Sec 3.6 Analyzing Arguments with Truth Tables

Some arguments are more easily analyzed to determine if they are valid or invalid using **Truth Tables** instead of **Euler Diagrams**.

One example of such an argument is:

```
If it rains, then the squirrels hide.
It is raining.
-------------------------------------
The squirrels are hiding.
```

Notice that in this case, there are no universal quantifiers such as **all**, **some**, or **every**, which would indicate we could use Euler Diagrams.

To determine the validity of this argument, we must first identify the **component statements** found in the argument. They are:

```
p = it rains / is raining
q = the squirrels hide / are hiding
```


Rewriting the Premises and Conclusion

Premise 1: $p \rightarrow q$
Premise 2: p

Conclusion: q

Thus, the argument converts to:

$$((p \rightarrow q) \wedge p) \rightarrow q$$

With Truth Table:

p	q	$((p \rightarrow q) \wedge p) \rightarrow q$
T	T	
T	F	
F	T	
F	F	

Are the squirrels hiding?

Testing Validity with Truth Tables

1. Break the argument down into **component statements**, assigning each a letter.
2. Rewrite the premises and conclusion **symbolically**.
3. Rewrite the argument as an **implication** with the **conjunction** of all the premises as the antecedent, and the conclusion as the consequent.
4. Complete a Truth Table for the resulting conditional statement. If it is a **tautology**, then the argument is **valid**; otherwise, it's **invalid**.

Recall

Direct Statement	$p \rightarrow q$
Converse	$q \rightarrow p$
Inverse	$\sim p \rightarrow \sim q$
Contrapositive	$\sim q \rightarrow \sim p$

Which are equivalent?


```
If you come home late, then you are grounded.
You come home late.
---------------------------------------------
You are grounded.

p =

q =
```

Premise 1:

Premise 2:

Conclusion:

Associated Implication:

p	q	
T	T	
T	F	
F	T	
F	F	

Are you grounded?

Modus Ponens — The Law of Detachment

Both of the prior example problems use a pattern for argument called **modus ponens**, or **The Law of Detachment**.

$p \rightarrow q$
p
------
q

or

$$((p \rightarrow q) \wedge p) \rightarrow q$$

Notice that **all** such arguments lead to **tautologies**, and therefore are **valid**.

```
If a knee is skinned, then it will bleed.
The knee is skinned.
----------------------------------------
It bleeds.

p =

q =
```

Premise 1:

Premise 2:

Conclusion:

Associated Implication:

p	q	
T	T	
T	F	
F	T	
F	F	

(Modus Ponens) – ***Did the knee bleed?***

Modus Tollens — Example

```
If Frank sells his quota, he'll get a bonus.
Frank doesn't get a bonus.
---------------------------------------
Frank didn't sell his quota.

p =

q =
```

Premise 1: $p \rightarrow q$ Premise 2: $\sim q$ Conclusion: $\sim p$

Thus, the argument converts to: $((p \rightarrow q) \wedge \sim q) \rightarrow \sim p$

p	q	$((p \rightarrow q) \wedge \sim q) \rightarrow \sim p$
T	T	
T	F	
F	T	
F	F	

Did Frank sell his quota or not?

Modus Tollens

An argument of the form:

$$\begin{array}{c} p \rightarrow q \\ \sim q \\ \hline \sim p \end{array}$$

or

$$((p \rightarrow q) \wedge \sim q) \rightarrow \sim p$$

is called **Modus Tollens**, and represents a **valid** argument.

Modus Tollens — Example II

```
If the bananas are ripe, I'll make banana bread.
I don't make banana bread.
---------------------------------------
The bananas weren't ripe.

p =

q =
```

Premise 1: $p \rightarrow q$ Premise 2: $\sim q$ Conclusion: $\sim p$

Thus, the argument converts to: $((p \rightarrow q) \wedge \sim q) \rightarrow \sim p$

p	q	$((p \rightarrow q) \wedge \sim q) \rightarrow \sim p$
T	T	
T	F	
F	T	
F	F	

Were the bananas ripe or not?

Fallacy of the Inverse — Example

```
If it rains, I'll get wet.
It doesn't rain.
---------------------------------------
I don't get wet.

p =

q =
```

Premise 1: $p \rightarrow q$ Premise 2: $\sim p$ Conclusion: $\sim q$

Thus, the argument converts to: $((p \rightarrow q) \wedge \sim p) \rightarrow \sim q$

p	q	$((p \rightarrow q) \wedge \sim p) \rightarrow \sim q$
T	T	
T	F	
F	T	
F	F	

Did I get wet?

Fallacy of the Inverse

An argument of the form:

$$\begin{array}{c} p \rightarrow q \\ \sim p \\ \hline \sim q \end{array}$$

or

$$((p \rightarrow q) \wedge \sim p) \rightarrow \sim q$$

is called the **Fallacy of the Inverse**, and represents an **invalid** argument.

Fallacy of the Inverse — Example II

```
If you're good, you'll be rewarded.
You aren't good.
---------------------------------------
You aren't rewarded.

p =

q =
```

Premise 1: $p \rightarrow q$ Premise 2: $\sim p$ Conclusion: $\sim q$

Thus, the argument converts to: $((p \rightarrow q) \wedge \sim p) \rightarrow \sim q$

p	q	$((p \rightarrow q) \wedge \sim p) \rightarrow \sim q$
T	T	
T	F	
F	T	
F	F	

Are you rewarded?

Another Type of (Invalid) Argument

```
If it rains, then the squirrels hide.
The squirrels are hiding.
-------------------------------------
It is raining.

p = it rains / is raining
q = the squirrels hide / are hiding
```

Premise 1: $p \rightarrow q$ Premise 2: q Conclusion: p

Thus, the argument converts to: $((p \rightarrow q) \wedge q) \rightarrow p$

p	q	$((p \rightarrow q) \wedge q) \rightarrow p$
T	T	
T	F	
F	T	
F	F	

(**Fallacy of the Converse**) — ***Is it raining?***

Fallacy of the Converse

An argument of the form:

$$\begin{array}{c} p \rightarrow q \\ q \\ \hline p \end{array}$$

or

$$((p \rightarrow q) \wedge q) \rightarrow p$$

is sometimes called the **Fallacy of the Converse**, and represents an **invalid** argument.


```
If you like me, then I like you.
I like you.
-------------------------------------
You like me.

p =

q =
```

Premise 1:

Premise 2:

Conclusion:

Associated Implication:

p	q	
T	T	
T	F	
F	T	
F	F	

(**Fallacy of the Converse**) — ***Do you like me?***

Disjunctive Syllogism — Example

```
Either you get home by midnight, or you're grounded.
You aren't grounded.
---------------------------------------
You got home by midnight.

p =

q =
```

Premise 1: $p \vee q$ Premise 2: $\sim q$ Conclusion: p

Thus, the argument converts to: $((p \vee q) \wedge \sim q) \rightarrow p$

p	q	$((p \vee q) \wedge \sim q) \rightarrow p$
T	T	
T	F	
F	T	
F	F	

Did you get home by midnight?

Disjunctive Syllogism

An argument of the form:

$$\begin{array}{c} p \vee q \\ \sim q \\ \hline p \end{array}$$

or

$$((p \vee q) \wedge \sim q) \rightarrow p$$

is called a **Disjunctive Syllogism**, and represents a **valid** argument.

Disjunctive Syllogism — Example II

```
Either this milk has soured, or I have the flu.
The milk has not soured.
--------------------------------------
I have the flu.

p =

q =
```

Premise 1: $p \vee q$ Premise 2: $\sim p$ Conclusion: q

Thus, the argument converts to: $((p \vee q) \wedge \sim p) \rightarrow q$

p	q	$((p \vee q) \wedge \sim p) \rightarrow q$
T	T	
T	F	
F	T	
F	F	

Do I have the flu?

Reasoning by Transitivity — Example

```
If you're kind to people, you'll be well liked.
If you're well liked, you'll get ahead in life.
---------------------------------------------------
If you're kind to people, you'll get ahead in life.

p = you're kind to people
q = you're well liked
r = you get ahead in life
```

Premise 1: $p \rightarrow q$ Premise 2: $q \rightarrow r$ Conclusion: $p \rightarrow r$

Thus, the argument converts to: $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

p	q	r	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Reasoning by Transitivity

An argument of the form:

$p \rightarrow q$
$q \rightarrow r$

$p \rightarrow r$

or

$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

is called **Reasoning by Transitivity**, and represents a **valid** argument.

Reasoning by Transitivity — Example

```
If it purrs, it's a cat.
If it's a cat, I'm allergic to it.
----------------------------------
If it purrs, I'm allergic to it.

p =

q =

r =
```

Valid or Invalid?

Argument Forms

VALID			
Modus Ponens	**Modus Tollens**	**Disjunctive Syllogism**	**Reasoning by Transitivity**
$p \rightarrow q$ p	$p \rightarrow q$ $\sim q$	$p \vee q$ $\sim p$	$p \rightarrow q$ $q \rightarrow r$
q	$\sim p$	q	$p \rightarrow r$

INVALID	
Fallacy of the Converse	**Fallacy of the Inverse**
$p \rightarrow q$ q	$p \rightarrow q$ $\sim p$
p	$\sim q$

Valid or Invalid?

```
If you stay in, your roommate goes out.
If your roommate doesn't go out, s/he will finish
        their math homework.
Your roommate doesn't finish their math homework.
Therefore, you do not stay in.
```


Determine a Valid Conclusion

```
It is either day or night.
If it is daytime, then the squirrels are scurrying.
It is not nighttime.
```


Determine a Valid Conclusion

```
If it is cold, you wear a coat.
If you don't wear a coat, you are dashing.
You aren't dashing.
```


Valid or Fallacy? Which Form?

```
If you use binoculars, then you get a glimpse of the comet.
If you get a glimpse of the comet, then you'll be amazed.
____________________________________________________________
If you use binoculars, then you'll be amazed.
```



```
If he buys another toy, his toy chest will overflow.
His toy chest overflows.
____________________________________________________________
He bought another toy.
```



```
If Ursula plays, the opponent gets shut out.
The Opponent does not get shut out.
____________________________________________________________
Ursula does not play.
```

If we evolved a race of Isaac Newtons, that
would be progress. (A. Huxley)
We have not evolved a race of Isaac Newtons.

That is progress.

Alison pumps iron or Tom jogs.
Tom doesn't jog.

Alison pumps iron.

Jeff loves to play gold. If Joan likes to sew, then Jeff does not love to play golf. If Joan does not like to sew, then Brad sings in the choir. Therefore, Brad sings in the choir.

If the Bobble head doll craze continues, then Beanie Babies will remain popular. Barbie dolls continue to be favorites or Beanie Babies will remain popular. Barbie dolls do not continue to be favorites. Therefore, the Bobble head doll craze does not continue.

If Jerry is a DJ, then he lives in Lexington. He lives in Lexington and is a history buff. Therefore, if Jerry is not a history buff, then he is not a DJ.

If I've got you under my skin, then you are deep in the heart of me. If you are deep in the heart of me, then you are not really a part of me. You are deep in the heart of me, or you are really a part of me. Therefore, if I've got you under my skin, then you are really a part of me.