Mat 1160
WEEK 7

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Student Responsibilities – Week 7

- Thursday, **Exam 2**
- **Reading**:

  This week: Textbook, Sections 3.1–3.2: Logic and Truth Tables
  Next week: Textbook, Section 3.3–3.4: Conditionals, Circuits
- Summarize Sections & Work Examples
- Attendance
- Recommended exercises:
  - Section 3.1: evens 2–78
  - Section 3.2: evens 2–80

Sec 2.5 Infinite Sets & Their Cardinalities—Review

- **Cardinality**
- **One–to–one (1–1) Correspondence**
- $\aleph_0$, Aleph–naught or Aleph–null
- If we can show a 1–1 correspondence between some set, A, and the natural numbers, we say that A also has cardinality $\aleph_0$.
- Thus to show a set has cardinality $\aleph_0$, we need to find a 1–1 correspondence between the set and $\mathbb{N}$, the set of natural numbers

Show the Cardinality of each set is $\aleph_0$

- the positive even integers, $\{2, 4, 6, \ldots\}$

- The negative integers, $\{-1, -2, -3, \ldots\}$

- The positive odd integers, $\{1, 3, 5, \ldots\}$

Chapter 3. Introduction to Logic

- How can we draw logical conclusions from the facts we have at hand?
- How can we know when someone is making a valid argument?
- How can we determine the veracity of statements with many parts?
- Why should we care about these things?

Sec 3.1 Statements and Quantifiers

- The Greek philosopher Aristotle was one of the first to attempt to codify "right thinking," or **irrefutable reasoning processes**.
- His syllogisms provided **patterns** for **argument structures** that **always** gave **correct conclusions** given correct **premises**.

For example: SOCRATES IS A MAN

ALL MEN ARE MORTAL

THEREFORE, SOCRATES IS MORTAL.

Laws of Logic

- These laws of thought were supposed to govern the operation of the mind, and initiated the field of **logic**.
- **Logic** is based on **knowledge/facts** and **reasoning**.
- We have some facts and from them draw **conclusions**, perhaps about our next course of **action** or to extend our **knowledge**.
- **Logic** consists of:
  1. a **formal language** (such as mathematics) in which knowledge can be expressed
  2. a means of carrying out **reasoning** in such a language

Logic Values

- **Logic values**: `True` and `False`
- **Statement**: a declarative (factual) sentence that is either TRUE or FALSE, but **not** both. Examples:
  - Salt lowers the melting point of ice.
  - $3 + 5 = 9$
  - The outdoor temperature in Charleston today is 26° F
- Some sentences are not statements. For example:
  - The best way to melt ice is to move to Florida.
  - Get outta here!
  - Are you feeling okay today?
  - This sentence is false.

  Opinions, commands, questions, and *paradoxes* are not statements.

Compound Statements

- **Compound Statement**: a statement formed by combining two or more statements.

  Ex: You are my student and we are studying mathematics.

- **Component Statements**: the statements used to form a compound statement.

  In the above example, **You are my student** and **we are studying mathematics** are the two component statements.

Logical Connectives

Logical Connectives (or **connectives**) are used to form compound statements: `and, or, not`, and `if` ... `then`

- Today it is sunny **and** there is a slight breeze.
- Yesterday it was raining **or** snowing.
- The back tire on my bicycle is**n't** flat.
- **If** the moon is made of green cheese, **then** so am I.

Negation

Negation: an opposite statement.

The **negation** of a TRUE statement is FALSE

The **negation** of a FALSE statement is TRUE

Statement	**Negation**
My car is red.	My car is not red.
My car is not red.	My car is red.
The pen is broken.	The pen isn't broken.
Four is less than nine.	Four is not less than nine (i.e., $4 \geq 9$).
$a \geq b$	$a < b$

Remember: a negation must have the **opposite** truth value from the original statement.

Symbolic Logic

- **Symbolic logic** uses `letters` to represent statements, and `symbols` for words such as **and, or**, and **not**.

- The letters used are often p and q. They will represent **statements**.

Connective	**Symbol**	**Statement Type**
and	$\wedge$	Conjunction
or	$\vee$	Disjunction
not	$\sim$	Negation

Symbolic Logic to English Statements

If p represents "**Today is Tuesday**," and
q represents "**It is sunny**,"
translate each of the following into an English sentence:

1. $p \wedge q$
2. $p \vee q$
3. $\sim p \ \wedge \ q$
4. $p \vee \sim q$
5. $\sim (p \vee q)$
6. $\sim p \ \wedge \ \sim q$

Quantifiers, Universal Quantifiers

Quantifiers in mathematics indicate **how many** cases of a particular situation exist.

Universal Quantifier: indicates the property applies to **all** or **every** case. Universal quantifiers are:

all, each, every, no, and **none**

- **All** athletes must attend the meeting.
- **Every** math student enjoys the subject.
- There are **no** groundhogs which are purple.

Existential Quantifiers

Existential Quantifier: indicates the property applies to **one or more** cases. Existential quantifiers include:

some, **there exists**, and **(for) at least one**

- **Some** `athletes must attend the meeting.`
- **At least one** `math student enjoys the subject.`
- **There exists** `a groundhog which is brown.`

Negating Quantifiers

Care must be taken when negating statements with quantifiers.

Negations of Quantified Statements	
Statement	**Negation**
All do	**Some do not** (Equivalently: **Not all do**)
Some do	**None do** (Equivalently: **All do not**)

Practice with Negation

What is the negation of each statement?

1. **Some** people wear glasses.
2. **Some** people **do not** wear glasses.
3. **Nobody** wears glasses.
4. **Everybody** wears glasses.
5. **Not everybody** wears glasses.

Practice with Quantifiers: **True** or **False**?

1. **All** Whole numbers are Natural numbers.

2. **Some** Whole number **isn't** a Natural number.

3. **Every** Integer is a Natural number.

4. **No** Integer is a Natural number.

5. **Every** Natural number is a Rational number.

6. **There exists** an Irrational number that **is not** Real.

Sec 3.2 Truth Tables and Equivalent Statements

Conjunction: Given two statements p and q, their conjunction is $p \wedge q$.

Conjunction Truth Table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction Examples

Determine the truth values (T/F):

1. ___ Today is Tuesday **and** it is sunny.
2. ___ Today is Wednesday **and** it is sunny.
3. ___ The moon is made of green cheese **and** some violets are blue.
4. ___ It is daytime here **and** there are **not** 1000 desks in this classroom.
5. ___ This course is `MAT 1160` **and** we are learning calculus.
6. ___ This course is `MAT 4870` **and** we are learning physics.
7. ___ $3 < 5 \ \wedge \ 5 < 3$
8. ___ $3 < 5 \ \wedge \ 5 < 8$

Disjunction

Disjunction. Given two statements p and q, their (inclusive) **disjunction** is $p \lor q$.

Inclusive disjunctions are TRUE if either or both components are TRUE.

Disjunction Truth Table

p	q	$p \lor q$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction Examples

Determine the truth values:

1. ___ Today is Tuesday **or** it is sunny.
2. ___ Today is Wednesday **or** it is sunny.
3. ___ The moon is made of green cheese **or** some violets are blue.
4. ___ It is daytime here **or** there are not 100 desks in this classroom.
5. ___ This course is `MAT 1160` **or** we are learning calculus.
6. ___ This course is `MAT 4870` **or** we are learning physics.
7. ___ $3 < 5 \;\vee\; 5 < 3$
8. ___ $3 < 5 \;\vee\; 5 < 8$

Mathematical Examples

Using `or`

Statement	Reason It's True
$7 \geq 7$	$7 = 7$
$8 \geq 5$	$8 > 5$
$-7 \leq -3$	$-7 < -3$
$-3 \leq -3$	$-3 = -3$

The Porsche & The Tiger

A prisoner must make a choice between two doors: behind one is a beautiful red Porsche, and behind the other is a hungry tiger. Each door has a sign posted on it, but only one sign is true.

Door #1. In this room there is a Porsche and in the other room there is a tiger.

Door #2. In one of these rooms there is a Porsche and in one of these rooms there is a tiger.

With this information, the prisoner is able to choose the correct door. . . Which one is it?

Negation

Negation. Given a statement p, its negation is $\sim p$.

Negation Truth Table

p	$\sim p$
T	F
F	T

Negation Examples

Determine the truth values

Assume p is **true**, q is **false**, and r is **false**

1. ___ p
2. ___ $\sim p$
3. ___ q
4. ___ $\sim q$
5. ___ r
6. ___ $\sim r$
7. ___ $\sim p \wedge p$
8. ___ $p \vee \sim p$
9. ___ $p \wedge \sim q$
10. ___ $p \vee \sim q$
11. ___ $\sim p \wedge (q \vee \sim r)$
12. ___ $p \wedge (\sim q \vee r)$

More Examples

Determine the truth values

Let p represent the statement $3 > 2$

q represent the statement $5 < 4$

r represent the statement $3 \leq 8$

1. ___ p
2. ___ $\sim p$
3. ___ q
4. ___ $\sim q$
5. ___ r
5. ___ $\sim r$
7. ___ $\sim p \wedge q$
8. ___ $\sim (p \wedge q)$
9. ___ $\sim p \vee (\sim q \vee r)$
10. ___ $(\sim p \wedge r) \vee (\sim q \wedge \sim p)$

Yet More Examples

11. ___ For some real number x, $x > 2$ and $x < 8$
12. ___ There exists a real number b, $b < 8$ or $b > 2$
13. ___ For at least one real number y, $y < 8$ and $y > 12$
14. ___ There is a real number m, $m < 8$ or $m > 12$
15. ___ For all real numbers x, $x < 8$ and $x > 2$
16. ___ For every real number b, $b < 8$ or $b > 2$
17. ___ For all real numbers y, $y < 8$ and $y > 12$
18. ___ For every real number m, $m < 8$ or $m > 12$
19. ___ For every real number n, $n^2 > 0$
20. ___ For every real number n, $n^2 \geq 0$

Constructing Truth Tables

Construct a Truth Table for: $(\sim p \land q) \lor \sim q$

p	q	$(\sim p \land q) \lor \sim q$
T	T	
T	F	
F	T	
F	F	

Construct a Truth Table for: $p \wedge (\sim p \vee \sim q)$

p	q	$p \wedge (\sim p \vee \sim q)$
T	T	
T	F	
F	T	
F	F	

Complete the Truth Table

p	q	r	$\sim p \wedge (q \vee \sim r)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Complete the Truth Table

p	q	r	$(\sim p \land r) \lor (\sim q \land \sim p)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Some Notes of Interest

A logical statement having n component statements will have 2^n rows in its truth table.

Two statements are **equivalent** if they have the same truth value in **every** possible situation.

In other words, two statements are **equivalent** if their columns in the same truth table have the same truth values.

De Morgan's Laws

p	q	$\sim p \wedge \sim q$	$\sim (p \vee q)$
T	T		
T	F		
F	T		
F	F		

p	q	$\sim p \vee \sim q$	$\sim (p \wedge q)$
T	T		
T	F		
F	T		
F	F		