Mat 1160 WEEK 7

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Student Responsibilities - Week 7

- Thursday, Exam 2
- ► Reading:

This week: Textbook, Sections 3.1–3.2: Logic and Truth Tables Next week: Textbook, Section 3.3–3.4: Conditionals, Circuits

- Summarize Sections & Work Examples
- Attendance
- Recommended exercises:
 - Section 3.1: evens 2–78
 - Section 3.2: evens 2–80

Sec 2.5 Infinite Sets & Their Cardinalities—Review

Cardinality

- ► One-to-one (1-1) Correspondence
- ▶ ℵ₀, Aleph–naught or Aleph–null
- If we can show a 1−1 correspondence between some set, A, and the natural numbers, we say that A also has cardinality ℵ₀.
- Thus to show a set has cardinality ℵ₀, we need to find a 1–1 correspondence between the set and ℕ, the set of natural numbers

Show the Cardinality of each set is \aleph_0

• the positive even integers, $\{2, 4, 6, \ldots\}$

• The negative integers, $\{-1, -2, -3, \ldots\}$

▶ The positive odd integers, {1, 3, 5, ...}

Chapter 3. Introduction to Logic

How can we draw logical conclusions from the facts we have at hand?

▶ How can we know when someone is making a valid argument?

How can we determine the veracity of statements with many parts?

Why should we care about these things?

Sec 3.1 Statements and Quantifiers

The Greek philosopher Aristotle was one of the first to attempt to codify "right thinking," or irrefutable reasoning processes.

His syllogisms provided patterns for argument structures that always gave correct conclusions given correct premises.

For example: SOCRATES IS A MAN

All men are mortal

THEREFORE, SOCRATES IS MORTAL.

Laws of Logic

- These laws of thought were supposed to govern the operation of the mind, and initiated the field of logic.
- Logic is based on knowledge/facts and reasoning.
- ► We have some facts and from them draw conclusions, perhaps about our next course of action or to extend our knowledge.

Logic consists of:

- 1. a **formal language** (such as mathematics) in which knowledge can be expressed
- 2. a means of carrying out reasoning in such a language

Logic Values

- ▶ Logic values: True and False
- ► **Statement**: a declarative (factual) sentence that is either TRUE or FALSE, but not both. Examples:
 - Salt lowers the melting point of ice.
 - ▶ 3 + 5 = 9
 - \blacktriangleright The outdoor temperature in Charleston today is 26° F
- Some sentences are not statements. For example:
 - The best way to melt ice is to move to Florida.
 - Get outta here!
 - Are you feeling okay today?
 - This sentence is false.

Opinions, commands, questions, and *paradoxes* are not statements.

Compound Statements

 Compound Statement: a statement formed by combining two or more statements.

Ex: You are my student and we are studying mathematics.

Component Statements: the statements used to form a compound statement.

In the above example, **You are my student** and **we are studying mathematics** are the two component statements.

Logical Connectives

Logical Connectives (or connectives) are used to form compound statements: and, or, not, and if ...then

- Today it is sunny and there is a slight breeze.
- Yesterday it was raining or snowing.
- The back tire on my bicycle isn't flat.
- If the moon is made of green cheese, then so am I.

Negation

Negation: an opposite statement.

The negation of a TRUE statement is FALSE The negation of a FALSE statement is TRUE

Statement	Negation
My car is red.	My car is not red.
My car is not red.	My car is not red. My car is red.
The pen is broken.	The pen isn't broken.
Four is less than nine.	Four is not less than nine (i.e., $4 \ge 9$).
$a \geq b$	a < b

Remember: a negation must have the **opposite** truth value from the original statement.

Symbolic Logic

Symbolic logic uses letters to represent statements, and symbols for words such as and, or, and not.

The letters used are often p and q. They will represent statements.

Connective	Symbol	Statement Type
and	\wedge	Conjunction
or	\vee	Disjunction
not	\sim	Negation

Symbolic Logic to English Statements

If *p* represents "**Today is Tuesday**," and *q* represents "**It is sunny**," translate each of the following into an English sentence:

1. $p \wedge q$

- 2. $p \lor q$
- 3. $\sim p \wedge q$
- 4. $p \lor \sim q$
- 5. $\sim (p \lor q)$
- 6. $\sim p \wedge \sim q$

Quantifiers, Universal Quantifiers

Quantifiers in mathematics indicate **how many** cases of a particular situation exist.

Universal Quantifier: indicates the property applies to all or every case. Universal quantifiers are: all, each, every, no, and none

- ▶ All athletes must attend the meeting.
- **Every** math student enjoys the subject.
- ▶ There are **no** groundhogs which are purple.

Existential Quantifiers

Existential Quantifier: indicates the property applies to **one or more** cases. Existential quantifiers include:

some, there exists, and (for) at least one

- **Some** athletes must attend the meeting.
- > At least one math student enjoys the subject.
- There exists a groundhog which is brown.

Negating Quantifiers

Care must be taken when negating statements with quantifiers.

Negations of Quantified Statements		
Statement	Negation	
All do	Some do not (Equivalently: Not all do)	
Some do	None do (Equivalently: All do not)	

Practice with Negation

What is the negation of each statement?

1. **Some** people wear glasses.

2. Some people do not wear glasses.

3. **Nobody** wears glasses.

4. Everybody wears glasses.

5. Not everybody wears glasses.

Practice with Quantifiers: True or False?

- 1. All Whole numbers are Natural numbers.
- 2. **Some** Whole number **isn't** a Natural number.
- 3. Every Integer is a Natural number.
- 4. No Integer is a Natural number.
- 5. Every Natural number is a Rational number.
- 6. There exists an Irrational number that is not Real.

Sec 3.2 Truth Tables and Equivalent Statements

Conjunction: Given two statements p and q, their conjunction is $p \land q$.

Conjunction Truth Table

р	q	$p \land q$
Т	Т	Т
Т	F	F
F	Т	F
F	F	F

Conjunction Examples

Determine the truth values (T/F):

- 1. ____ Today is Tuesday and it is sunny.
- 2. ____ Today is Wednesday and it is sunny.
- 3. ____ The moon is made of green cheese and some violets are blue.
- 4. ____ It is daytime here **and** there are **not** 1000 desks in this classroom.
- 5. ____ This course is MAT 1160 and we are learning calculus.
- 6. ____ This course is MAT 4870 and we are learning physics.
- 7. ____ $3 < 5 \land 5 < 3$
- 8. ____ $3 < 5 \land 5 < 8$

Disjunction

Disjunction. Given two statements p and q, their (inclusive) **disjunction** is $p \lor q$.

Inclusive disjunctions are TRUE if either or both components are TRUE.

Disjunction Truth Table

р	q	$p \lor q$
Т	Т	Т
Т	F	Т
F	Т	Т
F	F	F

Disjunction Examples

Determine the truth values:

- 1. ____ Today is Tuesday or it is sunny.
- 2. ____ Today is Wednesday or it is sunny.
- 3. ____ The moon is made of green cheese **or** some violets are blue.
- 4. ____ It is daytime here **or** there are not 100 desks in this classroom.
- 5. ____ This course is MAT 1160 or we are learning calculus.
- 6. ____ This course is MAT 4870 or we are learning physics.
- 7. ____ $3 < 5 \lor 5 < 3$
- 8. ____ 3 < 5 V 5 < 8

Mathematical Examples

Using or

Statement	Reason It's True
$7 \ge 7$	7 = 7
8 ≥ 5	8 > 5
$-7 \leq -3$	-7 < -3
$-3 \leq -3$	-3 = -3

The Porsche & The Tiger

A prisoner must make a choice between two doors: behind one is a beautiful red Porsche, and behind the other is a hungry tiger. Each door has a sign posted on it, but only one sign is true.

Door #1. IN THIS ROOM THERE IS A PORSCHE AND IN THE OTHER ROOM THERE IS A TIGER.

Door #2. IN ONE OF THESE ROOMS THERE IS A PORSCHE AND IN ONE OF THESE ROOMS THERE IS A TIGER.

With this information, the prisoner is able to choose the correct door... Which one is it?



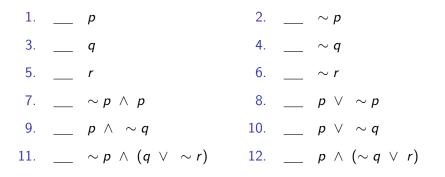
Negation. Given a statement p, its negation is $\sim p$.

Negation Truth Table

р	\sim p
Т	F
F	Т

Negation Examples

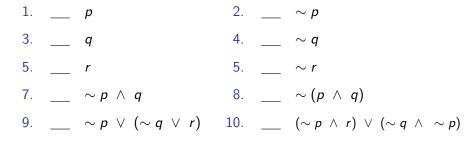
Determine the truth values Assume p is **true**, q is **false**, and r is **false**



More Examples

Determine the truth values

Let p represent the statement 3 > 2q represent the statement 5 < 4r represent the statement $3 \le 8$



Yet More Examples

- 11. ____ For some real number x, x > 2 and x < 8
- 12. ____ There exists a real number b, b < 8 or b > 2
- 13. ____ For at least one real number y, y < 8 and y > 12
- 14. ____ There is a real number m, m < 8 or m > 12
- 15. ____ For all real numbers x, x < 8 and x > 2
- 16. ____ For every real number b, b < 8 or b > 2
- 17. ____ For all real numbers y, y < 8 and y > 12
- 18. ____ For every real number m, m < 8 or m > 12
- 19. ____ For every real number *n*, $n^2 > 0$
- 20. ____ For every real number *n*, $n^2 \ge 0$

Constructing Truth Tables

Construct a Truth Table for: $(\sim p \land q) \lor \sim q$

р	q	$(\sim p \land q) \lor \sim q$
Т	Т	
Т	F	
F	Т	
F	F	

Construct a Truth Table for: $p \land (\sim p \lor \sim q)$

р	q	$p \ \land \ (\sim p \ \lor \ \sim q)$
Т	Т	
Т	F	
F	Т	
F	F	

Complete the Truth Table

р	q	r	$\sim p \land (q \lor \sim r)$
Т	Т	Т	
Т	Т	F	
Т	F	Т	
Т	F	F	
F	Т	Т	
F	Т	F	
F	F	Т	
F	F	F	

Complete the Truth Table

р	q	r	$(\sim p \land r) \lor (\sim q \land \sim p)$
Т	Т	Т	
Т	Т	F	
Т	F	Т	
Т	F	F	
F	Т	Т	
F	Т	F	
F	F	Т	
F	F	F	

A logical statement having n component statements will have 2^n rows in its truth table.

Two statements are **equivalent** if they have the same truth value in **every** possible situation.

In other words, two statements are **equivalent** if their columns in the same truth table have the same truth values.

De Morgan's Laws

р	q	$\sim p \ \wedge \ \sim q$	$\sim (p ~ee ~q)$
Т	Т		
Т	F		
F	Т		
F	F		

p	q	$\sim p ~ee~ \sim q$	$\sim (p ~\wedge~ q)$
Т	Т		
Т	F		
F	Т		
F	F		