Mat 1160
WEEK 5

Dr. N. Van Cleave

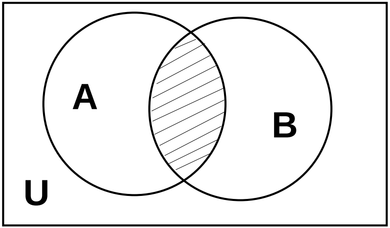
Spring 2010

Student Responsibilities – Week 5

- **Reading**:
  This week: Textbook, Sections 2.3 & 2.4
  Next week: Textbook, Sections 2.5
- Summarize Sections & Work through Examples
- Attendance
- Recommended exercises:
  - Section 2.3: all 1 - 6, every other even 8 - 132
  - Section 2.4: 2 - 30 (even)

Sec 2.3 Set Operations & Cartesian Products

- **Intersection** of sets: $\mathrm{A} \cap \mathrm{B}$ is the set of elements common to both: $\mathrm{A} \cap \mathrm{B} = \{ x \mid x \in A \text{ and } x \in B \}$



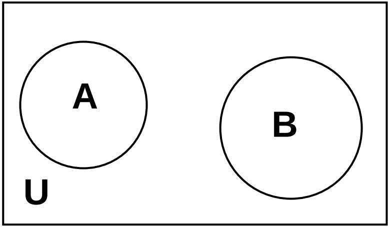
- Find the intersections of the following sets:

{ a, b, c }	and	{ b, f, g }	________________
{ a, b, c }	and	{ a, b, c }	________________
{ a, b, c }	and	{ a, b, z }	________________
{ a, b, c }	and	{ x, y, z }	________________
{ a, b, c }	and	$\emptyset$	________________

Disjoint Sets

- **Disjoint** sets: two sets which have no elements in common.

  I.e., their intersection is empty: $A \cap B = \emptyset$



- Are the following sets disjoint?

  { a, b, c } and { d, e, f, g } ________________

  { a, b, c } and { a, b, c } ________________

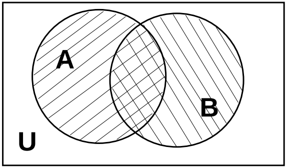
  { a, b, c } and { a, b, z } ________________

  { a, b, c } and { x, y, z } ________________

  { a, b, c } and $\emptyset$ ________________

Set Union

- **Union** of sets: $\mathrm{A} \cup \mathrm{B}$ is the set of elements belonging to either of the sets: $\mathrm{A} \cup \mathrm{B} = \{ x \mid x \in A \text{ or } x \in B \}$



Note: an element in the union of sets A and B may be a member of A, a member of B, *or* a member of **both** sets.

- Find the unions of the following sets:

{ a, b, c } and { b, f, g } ______________

{ a, b, c } and { a, b, c } ______________

{ a, b, c } and { a, b, z } ______________

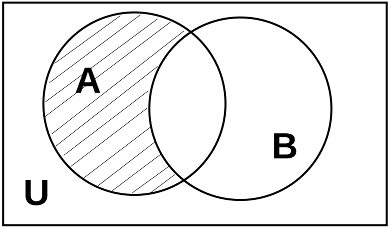
{ a, b, c } and { x, y, z } ______________

{ a, b, c } and $\emptyset$ ______________

Set Difference

- **Difference** of two sets: $\mathrm{A} - \mathrm{B}$ is the set of all elements **belonging** to set A and **not** to set B.

$$\mathrm{A} - \mathrm{B} = \{\ x \mid x \in A \text{ and } x \notin B\ \}$$



$$\{\ 1, 2, 3, 4, 5\ \} - \{\ 2, 4, 6\ \} = \{\ 1, 3, 5\ \}$$

but $\{\ 2, 4, 6\ \} - \{\ 1, 2, 3, 4, 5\ \} = \{\ 6\ \}$

Note: $x \notin B \rightarrow x \in B'$ (the complement of B)

$$\begin{aligned} \text{Thus, } \mathrm{A} - \mathrm{B} &= \{\ x \mid x \in A \text{ and } x \notin B\ \} \\ &= \{\ x \mid x \in A \text{ and } x \in B'\ \} \\ &= A \cap B' \end{aligned}$$

Given the sets:

$$\begin{aligned} \text{U} &= \{1, 2, 3, 4, 5, 6, 9\} \\ \text{A} &= \{1, 2, 3, 4\} \\ \text{B} &= \{2, 4, 6\} \\ \text{C} &= \{1, 3, 6, 9\} \end{aligned}$$

Find each of these sets:

- $A \cup B =$
- $A \cap B =$
- $A \cap U =$
- $A \cup U =$

$$
\begin{aligned}
\text{U} &= \{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 9\} \\
\text{A} &= \{1,\ 2,\ 3,\ 4\} \\
\text{B} &= \{2,\ 4,\ 6\} \\
\text{C} &= \{1,\ 3,\ 6,\ 9\}
\end{aligned}
$$

- $A' =$
- $A' \cap B =$
- $A' \cup B =$
- $A \cup B \cup C =$
- $A \cap B \cap C =$

Describe each of the following sets in words

- $A' \cup B' =$ *the set of all elements not in set* A *or not in set* B.
- $(A' \cup C) \cap B =$ *the set of all elements in* B *that are either not in set* A *or are in set* C.
- $A' \cap B' =$
- $A \cap (B \cup C) =$

Given the sets

$$
\begin{aligned}
\text{U} &= \{1, 2, 3, 4, 5, 6, 7\} \\
\text{A} &= \{1, 2, 3, 4, 5, 6\} \\
\text{B} &= \{2, 3, 6\} \\
\text{C} &= \{3, 5, 7\}
\end{aligned}
$$

Find each set:

- $A - B =$
- $B - A =$
- $(A - B) \cup C' =$

Note, in general, $A - B \neq B - A$

Ordered Pairs

- **Ordered Pair**: a group of two objects designated as **first** and **second** components.

  In the **ordered pair** (a, b):
  a is called the **first component**
  b is called the **second component**

- In general $(a, b) \neq (b, a)$, and **order** is **important**!
- Two ordered pairs (a, b) and (c, d) are **equal** provided $a = c$ and $b = d$

  $(1, 3) = (1, 3)$ $\qquad$ $(1, 3) \neq (3, 1)$
  $(4, 9) = (4, 9)$ $\qquad$ $(9, 4) \neq (4, 9)$
  $$(2+2, 3\times 3) = (2\times 2, 6+3)$$

- Sets can contain ordered pairs:

  $\{ (-3, 3), (-12, -6), (13, 29), (8, 7) \}$
  $\{ (1, 3), (2, 6), (3, 9), \ldots \}$

Cartesian Products

The **Cartesian product** of sets A and B is:

$$A \times B = \{(\text{a,b}) \mid a \in A \text{ and } b \in B\}$$

The Cartesian product of $\{$a, b, c$\} \times \{1, 2\} =$

$\{$ (a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2) $\}$

The Cartesian product of $\{1, 2\} \times \{$a, b, c$\} =$

$\{$ (1, a), (1, b), (1, c), (2, a), (2, b), (2, c) $\}$

What's the difference between the two resulting sets above?

If set A = $\{$x, y, z$\}$, what is $A \times A$?

Cardinality of Cartesian Products

If set A has cardinality 5 and set B has cardinality 4, what is the cardinality of:

$A \times B$?

$B \times A$?

If $|A| = n$ and $|B| = m$, what is $|A \times B|$?

Set Operations

- Finding intersections, unions, differences, Cartesian products, and complements of sets are examples of **set operations**

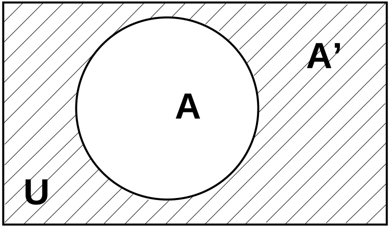
- An **operation** is a **rule** or **procedure** by which one or more objects are used to obtain another object (usually a set or number).

Common Set Operations

Let A and B be any sets, with U the universal set.

Complement of A is:

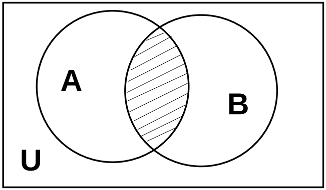
$$A' = \{x \mid x \in U \text{ and } x \notin A\}$$



Set Intersection and Union

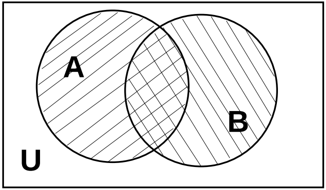
- **Intersection** of A and B is:

  $$A \cap B \;=\; \{x \mid x \in A \text{ and } x \in B\}$$



- **Union** of A and B is:

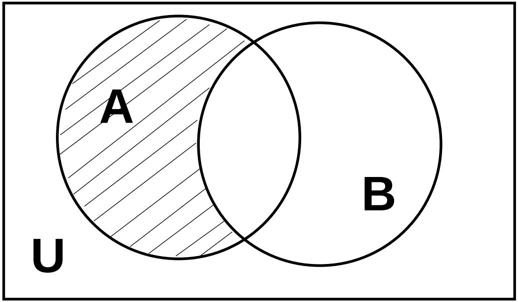
  $$A \cup B \;=\; \{x \mid x \in A \text{ or } x \in B\}$$



Set Difference and Cartesian Product

- **Difference** of A and B is:

  $$A - B \;=\; \{x \mid x \in A \text{ and } x \notin B\}$$

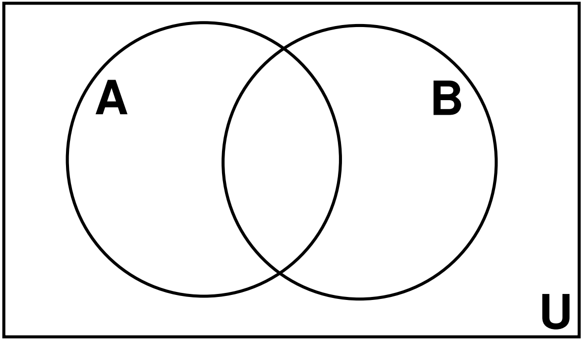


- The **Cartesian product** of A and B is:

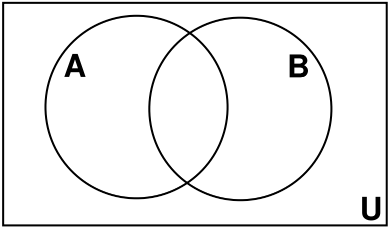
  $$A \;\times\; B = \{(\text{x,y}) \mid x \in A \text{ and } y \in B\}$$

Complete the Venn Diagram to represent U, A, and B

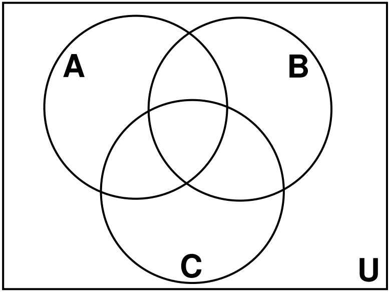
Let $U = \{q, r, s, t, u, v, w, x, y, z\}$
$A = \{r, s, t, u, v\}$ $B = \{t, v, x\}$



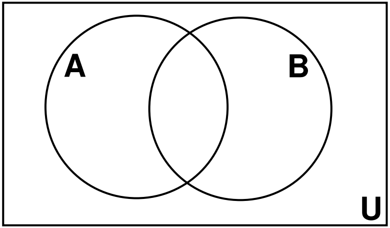
- Shade the Diagram for: $A \cap B$



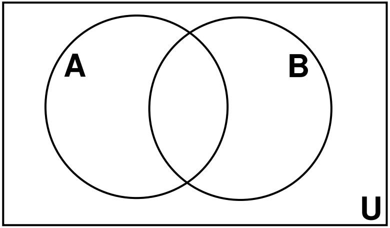
- Shade the Diagram for: $(A' \cap B') \cap C$



- Shade the Diagram for: $(A \cap B)'$



- Shade the Diagram for: $A' \cup B'$



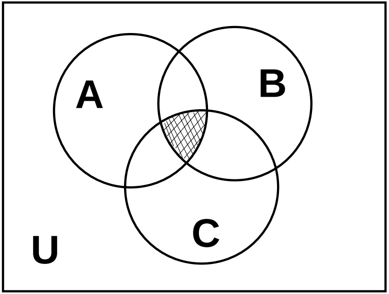
Did we get these last two correct? They look the same!

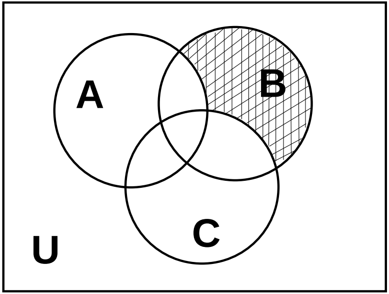
De Morgan's Laws

De Morgan's Laws. For any sets A and B

- $(A \cap B)' = A' \cup B'$
- $(A \cup B)' = A' \cap B'$

Using A, B, C, $\cap$, $\cup$, $-$, and $'$, give a symbolic description of the shaded area in each of the following diagrams. Is there more than one way to describe each?

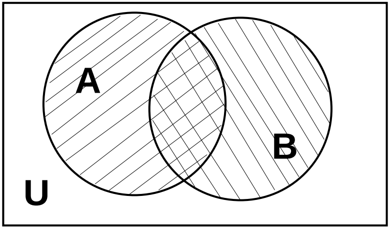




Cardinal Number Formula

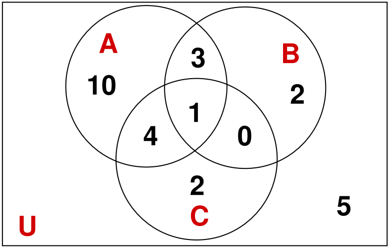
For any two sets A and B:

$$\mathbf{n}(A \cup B) = \mathbf{n}(A) + \mathbf{n}(B) - \mathbf{n}(A \cap B)$$



I.e., the number of elements in the union of two sets is the sum of the number of elements in each of the sets minus the number of elements in their intersection.

Example I



Find the cardinality of the sets:

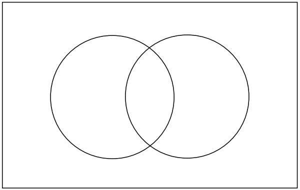
The numbers in the Venn diagram represent the cardinality of each subset.

$A \cap B \cap C$	_____	$A \cap B \cap C'$	_____
$A \cap B' \cap C$	_____	$A' \cap B \cap C$	_____
$A' \cap B' \cap C$	_____	$A \cap B' \cap C'$	_____
$A' \cap B \cap C'$	_____	$A' \cap B' \cap C'$	_____

Ex. 2 — Using Venn diagrams to display survey data

Kim is a fan of the music of Paul Simon and Art Garfunkel. In her collection of 22 CDs, she has the following:

- 5 on which both Simon and Garfunkel sing
- 8 total on which Simon sings
- 7 total on which Garfunkel sings
- 12 on which neither Simon nor Garfunkel sings



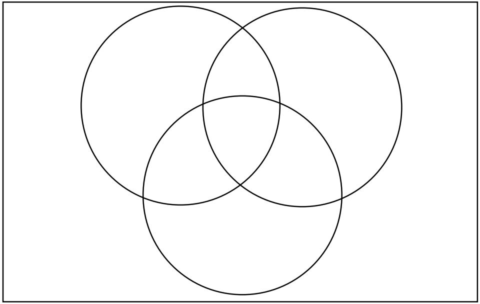
1. How many of her CDs feature only Paul Simon?
2. How many of her CDs feature only Art Garfunkel?
3. How many feature at least one of these two artists?

Ex. 3 — Love, Prison, and Trucks

It was once said that Country–Western songs emphasize three basic themes: love, prison, and trucks. A survey of the local Country–Western radio station produced the following data of songs about:

- 12 truck drivers in love while in prison
- 13 prisoners in love
- 28 people in love
- 18 truck drivers in love
- 3 truck drivers in prison who are not in love
- 2 prisoners not in love and not driving trucks
- 8 people who are out of prison, are not in love, and do not drive trucks
- 16 truck drivers who are not in prison

Number	L / P / T	Set Expression
12	TLP	
13	LP	
28	L	
18	TL	
3	TP L'	
2	P L' T'	
8	P' L' T'	
16	T P'	



How many songs were. . .

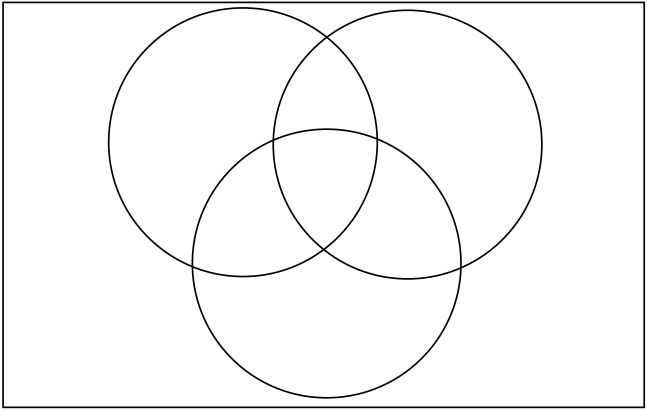
1. Surveyed?
2. About truck drivers?
3. About prisoners?
4. About truck drivers in prison?
5. About people not in prison?
6. About people not in love?

Ex. 4 — The Peter Principle

Jim Donahue was a section chief for an electric utility company. The employees in his section cut down tall trees (T), climbed poles (P), and spliced wire (W). Donahue submitted the following report to the his manager:

$$\begin{aligned} n(T) &= 45 & \quad n(P \cap W) &= 20 \\ n(P) &= 50 & \quad n(T \cap W) &= 25 \\ n(W) &= 57 & \quad n(T \cap P \cap W) &= 11 \\ n(T \cap P) &= 28 & \quad n(T' \cap P' \cap W') &= 9 \end{aligned}$$

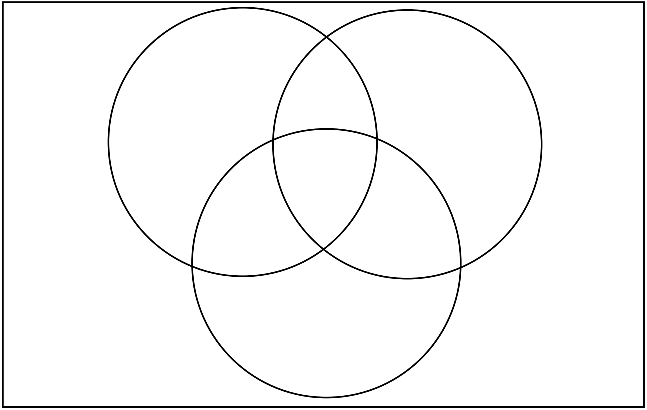
Donahue also stated that 100 employees were included in the report. Why did management reassign him to a new section?



Ex. 5 — Round II

Jim Donahue was reassigned to the home economics department of the electric utility company. he interviewed 140 people in a suburban shopping center to find out some of their cooking habits. He obtained the following results. There is a job opening in the antarctic. Should he be reassigned yet again?

- 58 use microwave ovens
- 63 use electric ranges
- 58 use gas ranges
- 19 use microwave ovens and electric ranges
- 17 use microwave ovens and gas ranges
- 4 use both gas and electric ranges
- 1 uses all three
- 2 cook only with solar energy



Ex. 6 — Student Values

Julie Ward, who sells college textbooks, interviewed freshmen on a community college campus to determine what is important to today's students. She found that **Wealth**, **Family**, and **Expertise** topped the list. Her findings can be summarized as:

$$
\begin{aligned}
n(W) &= 160 & n(E \cap F) &= 90 \\
n(F) &= 140 & n(W \cap F \cap E) &= 80 \\
n(E) &= 130 & n(E') &= 95 \\
n(W \cap F) &= 95 & n[(W \cup F \cup E)'] &= 10
\end{aligned}
$$

How many students were interviewed?