MAT 1160 — WEEK 12

Dr. N. Van Cleave

Spring 2010

Student Responsibilities – Week 12

- **Reading**:

  This week: Textbook, Sections 3.5, 3.6
  Next week: Fallacies, Sudoku

- Summarize Sections

- Work through Examples

- Recommended exercises:
  - Section 3.5: evens, 2 – 32
  - Section 3.6: evens, 2 – 52

3.5 Analyzing Arguments with Euler Diagrams

— Recall —

- Two types of reasoning: **inductive** and **deductive**.
- Inductive reasoning observed patterns to solve problems.
- Deductive reasoning involves drawing specific conclusions from given general premises.

Parts of an Arguments

A **logical argument** is composed of:

1. **premises** (assumptions, laws, rules, widely held ideas, or observations) and
2. **conclusion**

Valid and Invalid Arguments

- An argument is **valid** if the fact that **all the premises are true** forces the **conclusion to be true**.

- An argument that is not valid is said to be **invalid** or a **fallacy**.

- Deductive reasoning can be used to determine whether logical arguments are **valid** or **invalid**.

- **Note**: **valid** and `true` are not the same — an argument can be valid even though the conclusion is false, as we shall see later.

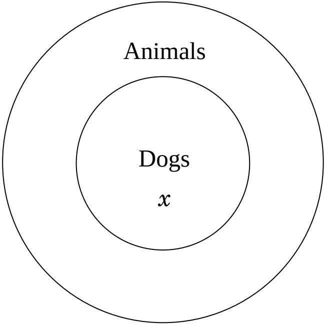
Euler diagrams

- One method for verifying the validity of an argument is the visual technique based on **Euler diagrams**
- This technique is similar to Venn diagrams, in that circles are used to denote sets, with
  - **overlap** indicating shared elements
  - **disjoint** circles indicating no shared elements
  - a circle **contained within** another circle indicating a subset
- An **x** may be used to indicate a single element
- This is like a game — if possible, we want to show the argument is **invalid** ! As long as the circles and x's do not contradict the premises, we can position them to win the game.

Example 1. Is the following argument valid?

```
All dogs are animals.
Fred is a dog.
--------------------
Fred is an animal.
```

Draw regions to represent the premise. Let x represent `Fred`.



Since:

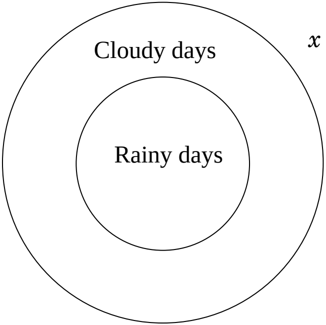
- the set of all animals contains the set of all dogs, and
- that set contains `Fred`
- `Fred` is also inside the regions for animals.

Therefore, if both premises are true, the conclusion that `Fred` is an animal must be true also.

The argument is valid as checked by the Euler diagram.

Example 2. Is the following argument valid?

```
All rainy days are cloudy.
Today is not cloudy.
----------------------
Today is not rainy.
```



Draw regions to represent the premise. Let x represent `today`.

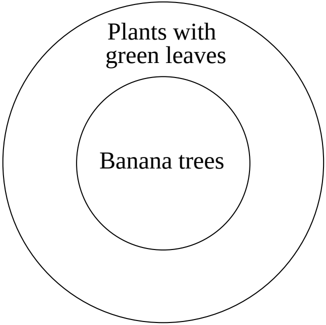
Placing the x for `today` outside the `cloudy days` region forces it to also be outside the `rainy days` region.

Thus, if both premises are true, the conclusion that `today` is not rainy is also true.

The argument is valid.

Example 3. Is the following argument valid?

```
All banana trees have green leaves
That plant has green leaves.
--------------------
That plant is a banana tree.
```



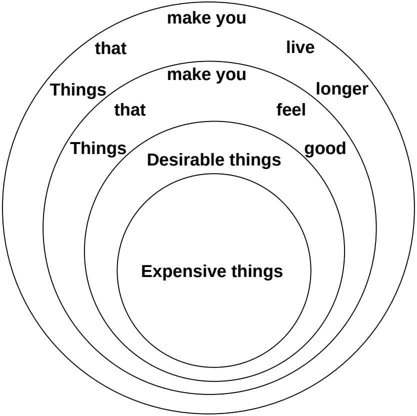
Where does the x go?

Draw regions to represent the premise. Let x represent `that plant`.

Rule: Place the x to make the argument **invalid** if possible.

Example 4. Is the following argument valid?

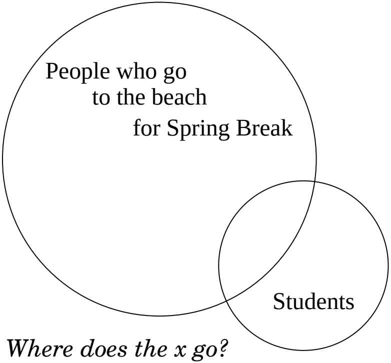
```
All expensive things are desirable.
All desirable things make you feel good.
All things that make you feel good make you live longer.
--------------------------------------------------------
All expensive things make you live longer.
```



Example of a **valid** argument which need **not** have a true conclusion.

Example 5. Is the following argument valid?

```
Some students go to the beach
               for Spring Break.
I am a student.
------------------------------
I go to the beach for
                  Spring Break.
```



Can we place the x to make the argument **invalid**?

Valid or Invalid Arguments?

1.
```
All boxers wear trunks.
Steve Tomlin is a boxer.
--------------------------
Steve Tomlin wears trunks.
```

2.
```
All residents of NYC love Coney Island hot dogs.
Ann Stypuloski loves Coney Island hot dogs.
------------------------------------------------
Ann Stypuloski is a resident of NYC.
```

3.
```
All politicians lie, cheat, and steal.
That man lies, cheats, and steals.
--------------------------------------
That man is a politician.
```

1. All contractors use cell phones.
   Laura Boyle does not use a cell phone.

   Laura Boyle is not a contractor.

2. Some trucks have sound systems.
   Some trucks have gun racks.

   Some trucks with sound systems have gun racks.

Each of these arguments has a `true` conclusion—determine if the argument is **valid** or **invalid**.

1. ```
   All cars have tires.
   All tires are rubber.
   --------------------
   All cars have rubber.
   ```

2. ```
   All chickens have beaks.
   All birds have beaks.
   -----------------------
   All chickens are birds.
   ```

3. ```
   Veracruz is south of Tampico.
   Tampico is south of Monterrey.
   ------------------------------
   Veracruz is south of Monterrey.
   ```
   ```

1. ```
   All chickens have beaks.
   All hens are chickens.
   ------------------------
   All hens have beaks.
   ```

2. ```
   No whole numbers are negative.
   -4 is negative.
   ------------------------------
   -4 is not a whole number.
   ```

Given the premises:

1. **All people who drive contribute to air pollution.**
2. **All people who contribute to air pollution make life a little worse.**
3. **Some people who live in a suburb make life a little worse.**

Which of the following conclusions are valid?

a) Some people who live in a suburb drive.

b) Some people who contribute to air pollution live in a suburb.

c) Suburban residents never drive.

d) All people who drive make life a little worse.

3.6 Analyzing Arguments with Truth Tables

Some arguments are more easily analyzed to determine if they are valid or invalid using **Truth Tables** instead of **Euler Diagrams**.

One example of such an argument is:

```
If it rains, then the squirrels hide.
It is raining.
--------------------------------------
The squirrels are hiding.
```

Notice that in this case, there are no universal quantifiers such as `all`, `some`, or `every`, which would indicate we could use Euler Diagrams.
To determine the validity of this argument, we must first identify the **component statements** found in the argument. They are:

```
p = it rains / is raining
q = the squirrels hide / are hiding
```

Rewriting the Premises and Conclusion

Premise 1: $p \rightarrow q$
Premise 2: p

Conclusion: q

Thus, the argument converts to:

$$((p \rightarrow q) \ \wedge \ p) \ \rightarrow \ q$$

With Truth Table:

p	q	$((p \rightarrow q) \ \wedge \ p) \ \rightarrow \ q$
T	T	
T	F	
F	T	
F	F	

Are the squirrels hiding?

Testing Validity with Truth Tables

1. Break the argument down into **component statements**, assigning each a letter.

2. Rewrite the premises and conclusion **symbolically**.

3. Rewrite the argument as an **implication** with the **conjunction** of all the premises as the antecedent, and the conclusion as the consequent.

4. Complete a Truth Table for the resulting conditional statement. If it is a **tautology**, then the argument is **valid**; otherwise, it's **invalid**.

Recall

Direct Statement	$p \rightarrow q$
Converse	$q \rightarrow p$
Inverse	$\sim p \rightarrow \sim q$
Contrapositive	$\sim q \rightarrow \sim p$

Which are equivalent?

```
If you come home late, then you are grounded.
You come home late.
---------------------------------------------
You are grounded.
```

```
p =

q =
```

Premise 1:

Premise 2:

Conclusion:

Associated Implication:

p	q	
T	T	
T	F	
F	T	
F	F	

Are you grounded?

Modus Ponens — The Law of Detachment

Both of the prior example problems use a pattern for argument called **modus ponens**, or **The Law of Detachment**.

$$
\begin{array}{c}
p \rightarrow q \\
p \\
\text{------} \\
q
\end{array}
$$

or

$$((p \rightarrow q) \wedge p) \rightarrow q$$

Notice that **all** such arguments lead to **tautologies**, and therefore are **valid**.

```
If a knee is skinned, then it will bleed.
The knee is skinned.
----------------------------------------
It bleeds.

p =

q =
```

Premise 1:

Premise 2:

Conclusion:

Associated Implication:

p	q	
T	T	
T	F	
F	T	
F	F	

(**Modus Ponens**) – *Did the knee bleed?*

Modus Tollens — Example

```
If Frank sells his quota, he'll get a bonus.
Frank doesn't get a bonus.
----------------------------------------
Frank didn't sell his quota.

p =

q =
```

Premise 1: $p \rightarrow q$ Premise 2: $\sim q$ Conclusion: $\sim p$

Thus, the argument converts to: $((p \rightarrow q) \wedge \sim q) \rightarrow \sim p$

p	q	$((p \rightarrow q) \wedge \sim q) \rightarrow \sim p$
T	T	
T	F	
F	T	
F	F	

Did Frank sell his quota or not?

Modus Tollens

An argument of the form:

$$p \rightarrow q$$
$$\sim q$$

$$\sim p$$

or

$$((p \rightarrow q) \wedge \sim q) \rightarrow \sim p$$

is called **Modus Tollens**, and represents a **valid** argument.

Modus Tollens — Example II

```
If the bananas are ripe, I'll make banana bread.
I don't make banana bread.
---------------------------------------
The bananas weren't ripe.

p =

q =
```

Premise 1: $p \rightarrow q$ Premise 2: $\sim q$ Conclusion: $\sim p$

Thus, the argument converts to: $((p \rightarrow q) \wedge \sim q) \rightarrow \sim p$

p	q	$((p \rightarrow q) \wedge \sim q) \rightarrow \sim p$
T	T	
T	F	
F	T	
F	F	

Were the bananas ripe or not?

Fallacy of the Inverse — Example

```
If it rains, I'll get wet.
It doesn't rain.
----------------------------------------
I don't get wet.

p =

q =
```

Premise 1: $p \rightarrow q$ Premise 2: $\sim p$ Conclusion: $\sim q$

Thus, the argument converts to: $((p \rightarrow q) \wedge \sim p) \rightarrow \sim q$

p	q	$((p \rightarrow q) \wedge \sim p) \rightarrow \sim q$
T	T	
T	F	
F	T	
F	F	

Did I get wet?

Fallacy of the Inverse

An argument of the form:

$$p \rightarrow q$$
$$\sim p$$

$$\sim q$$

or

$$((p \rightarrow q) \wedge \sim p) \rightarrow \sim q$$

is called the **Fallacy of the Inverse**, and represents an **invalid** argument.

Fallacy of the Inverse — Example II

```
If you're good, you'll be rewarded.
You aren't good.
-------------------------------------
You aren't rewarded.

p =

q =
```

Premise 1: $p \rightarrow q$ Premise 2: $\sim p$ Conclusion: $\sim q$

Thus, the argument converts to: $((p \rightarrow q) \wedge \sim p) \rightarrow \sim q$

p	q	$((p \rightarrow q) \wedge \sim p) \rightarrow \sim q$
T	T	
T	F	
F	T	
F	F	

Are you rewarded?

Another Type of (Invalid) Argument

```
If it rains, then the squirrels hide.
The squirrels are hiding.
---------------------------------------
It is raining.

p = it rains / is raining
q = the squirrels hide / are hiding
```

Premise 1: $p \rightarrow q$ Premise 2: q Conclusion: p

Thus, the argument converts to: $((p \rightarrow q) \wedge q) \rightarrow p$

p	q	$((p \rightarrow q) \wedge q) \rightarrow p$
T	T	
T	F	
F	T	
F	F	

(**Fallacy of the Converse**) — *Is it raining?*

Fallacy of the Converse

An argument of the form:

$$\begin{array}{c} p \rightarrow q \\ q \\ \hline p \end{array}$$

or

$$((p \rightarrow q) \wedge q) \rightarrow p$$

is sometimes called the **Fallacy of the Converse**, and represents an **invalid** argument.

```
If you like me, then I like you.
I like you.
---------------------------------------
You like me.

p =

q =
```

Premise 1:

Premise 2:

Conclusion:

Associated Implication:

p	*q*	
T	T	
T	F	
F	T	
F	F	

(**Fallacy of the Converse**) — *Do you like me?*

Disjunctive Syllogism — Example

```
Either you get home by midnight, or you're grounded.
You aren't grounded.
-----------------------------------------
You got home by midnight.

p =

q =
```

Premise 1: $p \vee q$ Premise 2: $\sim q$ Conclusion: p

Thus, the argument converts to: $((p \vee q) \wedge \sim q) \rightarrow p$

p	q	$((p \vee q) \wedge \sim q) \rightarrow p$
T	T	
T	F	
F	T	
F	F	

Did you get home by midnight?

Disjunctive Syllogism

An argument of the form:

$$p \vee q$$
$$\sim q$$
$$------$$
$$p$$

or

$$((p \vee q) \wedge \sim q) \rightarrow p$$

is called a **Disjunctive Syllogism**, and represents a **valid** argument.

Disjunctive Syllogism — Example II

```
Either this milk has soured, or I have the flu.
The milk has not soured.
---------------------------------------
I have the flu.

p =

q =
```

Premise 1: $p \vee q$ Premise 2: $\sim p$ Conclusion: q

Thus, the argument converts to: $((p \vee q) \wedge \sim p) \rightarrow q$

p	q	$((p \vee q) \wedge \sim p) \rightarrow q$
T	T	
T	F	
F	T	
F	F	

Do I have the flu?

Reasoning by Transitivity — Example

```
If you're kind to people, you'll be well liked.
If you're well liked, you'll get ahead in life.
---------------------------------------------------
If you're kind to people, you'll get ahead in life.
```

```
p = you're kind to people
q = you're well liked
r = you get ahead in life
```

Premise 1: $p \rightarrow q$ Premise 2: $q \rightarrow r$ Conclusion: $p \rightarrow r$

Thus, the argument converts to:
$((p \rightarrow q) \land (q \rightarrow r)) \rightarrow (p \rightarrow r)$

p	q	r	$((p \to q) \ \wedge \ (q \to r)) \ \to \ (p \to r)$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Reasoning by Transitivity

An argument of the form:

$$\begin{array}{c} p \rightarrow q \\ q \rightarrow r \\ \text{------} \\ p \rightarrow r \end{array}$$

or

$$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$$

is called **Reasoning by Transitivity**, and represents a **valid** argument.

Reasoning by Transitivity — Example

```
If it purrs, it's a cat.
If it's a cat, I'm allergic to it.
----------------------------------
If it purrs, I'm allergic to it.

p =

q =

r =
```

Valid or Invalid?

Argument Forms

VALID			
Modus Ponens	**Modus Tollens**	**Disjunctive Syllogism**	**Reasoning by Transitivity**
$p \rightarrow q$ p	$p \rightarrow q$ $\sim q$	$p \vee q$ $\sim p$	$p \rightarrow q$ $q \rightarrow r$
q	$\sim p$	q	$p \rightarrow r$

INVALID	
Fallacy of the Converse	**Fallacy of the Inverse**
$p \rightarrow q$ q	$p \rightarrow q$ $\sim p$
p	$\sim q$

Valid or Invalid?

```
If you stay in, your roommate goes out.
If your roommate doesn't go out, s/he will finish
         their math homework.
Your roommate doesn't finish their math homework.
Therefore, you do not stay in.
```

Determine a Valid Conclusion

`It is either day or night.`
`If it is daytime, then the squirrels are scurrying.`
`It is not nighttime.`

Determine a Valid Conclusion

```
If it is cold, you wear a coat.
If you don’t wear a coat, you are dashing.
You aren’t dashing.
```

Valid or Fallacy? Which Form?

If you use binoculars, then you get a glimpse of the comet.
If you get a glimpse of the comet, then you'll be amazed.

If you use binoculars, then you'll be amazed.

```
If he buys another toy, his toy chest will overflow.
His toy chest overflows.
____________________________________________________
He bought another toy.
```

If Ursula plays, the opponent gets shut out.
The Opponent does not get shut out.

Ursula does not play.

```
If we evolved a race of Isaac Newtons, that
          would be progress.  (A. Huxley)
We have not evolved a race of Isaac Newtons.
____________________________________________
That is progress.
```

Alison pumps iron or Tom jogs.
Tom doesn't jog.

Alison pumps iron.

Jeff loves to play golf. If Joan likes to sew, then Jeff does not love to play golf. If Joan does not like to sew, then Brad sings in the choir. Therefore, Brad sings in the choir.

If the Bobble head doll craze continues, then Beanie Babies will remain popular. Barbie dolls continue to be favorites or Beanie Babies will remain popular. Barbie dolls do not continue to be favorites. Therefore, the Bobble head doll craze does not continue.

If Jerry is a DJ, then he lives in Lexington. He lives in Lexington and is a history buff. Therefore, if Jerry is not a history buff, then he is not a DJ.

If I've got you under my skin, then you are deep in the heart of me. If you are deep in the heart of me, then you are not really a part of me. You are deep in the heart of me, or you are really a part of me. Therefore, if I've got you under my skin, then you are really a part of me.