MAT 1160 – Chapter 1
1.1 Solving Problems by Inductive Reasoning

Summary

This section introduces solving problems by various types of reasoning: inductive and deductive.

Definitions

- Conjecture: an educated guess based upon repeated observations of a particular process or pattern.
- Inductive Reasoning: characterized by drawing a general conclusion (make a conjecture) from repeated observations of specific examples. The conjecture may or may not be true.
- Counterexample: an example that does not work, used in order to prove a conjecture incorrect.
- Deductive Reasoning: characterized by applying general principles to specific examples.
- Natural or Counting Numbers: $\{1, 2, 3, \ldots\}$
- Premise: an assumption, law, rule, widely held idea, or observation (upon which an argument is based)
- Conclusion: the result of apply reasoning to a premise.
- Logical Argument: the premises and conclusion drawn from them by applying reasoning to the premises.

Examples

Ex 1 (a) Our house is made of redwood. Both of my next–door neighbors have redwood houses. Therefore, all houses in our neighborhood are made of redwood.

premise: my house and next–door neighbors' houses are redwood.
conclusion: all neighborhood houses are redwood
inductive reasoning

(b) All word processors will type the `@`. I have a word processor. I can type the symbol `@`.

premise: All word processors have the `@` symbol and I have a word processor.
conclusion: I can type the symbol `@`.
deductive reasoning

(c) Today is Friday. Tomorrow will be Saturday.

premise: Today is Friday.
conclusion: Tomorrow will be Saturday
deductive reasoning

Ex 2 Use inductive reasoning to determine the *probable* next number in each list:

(a) `3, 7, 11, 15, 19, 23`: next is 27 since adding 4 to previous term to get next
(b) `1, 1, 2, 3, 5, 8, 13, 21`: next is 34 since adding two previous terms
(c) `1, 2, 4, 8, 16`: next is 32 since doubling previous term

Ex 3 Consider the list of equations; predict the next one:

```
37 x  3 = 111
37 x  6 = 222
37 x  9 = 333
37 x 12 = 444
=============
37 x 15 = 555
```

Max of two more problems from this section

1.2 An Application of Inductive Reasoning: Number Patterns

Summary

This section looks at sequences of numbers and how to use patterns to predict the next term. It includes several formulas for summing special sequences and figurate numbers.

Definitions & Formulas

- Sequence: an ordered list of numbers, such as: 3, 9, 15, 21, 27, ...
- Number Sequence: a list of numbers having a first number, a second number, a third number, and so on.
- Term: a number in a sequence
- Arithmetic Sequence: a number sequence with a common difference between terms
- Geometric Sequence: a number sequence with a common ratio between terms
- Method of Successive Differences: find the differences between terms, until all are the same, then add along the last diagonal to find the next term in the sequence.
- Sum of the First n Odd Counting Numbers: $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.
- Special Sum Formula I, sum of first n integers, squared: $(1+2+3+\ldots+n)^2 = 1^3+2^3+\ldots+n^3$
- Special Sum Formula II, sum of first n integers: $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

Triangular number: $T_n = \frac{n(n+1)}{2}$	Square number: $S_n = n^2$
Pentagonal number: $P_n = \frac{n(3n-1)}{2}$	Hexagonal number: $H_n = \frac{n(4n-2)}{2}$
Heptagonal number: $Hp_n = \frac{n(5n-3)}{2}$	Octagonal number: $O_n = \frac{n(6n-4)}{2}$

Examples

Ex 1 (a)

```
14        22        32        44      58
   \    /    \    /    \    /    \  /
     8        10        12        14
       \    /    \    /    \    /
         2         2         2
```

(b)

```
5        15        37        77        141     235
   \    /    \    /    \    /    \    /  \  /
     10        22        40        64     94
        \    /    \    /    \    /    \  /
          12        18        24        30
             \    /    \    /    \    /
                6         6         6
```

Ex 2 (a) $(1 + 2 + 3 + 4 + 5)^2 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3$

(b) $21 + 23 + 25 + 27 + 29 = 5^3$

(c) $1 + 2 + 3 + 4 + 5 = \frac{5 \times 6}{2}$

Ex 3 (a) $T_7 = \frac{7(7+1)}{2} = \frac{56}{2} = 28$

(b) $S_{12} = 12^2 = 144$

(c) $P_6 = \frac{6[3(6)-1]}{2} = \frac{6\times17}{2} = 51$

Ex 4 $P_6 = 51$, $T_5 = 15$, and $51 = (3 \times 15) + 6 = 45 + 6 = 51$

Ex 5

```
1    5    12     22    35    51
 \ / \ /   \   /   \ /   \ /
  8   7     10     13    16
   \ / \   /   \   / \   /
    3    3       3     3
```

Max of two more problems from this section