Mat 1160
WEEK 8

Dr. N. Van Cleave

Spring 2010

Student Responsibilities – Week 8

- **Reading**:

  This week: Textbook, Sections 3.3–3.4: Conditionals, Circuits
  Next week: Textbook, Sections 3.5–3.6 Analysis

- Summarize Sections & Work Examples
- Attendance
- Recommended exercises:
  - Section 3.3: evens 2–100
  - Section 3.4: evens 2–58

Sec 3.3 The Conditional & Circuits

- **Conditional** statement: a compound statement that uses the connective **if** . . . **then**.

- **Conditional statements** are also known as **implications**, and can be written as:

  $p \rightarrow q$ (pronounced "p implies q")

- The statement p is called the **antecedent**.

- The statement q is called the **consequent**.

Conditional Examples

- **If** you are not home by midnight,
  (**then**) you'll be grounded.

- **If** he hits a home run,
  (**then**) he'll beat the old record.

- **If** you scratch my back,
  (**then**) I'll scratch yours.

- **If** you exceed the speed limit,
  (**then**) you'll get a ticket.

- The English are bad cooks.
  translation: **If** you are English, **then** you are a bad cook.

- College students are immature.
  translation: **If** you are a student, **then** you are immature.

Truth Table for Conditional Statements

There are four possible combinations of truth values for the two component statements

p	q	$p \rightarrow q$
T	T	?
T	F	?
F	T	?
F	F	?

Let's consider: **If you are not home by midnight, then you'll be grounded**.

Is the implication true when:

1. ____ You are **not** home by midnight and you **are** grounded
2. ____ You are **not** home by midnight but you are **not** grounded
3. ____ You **are** home by midnight but you **are** grounded
4. ____ You **are** home by midnight and you are **not** grounded.

Another Example

Let's consider: **If he hits a home run,**

then he'll beat the old record.

p	q	$p \rightarrow q$ T or F?
he hits a home run	he beats the old record	
he hits a home run	he doesn't beat the old record	
he doesn't hit a home run	he beats the old record	
he doesn't hit a home run	he doesn't beat the old record	

Another Example

How about: **If you are English,**

then you are a bad cook.

p	q	$p \rightarrow q$ T or F?
you are English	you are a bad cook	
you are English	you are not a bad cook	
you aren't English	you are a bad cook	
you aren't English	you are not a bad cook	

Another Example

And finally: **If you are a college student,**

then you are immature.

p	q	$p \rightarrow q$ T or F?
you are a college student	you are immature	
you are a college student	you aren't immature	
you aren't a college student	you are immature	
you aren't a college student	you aren't immature	

Truth Table for the Conditional

If p, then q

p	q	$p \to q$
T	T	T
T	F	**F**
F	T	T
F	F	T

If the moon is made of green cheese, ...

If my name isn't $< My\ \ name\ \ here >$ ...

If I finish my homework, ...

If I had a million dollars, ...

If wishes were fishes, ...

Notes

- $p \longrightarrow q$ is `false` **only** when the antecedent is `true` and the consequent is `false`
- If the antecedent is `false`, then $p \longrightarrow q$ is automatically `true`
- If the consequent is `true`, then $p \longrightarrow q$ is automatically `true`

true or false?

____ true $\rightarrow$ $(6 = 6)$

____ true $\rightarrow$ $(6 = 3)$

____ false $\rightarrow$ $(6 = 6)$

____ false $\rightarrow$ $(6 = 3)$

____ $(6 = 6) \rightarrow$ true

____ $(6 = 3) \rightarrow$ true

____ $(6 = 6) \rightarrow$ false

____ $(6 = 3) \rightarrow$ false

Let p, q, and r be false

____ $(p \rightarrow q)$

____ $(\sim r \rightarrow q)$

____ $(p \rightarrow \sim q)$

____ $(p \rightarrow \sim q) \rightarrow (\sim r \rightarrow q)$

Exercises

Truth Table: $(\sim p \rightarrow \sim q) \rightarrow (\sim p \wedge q)$

p	q	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$	$\sim p \wedge q$	$(\sim p \rightarrow \sim q) \rightarrow (\sim p \wedge q)$
T	T					
T	F					
F	T					
F	F					

Truth Table: $(p \rightarrow q) \rightarrow (\sim p \vee q)$

p	q	$p \rightarrow q$	$\sim p$	$\sim p \vee q$	$(p \rightarrow q) \rightarrow (\sim p \vee q)$
T	T				
T	F				
F	T				
F	F				

Tautology: a statement that is **always** true, no matter what the truth values of the components.

Truth Table: $p \vee \sim p$

p	$\sim p$	$p \vee \sim p$
T		
F		

Truth Table: $p \rightarrow p$

p	$\sim p$	$p \rightarrow p$
T		
F		

Truth Table: $(\sim p \vee \sim q) \to \ \sim (q \wedge p)$

p	q	$\sim p \vee \sim q$	$\sim (q \wedge p)$	$(\sim p \vee \sim q) \to \ \sim (q \wedge p)$
T	T			
T	F			
F	T			
F	F			

Truth Table: Negation of $p \to q$

p	q	$p \to q$	$\sim (p \to q)$	$\sim q$	$p \wedge \sim q$
T	T				
T	F				
F	T				
F	F				

Recall: You are not home by midnight,
you are not grounded...
the only false result, and thus the negation

The **negation of** $p \rightarrow q$ is $p \wedge \sim q$

Write the negation of each statement

- *If* you are not home by midnight, *then* you'll be grounded.
- *If* he hits a home run, (*then*) he'll beat the old record.
- *If* you scratch my back, (*then*) I'll scratch yours.
- *If* you exceed the speed limit, (*then*) you'll get a ticket.

The **negation of** $p \rightarrow q$ is $p \wedge \sim q$

Write the negation of each statement

- *If* it's Smucker's, it's got to be good!

- *If* that is an authentic Persian rug, I'll be surprised.

- The English are bad cooks.
  translation: **If** you are English, **then** you are a bad cook.

- College students are immature.
  translation: **If** you are a student, **then** you are immature.

$p \rightarrow q$ is equivalent to $\sim p \vee q$

Rewrite as a statement that doesn't use the if. . . then connective

- *If* you are not home by midnight, *then* you'll be grounded.
- *If* he hits a home run, (*then*) he'll beat the old record.
- *If* you scratch my back, (*then*) I'll scratch yours.
- *If* you exceed the speed limit, (*then*) you'll get a ticket.

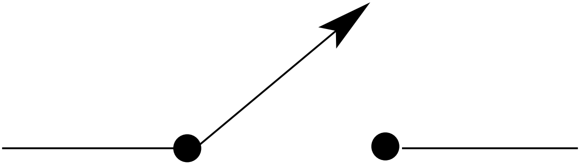
$p \longrightarrow q$ is equivalent to $\sim p \vee q$

Rewrite as a statement that doesn't use the if. . . then connective

- *If* it's Smucker's, it's got to be good!
- *If* that is an authentic Persian rug, I'll be surprised.
- *If* you give your plants tender, loving care, they flourish.
- *If* she doesn't, he will.
- *If* you are a student, then you are immature.

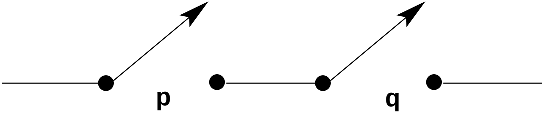
CIRCUITS

When will current flow through the switch and wire?

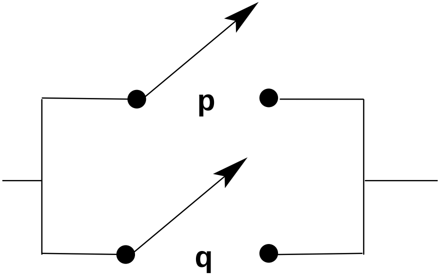


A Switch — On or Off?

Combining Circuits

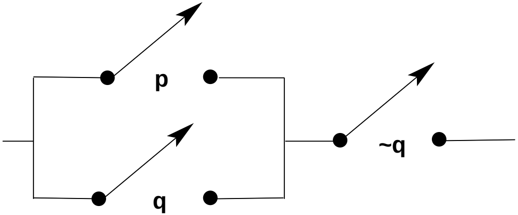


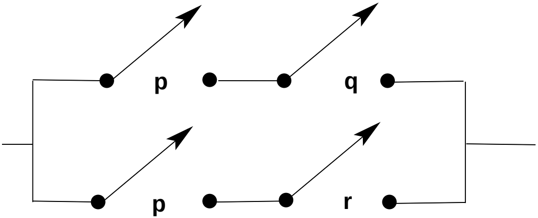
A Series Circuit



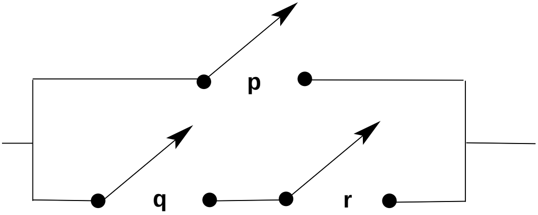
A Parallel Circuit

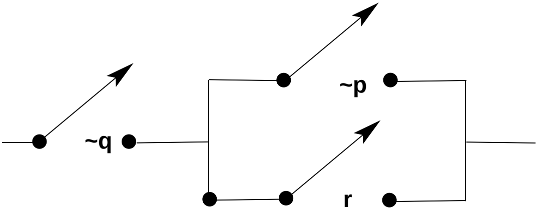
What is the corresponding logic statement?





What is the corresponding logic statement?

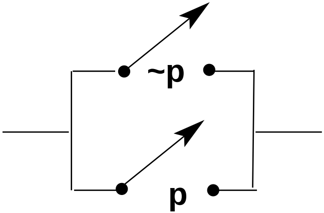


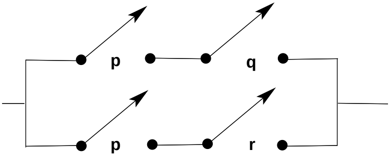


Equivalent Statements — Used to Simplify Circuits

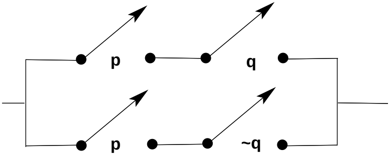
$$
\begin{aligned}
p \vee T &\equiv T \\
p \wedge F &\equiv F \\
p \vee \sim p &\equiv T \\
p \wedge \sim p &\equiv F \\
p \vee p &\equiv p \\
p \wedge p &\equiv p \\
\sim (p \wedge q) &\equiv \sim p \vee \sim q \\
\sim (p \vee q) &\equiv \sim p \wedge \sim q \\
p \vee (q \wedge r) &\equiv (p \vee q) \wedge (p \vee r) \\
p \wedge (q \vee r) &\equiv (p \wedge q) \vee (p \wedge r) \\
p \rightarrow q &\equiv \sim q \rightarrow \sim p \\
p \rightarrow q &\equiv \sim p \vee q
\end{aligned}
$$

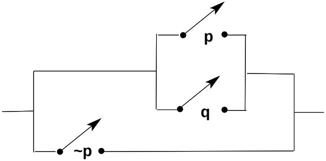
Rewrite as Boolean Expressions and Simplify

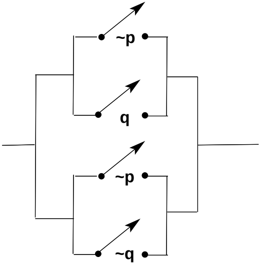




Rewrite as Boolean Expressions and Simplify







Draw Circuits:

- $p \vee (\sim q \wedge \sim r)$
- $p \rightarrow (q \wedge \sim r)$. (Rewrite it first)

Simplify and draw circuits

1. $p \wedge (q \vee \sim p)$

2. $(p \vee q) \wedge (\sim p \wedge \sim q)$

3. $[(p \vee q) \wedge r] \wedge \sim p$

Sec 3.3 Review

- A **conditional** statement uses implication ($\rightarrow$) or `if...else`
- $p \rightarrow q$ is `false` `only` when p is `true` and q is `false`.
- $p \rightarrow q$ is equivalent to $(\sim p \vee q)$
- The negation of $p \rightarrow q$ is $(p \wedge \sim q)$
- We can use Truth Tables to show two conditional expressions are equivalent (their truth values will be the same)
- A **tautology** is a statement which is always TRUE.
- Circuits in **series** correspond to conjunctions (`and`s)
- Circuits in **parallel** correspond to disjunctions (`or`s)
- Some circuits can be simplified.

Sec 3.4 More on the Conditional:

Converse, Inverse, and Contrapositive

Direct Statement	$p \to q$	If p, then q
Converse	$q \to p$	If q, then p
Inverse	$\sim p \to \sim q$	If not p, then not q
Contrapositive	$\sim q \to \sim p$	If not q, then not p

Let $p =$ "they stay" and $q =$ "we leave"
Direct Statement ($p \to q$):

Converse:

Inverse:

Contrapositive:

Let $p =$ "I surf the web" and $q =$ "I own a PC"

Direct Statement $(p \to q)$:

Converse:

Inverse:

Contrapositive:

Equivalent Conditionals

		Direct	**Converse**	**Inverse**	**Contrapositive**
		$p \to q$	$q \to p$	$\sim p \to \; \sim q$	$\sim q \to \; \sim p$
p	q	$\sim p \vee q$			
T	T	T	T		
T	F	F	T		
F	T	T	F		
F	F	T	T		

$\square \to \triangle$ is equivalent to $\sim \square \vee \triangle$

$$\sim \square \vee \triangle \;\equiv\; \square \to \triangle$$

$$\square \vee \triangle \;\equiv\; \sim \square \to \triangle$$

Tricky Question

For $p \vee q$, write each of the following:

Direct Statement:

Converse:

Inverse:

Contrapositive:

Alternate Conditional Forms

Common translations of $p \rightarrow q$

If p, then q	p is sufficient for q
If p, q	q is necessary for p
p implies q	All p's are q's
p only if q	q if p

These translations do not in any way depend upon the truth value of $p \rightarrow q$.

Translations of: "**If you get home late, then you are grounded**"

You are grounded if you get home late.

Getting home late is sufficient for you to get grounded.

Getting grounded is necessary when you get home late.

Getting home late implies that you are grounded.

Rewrite as `if...then` statements & give some alternatives for:

You'll be sorry if I go.

Today is Thursday only if yesterday was Wednesday.

All nurses wear white shoes.

A stitch in time saves nine.

Rolling stones gather no moss.

Birds of a feather flock together.

Let $p =$ "a triangle is equilateral" and $q =$ "a triangle has three equal sides"

Write in symbols:

A triangle is equilateral **if** it has three equal sides.

A triangle is equilateral **only if** it has three equal sides.

One of the following statements is **not** equivalent to the others...
Which one is it?

1. r only if s
2. r implies s
3. If r, then s
4. r is necessary for s

Consistent or Contrary?

Two statements about the same object are:

consistent — if they are both true.

contrary — if they cannot both be true.

1. The car is a Chevy. The car is a Toyota.
2. The car is a Chevy. The car is blue.
3. Elvis is alive. Elvis is dead.
4. The animal has four legs. The animal is a dog.
5. The cake is chocolate. The cake has two layers.
6. The clock is broken. The clock always has the right time.
7. The math class meets at noon. The math class lasts 50 minutes.

Consistent or Contrary?

1. The number is an integer. The number is irrational.
2. The punch is pink. The punch has juice in it.
3. President Obama is a registered Republican.
   President Obama is a registered Democrat.
4. The sofa is soft. The sofa is blue.
5. The plant is blooming. The plant is dead.
6. The dog ate my homework. The dog bites.
7. That rock is igneous. That rock is sedimentary.
8. That bird is a robin. That bird is blue.

Biconditional: compound statement of the form:

p if and only if q

written $p \leftrightarrow q$ or p `iff` q

$p \leftrightarrow q$ is equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$

or

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

Truth Table for $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

True or False?

A biconditional is `true` only when **both** statements are `true` or **both** statements are `false`.

`True` or `False`: $5 = 9 - 4$ if and only if $8 + 2 = 10$

`True` or `False`: Clinton was president IFF Carter wasn't president.

`True` or `False`: IBM sells computers IFF Pizza Hut sells Big Macs.

`True` or `False`: $8 + 7 \neq 15$ IFF $3 \times 5 \neq 9$.

Summary

$\sim p$	**negation** of p	truth value is opposite of p
$p \wedge q$	**conjunction**	true only when both p and q are true
$p \vee q$	**disjunction**	false only when both p and q are false
$p \rightarrow q$	**conditional**	false only when p is true and q is false
$p \leftrightarrow q$	**biconditional**	true only when p and q have the *same* truth value.