and Simplify



# Simplify and draw circuits

1. $p \wedge (q \vee \sim p)$

2. $(p \vee q) \wedge (\sim p \wedge \sim q)$

3. $[(p \vee q) \wedge r] \wedge \sim p$

# Sec 3.4 More on the Conditional: Converse, Inverse, and Contrapositive

| | | |
|---|---|---|
| **Direct Statement** | $p \rightarrow q$ | If p, then q |
| **Converse** | $q \rightarrow p$ | If q, then p |
| **Inverse** | $\sim p \rightarrow \sim q$ | If not p, then not q |
| **Contrapositive** | $\sim q \rightarrow \sim p$ | If not q, then not p |

Let p = "they stay" and q = "we leave"

**Direct Statement** ($p \rightarrow q$):

**Converse**:

**Inverse**:

**Contrapositive**:

Let p = "I surf the web" and q = "I own a PC"

**Direct Statement** ($p \rightarrow q$):

**Converse**:

**Inverse**:

**Contrapositive**:

# Equivalent Conditionals

| | | Direct | Converse | Inverse | Contrapositive |
|---|---|---|---|---|---|
| | | $p \rightarrow q$ | $q \rightarrow p$ | $\sim p \rightarrow \sim q$ | $\sim q \rightarrow \sim p$ |
| p | q | $\sim p \vee q$ | | | |
| T | T | T | T | | |
| T | F | F | T | | |
| F | T | T | F | | |
| F | F | T | T | | |

$\square \rightarrow \triangle$ is equivalent to $\sim \square \vee \triangle$

$$\sim \square \vee \triangle \equiv \square \rightarrow \triangle$$

$$\square \vee \triangle \equiv \sim \square \rightarrow \triangle$$

# Tricky Questions

For $p \vee q$, write each of the following:

**Direct Statement**:

**Converse**:

**Inverse**:

**Contrapositive**:

# Alternate Conditional Forms

## Common translations of $p \rightarrow q$

| | |
|---|---|
| If p, then q | p is sufficient for q |
| If p, q | q is necessary for p |
| p implies q | All p's are q's |
| p only if q | q if p |

These translations do not in any way depend upon the truth value of $p \rightarrow q$.

**Translations of**: "**If you get home late, then you are grounded**"

You are grounded if you get home late.

Getting home late is sufficient for you to get grounded.

Getting grounded is necessary when you get home late.

Getting home late implies that you are grounded.

## Rewrite as `if...then` statements & give some alternatives for

You'll be sorry if I go.

Today is Thursday only if yesterday was Wednesday.

All nurses wear white shoes.

A stitch in time saves nine.

Rolling stones gather no moss.

Birds of a feather flock together.

Let p = "a triangle is equilateral" and q = "a triangle has three equal sides"

Write in symbols:

A triangle is equilateral **if** it has three equal sides.

A triangle is equilateral **only if** it has three equal sides.

One of the following statements is **not** equivalent to the others... Which one is it?

1. r only if s
2. r implies s
3. If r, then s
4. r is necessary for s

# Consistent or Contrary?

Two statements about the same object are:

**consistent** — if they are both true.

**contrary** — if they cannot both be true.

1. The car is a Chevy. The car is a Toyota.

2. Elvis is alive. Elvis is dead.

3. The animal has four legs. The animal is a dog.

4. The cake is chocolate. The cake has two layers.

5. The clock is broken. The clock has the right time.

6. The math class meets at noon. The math class lasts 50 minutes.

7. The number is an integer. The number is irrational.

8. The punch is pink. The punch has juice in it.

9. President Bush is a Republican. President Bush is a Democrat.

10. The sofa is soft. The sofa is blue.

11. The plant is blooming. The plant is dead.

12. The dog ate my homework. The dog bites.

13. That rock is igneous. That rock is sedimentary.

14. That bird is a robin. That bird is blue.

# Biconditionals

**Biconditional**: compound statement of the form **p if and only if q**, written $p \leftrightarrow q$ or p **`iff`** q.

$$p \leftrightarrow q \text{ is equivalent to } (p \rightarrow q) \wedge (q \rightarrow p)$$

or

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

**Truth Table for** $p \leftrightarrow q$

| p | q | $p \leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

# True or False?

A biconditional is **true** only when **both** statements are **true** or **both** statements are **false**.

**True** or **False**: 5 = 9 – 4 if and only if 8 + 2 = 10

**True** or **False**: Clinton was president IFF Carter wasn't president.

**True** or **False**: IBM sells computers IFF Pizza Hut sells Big Macs.

**True** or **False**: 8 + 7 ≠ 15 IFF 3 × 5 ≠ 9.

# In Summary

| | | |
|---|---|---|
| $\sim p$ | **negation** of p | truth value is opposite of p |
| $p \wedge q$ | **conjunction** | true only when both p and q are true |
| $p \vee q$ | **disjunction** | false only when both p and q are false |
| $p \rightarrow q$ | **conditional** | false only when p is true and q is false |
| $p \leftrightarrow q$ | **biconditional** | true only when p and q have the *same* truth value. |